

## **Basic Vocational Knowledge – Introduction to Electrical Engineering**



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# Basic Vocational Knowledge – Introduction to Electrical Engineering

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## Textbook for Vocational Training

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## Preface

The present textbook is intended for trainees in the field of electrical engineering. It is also suitable for electricians who want to make themselves again acquainted with the theoretical fundamentals of electrical engineering by private study and qualify for higher positions.

The authors have spared no effort to represent the subject-matter scientifically exact and at the same time so that it is easily understood. For the derivation of formulas only fundamental knowledge of mathematics is required on the part of the reader. The grasping of the subject-matter is facilitated by the inclusion in this book of numerous sketches and illustrations. The practical use of the formulas is shown by demonstrations of examples of solutions and the trainee will thus be enabled to solve the problems presented without assistance. Each Section is followed by a summary of the subjects and problems for exercise and recapitulation.

We are indebted to the experts for their valuable suggestions. We wish all readers a successful completion of their vocational training and hope this textbook will be instrumental to this end; and we ask them to send suggestions for improving the textbook to the Institute for Vocational Development.

Institut für berufliche Entwicklung

## 1. Importance of Electrical Engineering

Our life would be unthinkable without the use of electrical energy. The growing utilisation of the latter is a decisive prerequisite for a rapid development of industry and agriculture.

A few examples will show the importance of electrical energy. Thus, electrical lighting is indispensable for working during the dark hours of the day. With increasing industrialisation, a growing proportion of electrical energy is used for the lighting of shops, offices, dwellings and for outdoor lighting. Man is relieved from heavy physical labour by the use of electrical devices. The drive of machines, hoisting gear and lifts is enabled in a simple form by the electromotor which in railway transport also has the advantage over internal combustion engines. There are many buildings where an air-conditioning system including heating, cooling and ventilation is installed for the operation of which electrical energy is required. At higher ambient temperatures, foodstuffs can only be kept for a prolonged period of time in refrigerators or cold-storage rooms which usually are also operated with electrical energy.

Without electrical energy, there would be no broadcasting and television systems, no telephone communication or telegraphy. In order to arrange automatic sequences of operation in production, devices of control and regulation engineering are required which, today, are driven almost exclusively by electrical energy. Table 1.1. shows a survey of the two large fields of electrical engineering, power electrical engineering and information electrical engineering.

The consumption of electrical energy by the various branches of economy is quite different. The chemical industry and metallurgy have a particularly high consumption. For the production of the electrical conductor materials electrolytic copper and aluminium, very large amounts of electrical energy are required.

Table 1.1. Sections of Electrical Engineering

Power Electrical Engineering		Information Electrical Engineering	
Section	Examples	Section	Examples
Generation of electrical energy	Power stations	Communication-engineering	Broadcasting, television, other telecommunication,
Transmission of electrical energy	Overhead lines, cables		telephone, telegraph, telewriter
Conversion of electrical energy	Motors, light sources, thermal devices, refrigerators, galvanic stations	Control and regulation engineering	Control of air-conditioning plants, control and regulation of production processes
Storage of electrical energy	accumulators	Electronic computer engineering	Pocket computers, data processing

In industrial countries, the consumption of electrical energy increases by 4 % to 7 % per year. For this purpose, considerable amounts of primary energy carriers such as coal, petroleum or natural gas must be provided. All over the world, a reduction of the resources of primary energy carriers takes place. At the same time, their prices have been increased continuously. Water power is not in all countries available to a sufficient degree.

The initial cost for the construction of a hydroelectric power station by far exceed the initial cost of a thermal power station.

According to the present developmental stage of engineering, nuclear power stations represent not yet a final solution of the problem. Therefore, it is absolutely necessary to use electrical energy sparingly. This also means that such devices and installations have to be developed and used which ensure a high net efficiency with as small a consumption of electrical energy as possible.

