INVESTIGATION OF THE OUTPUT RESPONSE OF A POSITIONAL SERVO SYSTEM

INTRODUCTION

These notes describe the simulation, with a desk-top-size PACE® TR-20 general purpose analog computer, of the output response of a positional servo system. It is the purpose of the simulation to investigate the behavior of this output response occurring as a result of changes in system parameters.

The physical system under consideration is best understood by reference to the simplified block diagram shown in Figure 1.

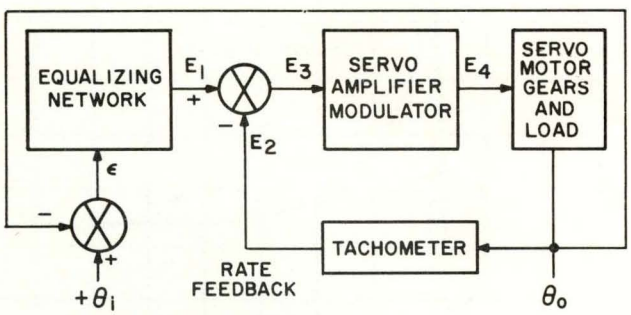

Figure 1. Functional Block Diagram of a Servo Mechanism

MATHEMATICAL MODEL

The equations which make up the mathematical model of the various devices used in the physical system include:

Physical Device	Transfer Functions	
Error Generator	$\epsilon = \theta_i - \theta_o$	(1)
Equalizing Network*	$E_1 = K_1 \left[\frac{1 + ST}{1 + S\alpha T}\right]$	(2)
Servo Amplifier	$E_4 = K_2 E_3 = K_2 \left[E_1 - E_3\right]$	(3)

*Note: *Equalizing Network is a synchro of gain K, in tandem with a phase lag network.*

Physical Device	Transfer Functions	
Servo Motor, Gears, and Load	$\theta_o = \left[\frac{K_3}{JS^2 + FS + K_4}\right] E_4$	(4)
Tachometer	$E_2 = \left[K_T S\right] \theta_o$	(5)

where

- F = Equivalent viscous friction referred to load shaft, ft-lb/rad/sec
- J = Equivalent inertia of motor and load referred to load shaft, slug/ft/ft
- K_1 = Gain of synchro, volts/rad
- K_2 = Gain of servo amplifier, volts/volt
- K_3 = Motor-developed torque constant referred to load shaft, ft-lb/volt
- $K_4 = K_1 \cdot K_2 \cdot K_3$
- K_T = Tachometer voltage constant, volts/rad/sec
- S = LaPlace transform variable
- T = Time constant of lag network, sec
- α = Gain constant of lag network
- θ_i = Input disturbance, radians
- θ_o = Output, radians

BENEFITS

A number of important benefits are provided through this low cost simulation. In particular, the computer solution makes possible a more complete understanding of the fundamental nature of the step response of the system. The immediate availability of a comprehensive picture of actual output response--displayed on an oscilloscope or plotted graphically--gives the engineer an excellent opportunity to get the 'feel' of system dynamic performance.

In terms of system design, the simulation provides the additional advantage of permitting the convenient preparation of parametric nomographs necessary in judging particular configurations. This is accomplished through the ease with which parameters can be changed: it is necessary only to manipulate hand-set potentiometers to change parametric values.

Printed in U.S.A. 364

Bulletin No. ALAC 64060

In this regard, the simulation technique used makes it especially easy to investigate the following conditions:

- change in output response as a result of varying the gain and time constant of the equalizing network
- change in output response as a result of introducing or removing rate feedback
- change in output response as a result of varying the physical parameters--inertia, friction, etc.--of the system

General benefits obtained through the application of analog simulation techniques to this problem include:

- high speed solutions
- dynamic, rather than purely static, analyses
- optimum resolution of results
- computational versatility

RESULTS

The two sets of curves shown in Figures 2 and 3 illustrate the graphic display of results obtained with this simulation program. Note the clearly defined variation in curve shape as a result of varying just one parameter, T. These plots show the effect of a step change in input voltage on error generator and servo motor outputs.