Finally, mention should be made of the fact that electrical energy can be transported conveniently through large distances at low losses. On the other hand, there is the disadvantage that electrical energy can be stored only in small amounts at high cost. Production and consumption must take place largely at the same time.

## 2. Fundamental Quantities of Electrical Engineering

### 2.1. Current

Flowing quantities of electricity cause effects which are utilised in practice, i.e. in electrical engineering. Since flowing quantities of air are called an air current, flowing quantities of water a water current, the phenomenon of flowing quantities of electricity is called electrical current.

The carriers of the quantities of electricity are called charge carriers. Mostly the latter are electrons, in rare cases ions. An electron has the smallest imaginable charge which, therefore, is called elementary charge. In electro-technology, electrons are considered as practically massless charge carriers because of their small volume and extremely small mass.

Electrons are constituents of atoms, the basic units of which the material world is constructed. An atom consists of a nucleus and the electrons surrounding it. Atoms or groups of atoms which have lost or gained one or more electrons are called ions. Ions are charge carriers having mass. When an ion – as compared with the chargeless neutral atom – has more electrons, then it is called negatively charged, when it has less electrons, it is called positively charged. The electron itself has a negative charge.

As a current consists of flowing quantities of electricity (charge quantities), it can only flow in such substances which possess freely mobile, non-stationary charge carriers. Substances with many mobile charge carriers are called conductors. They include all metals (especially silver, copper, aluminium and iron) and electrolytes (salt solutions). As the current in metals is carried by electrons, it is called electron current, whereas the current flowing through electrolytes is called ion current because the flowing charge carriers are ions.

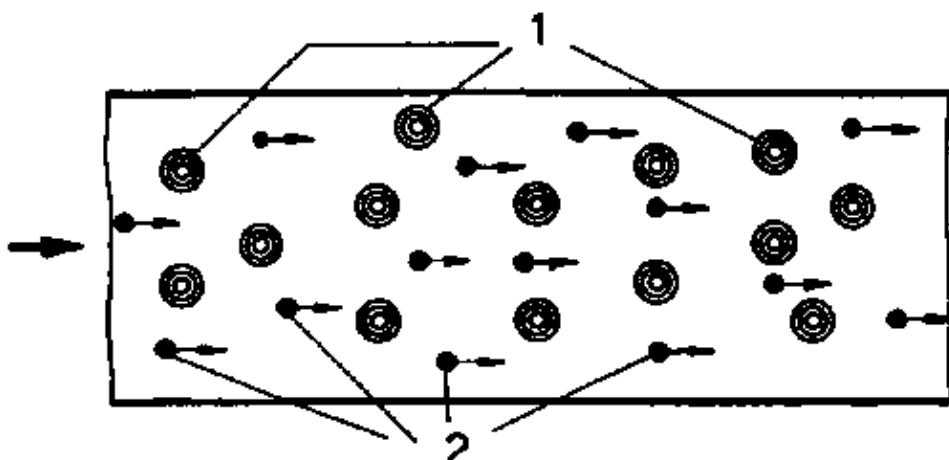


Fig. 2.1. Electron current; the free electrons move through the atomic lattice of the conductor

1 Atomic union; 2 Conductor electrons

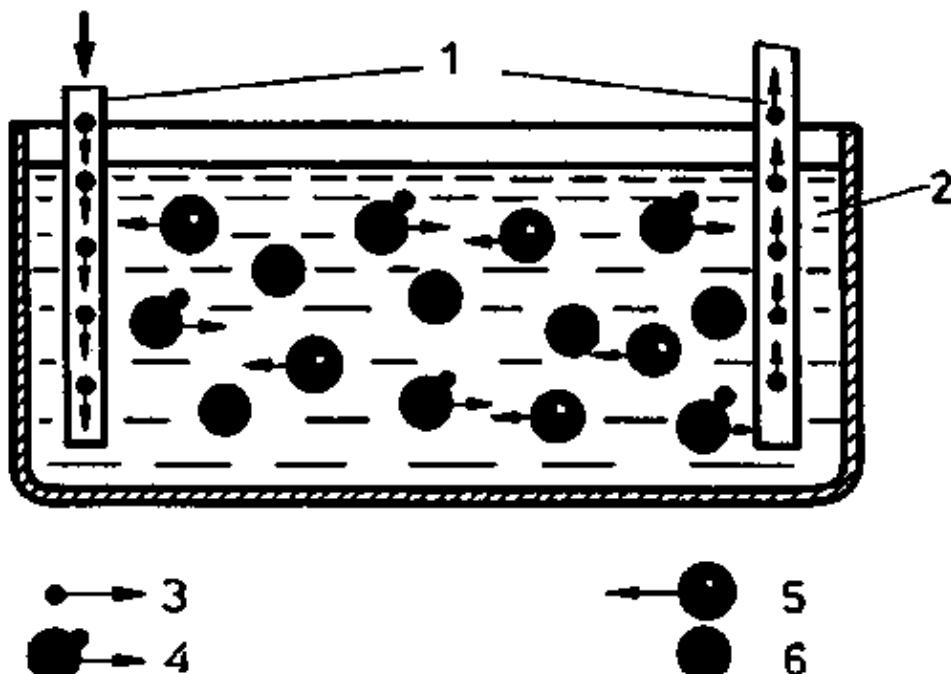


Fig. 2.2. Ion current

1 Metallic feed lines; 2 Electrolyte; 3 Electrons; 4 Negative ions; 5 Positive ions; 6 Neutral molecules

Substances in which the charge carriers are fixed or stationary, that is to say, they are not freely mobile, are called non-conductors or insulators. Current cannot flow through them. The most important non-conductors are porcelain, glass, plastics.

There are substances whose electrical conductivity is such that they are between conductor and non-conductor. They conduct current so badly that they cannot be termed as conductor but they allow a small current to flow so that they cannot be used as a non-conductor. These substances are called semiconductor. The most important semiconductors are silicon, germanium and selenium. Semiconductors are of particular practical importance to electrical engineering.

We cannot perceive electrical currents directly but only indirectly we become aware of three characteristic effects of current.

These are

1. the generation of heat in conductors through which current flows
2. the magnetic field associated with the current
3. transport of substance by ion currents

Re 1. – Every electric current generates electric heat in conductors. It is utilised in electric heating engineering, for example, cooking plate, flat iron. The generation of electric heat can be imagined in such a way that the flowing charge carriers collide with the stationary particles forming the skeleton of the material or substance. As a consequence, the energy of the braked charge carriers is converted into irregular oscillatory energy, namely thermal energy, of the stationary particles.