For complete details on this simulation, please write for Application Study: 8.4.6a, Bulletin No. ALAC 64059.

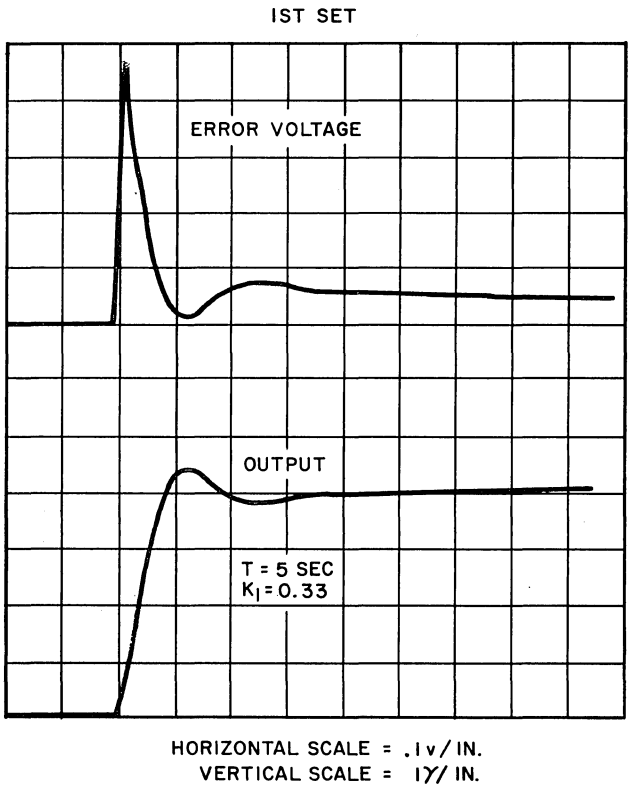

Figure 2. Plots Showing Relationship Between Error Voltage and Output, T = 5 Seconds

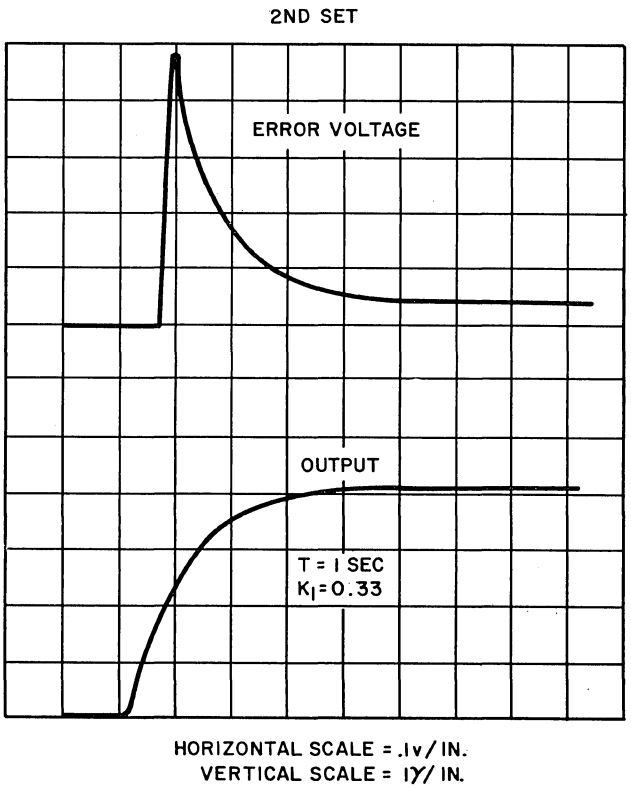

Figure 3. Plots Showing Relationship Between Error Voltage and Output, T = 1 Second