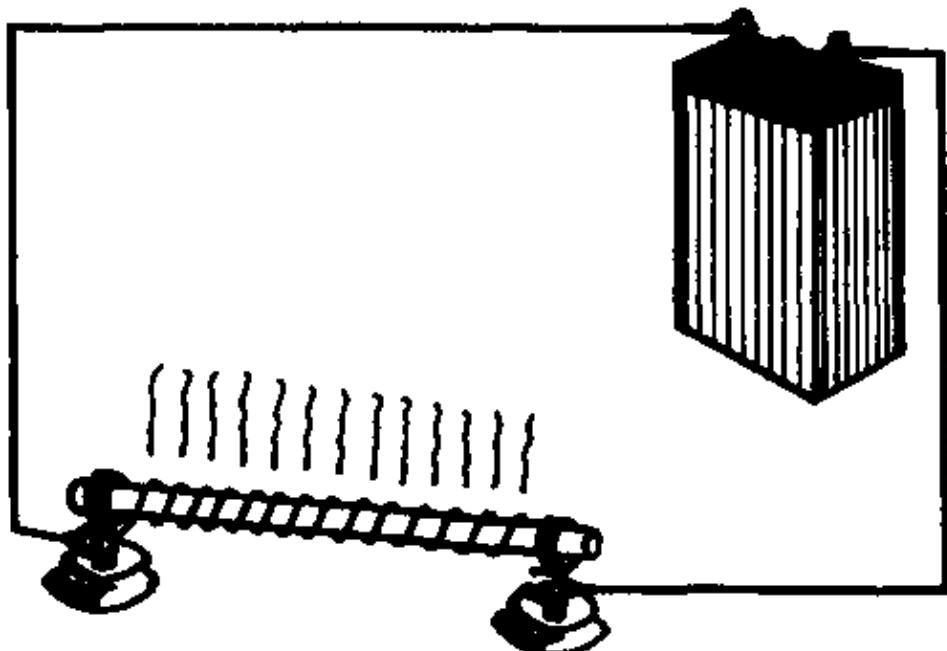


Fig. 2.3. Development of heat in the current carrying conductor

Re 2. – Every electric current is accompanied by a magnetic field. It surrounds the current spatially like an eddying fluid its axis of eddy. There is no current without a magnetic eddy and no magnetic field eddy without current. Proof of this can easily be given by means of a magnetic needle which with initial direction parallel to the current will be turned so that it is across to the current. The mutual coupling of current and magnetic field is of eminent practical importance, for example, for an economic production of electrical energy (see Section 5).

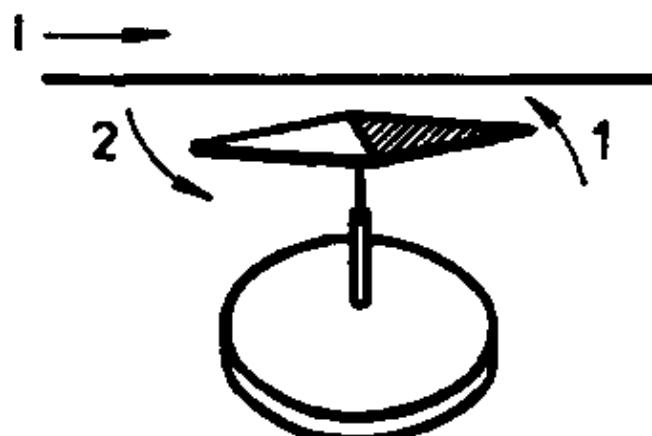


Fig. 2.4. Magnetic field associated with the current

1 North pole; south pole

Re 3. – When a fluid conductor with ions is interposed in a metallic current path, material changes will take place at the two feed wires when current flows. These material changes are the result of the material particles flowing with the ions, in other words, a consequence of the transport of substance associated with the current. From the ions, the electrons can migrate into the current supply leads or out of them; this cannot be effected by the material particles which, consequently, are deposited at these leads. If, for example, the fluid conductor is a copper sulphate solution, the copper particles are separated at one electrode in the form of a metallic coat. This process is called electrolysis. It is used for the winning of metals, especially metals in a pure state, for the deposition of metallic coats and protective coverings (galvanisation).

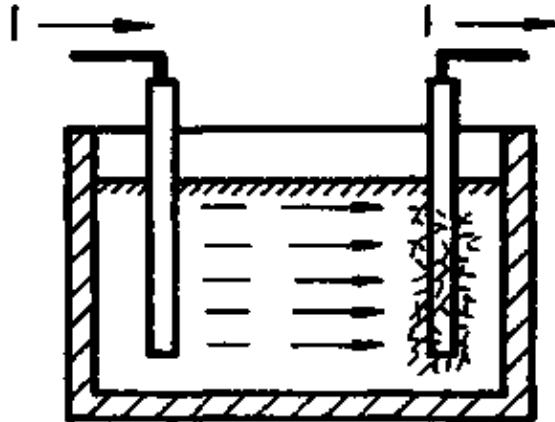


Fig. 2.5. Transport of matter in case of conduction by ions

In order to define the intensity of a current, the term current intensity (formula sign I) has been introduced.

Obviously, it is independent of the place of the line, the line material and the line cross-sectional area but it is only determined by the number of charge carriers (quantity of charge Q) flowing through the line in a certain time t. When, in a certain time, many charge carriers flow through the conductor, then the current intensity is high, vice versa it is low.

The following holds:

$$I = \frac{Q}{t} \quad (2.1.)$$

where

I current intensity

Q charge quantity

t time

The sign of the current intensity indicates the current direction. It is an arbitrarily established mathematical direction of counting and should not be confused with the actual flowing direction of the moved charge carriers.

One has defined:

The current intensity is positive when the current direction is equal to the direction of flow of the positive charge carriers or when it is opposite to the direction of flow of negative carriers (e.g. electrons).

The unit of current intensity is called ampere = A in honour of the French physicist Marie André Ampère (1775 – 1836).

$$[I] = A$$

Other usually used units of the ampere are

$$1 \text{ kA} = 1 \text{ kiloampere} = 10^3 \text{ A} = 1.000 \text{ A}$$

$$1 \text{ mA} = 1 \text{ milliampere} = 10^{-3} \text{ A} = 0.001 \text{ A}$$

$$1 \text{ ?A} = 1 \text{ microampere} = 10^{-6} \text{ A} = 0.000001 \text{ A}$$

In electrical engineering, current intensities may occur in largely different magnitudes. Table 2.1. shows a few values.

Table 2.1. Current Intensities for a Few Applications

Melting furnace	100,000 A	=	100 kA
Aluminium production	10,000 A	=	10 kA
Welding	1,000 A	=	1 kA
Starter for motor–car	100 A	=	
Household appliances up to	6 A	=	
Refrigerator	0,5 A	=	500 mA
Torch lamp	0,2 A	=	200 mA

After the establishment of the basic unit for the current intensity, units for the quantity of electricity can be derived from equation (2.1.), namely

$$Q = I \cdot t$$

$$[Q] = [I] \cdot [t]$$

$[Q] = A \cdot s$  and from  $1 A \cdot 1 s = 1 C = 1$  coulomb follows

$$[Q] = C$$

The product of  $A \cdot s$  is called coulomb in honour of the French physicist Charles Auguste de Coulomb (1736 – 1806).

A larger unit of the quantity of electricity is the ampere–hour ( $A \cdot h$ ). As 1 hour has 3,600 seconds, the following relation holds for the conversion of  $A \cdot s$  into  $A \cdot h$ :

$$1 A \cdot h = 1 A \cdot 3600 s = 3600 A \cdot s = 3600 C$$

The electrical current is the phenomenon of flowing quantities of electricity. The carriers of the quantities of electricity are called charge carriers; these are electrons and ions. As to their conductivity for electrical current, the various substances are divided into conductors, non–conductors (insulators) and semiconductors.

The three characteristic effects of current are

- generation of heat in conductors through which current passes
- the magnetic field associated with the current
- transport of substance by ion currents

The current intensity is determined by the quantity of charge flowing through the conductor during a certain

time. It results from the relation  $I = \frac{Q}{t}$ . The unit of current intensity is the ampere = A; the most frequently used sub–units are kA, mA and ?A.

From the definition equation of the current intensity, the basic unit for the quantity of electricity is derived; it is the ampere–second ( $A \cdot s$ ) = coulomb (C). A frequently used sub–unit is the ampere–hour ( $A \cdot h$ ).

Questions and problems:

1. How many A are

27 mA; 5,1 kA; 80 ?A; 1,000 mA; 6,300 ?A; 0,04 kA

2. How many C are 0,5 A·h; 84 A·h; 0,000278 A·h?

3. A quantity of electricity of 108,000 C is flowing through a line within 5 hours. Find the current intensity.

4. An electric current having the intensity of 2 A flows through a line for a period of 2 hours. Calculate the transported quantity of electricity in the units C and A·h.

## 2.2. Voltage

In order that a current flows through a conductor, an electrical “pressure” must be exerted on the freely mobile charge carriers. This “pressure” is the electrical drive phenomenon on the charge carriers which is called voltage. There is no current without an electrical voltage.

The original drive phenomenon for current is called primary electromotive force. It is generated in a voltage source. It imparts energy to the charge carriers which thus are driven through the conductor.

Since every conductor offers resistance more or less to the passage of current, the charge carriers lose energy when passing through. This loss can be characterised as voltage drop.

A current can only flow through a conductor; therefore, the current path formed by the conductor must be closed.

When a charge carrier has received drive energy from a voltage source, it passes through the conductor, completely transferring the energy taken up to this conductor. After exactly one circulation, the charge carrier differs by nothing from its state before it started the circulation, that is to say, it cannot have stored energy.

The primary electromotive force is designated by the formula sign E, the voltage drop by U. In practice, no difference is made between these two terms and they are called voltage in short. Primary electromotive force and voltage drop have the same unit which is called volt = V in honour of the Italian physicist Alessandro Volta (1745 – 1827).

$$[E] = V$$

$$[U] = V$$

Frequently used sub-units of volt are

1 MV = 1 megavolt	=	$10^6$ V	=	1,000,000 V
1 kV = 1 kilovolt	=	$10^3$ V	=	1,000 V
1 mV = 1 millivolt	=	$10^{-3}$ V	=	0.001 V
1 ?V = 1 microvolt	=	$10^{-6}$ V	=	0.000001 V

In electrical engineering, voltages may occur in quite different magnitudes. Table 2.2. shows some values.

Table 2.2. Voltage Values for a Few Applications

Lightning up to	10.000,000 V	=	10 MV
Extra-high voltage lines	600,000 V	=	600 kV
High-voltage lines	60,000 V	=	60 kV
Spark–plug in an internal combustion engine	15,000 v	=	15 kV
Lighting network	220 V		
Motor – car battery	12 V		

Receiving voltage of a wireless set

$$0.000,01 \text{ V} = 10 \text{ ?V}$$

The primary electromotive force is a prerequisite for an electrical current. Table 2.3. shows the various possibilities of producing a primary electromotive force, the designations of the respective voltage sources and their main applications.

For the winning of electrical energy, the generation of the primary electromotive force by chemical and magnetic-field actions is of particular importance. On principle, these voltage sources operate as follows

- Primary electromotive force by chemical action

When immersing two conductors of different kinds into an electrolyte, then one will find an excess of electrons at one conductor (negative pole) and an electron deficit at the other conductor (positive pole). This charge carrier difference externally acts as electrical primary electromotive force. Diluted sulphuric acid  $\text{H}_2\text{SO}_4$  is suitable as electrolyte; as conductor rods (electrodes), copper Cu and zinc Zn are particularly suitable (Fig. 2.6.).

Table 2.3. Ways of Producing Primary Electromotive Forces

<b>Causes of the production of the electromotive force</b>	<b>Designation of the voltage source</b>	<b>Examples of use</b>
chemical action	galvanic cell; battery; accumulator	voltage supply to portable devices; starting battery in motor-cars
thermal action	thermoelectric element (thermocouple)	measuring the temperature at points which are not readily accessible; remote temperature measurement
action of magnetic field (induction)	generator	economical generation of electrical energy in power stations
action of light	photovoltaic cell; solar cell	measuring the intensity of illumination
charge separation by		
– influence	influence machine	generation of high and extra-high voltages by means of which, for example, the properties of insulating materials are tested
– mechanical charge movement	belt-type generator	
displacement of charge (polarisation) on a non-conductor by means of pressure	piezoelectric element	measurement of pressure; sound pick-up for records; microphone

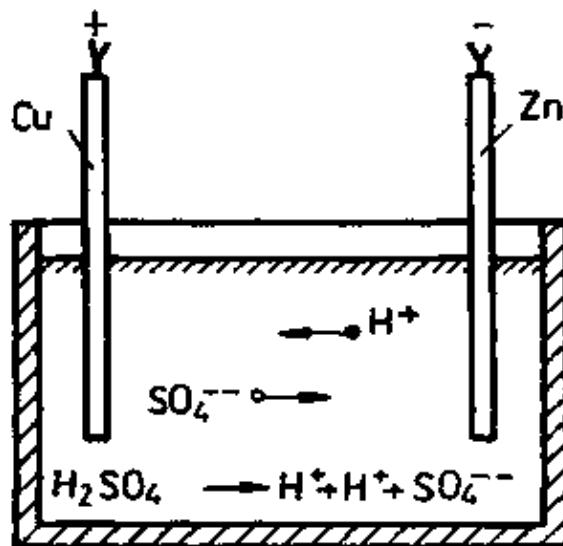


Fig. 2.6. Galvanic element also known as galvanic cell

Other substances are also suitable (especially coal and zinc in a thickened ammonium chloride solution).

In accordance with the general tendency to balance differences in concentration, the basic units of construction of the solid conductors are eager to migrate as ions in the electrolyte. On the other hand, the electrolyte tries to press its ions into the solid conductor. This impetus of motion is different in the different conductor materials so that, as a result, a primary electromotive force acts externally.

When current flows, these voltage sources disintegrate due to the transport of substance and become useless; this is also occurring when stored too long. Rechargeable voltages sources do not show this disadvantage; therefore, they are called accumulators (storage batteries). Lead accumulators and nickel–iron or nickel–cadmium accumulators are of particular importance.

- Primary electromotive force by magnetic–field action (induction)

This production of voltage is of greatest technical importance and it is used in all cases when primary electromotive force is to be generated by mechanical motion. According to a law of nature (law of induction) the following happens:

When the magnetic flux enclosed by a conductor loop is changed, the charge carriers in the conductor are subjected to an impetus to move. Then, the entire conductor loop is a primary electromotive force source.

The change of the magnetic flux may, for example, be due to the fact that the conductor loop is turned inside the magnetic field or the magnet is approached to or moved away from this loop.

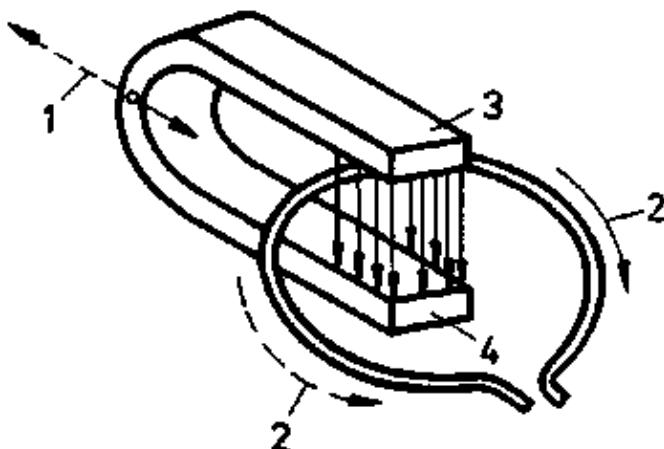


Fig. 2.7. Primary electromotive force generated by induction

1 Direction of motion; 2 Direction of the primary electromotive force; 3 North pole; 4 South pole

As symbol of a voltage source, the graphical symbol shown in Fig. 2.8. is used. The electrode with an excess of electrons is called negative pole (-); the electrode with an electron deficit is called positive pole (+). The direction of voltage corresponds to the direction of current defined in Section 2.1.; thus, the primary electromotive force E is directed from - to + whereas the voltage drop U runs from + to -.

The voltage direction is indicated by an arrow.

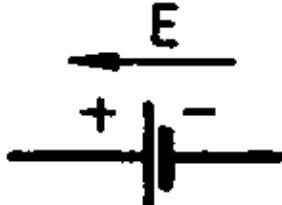


Fig. 2.8. Graphical symbol of a (direct) voltage source; the arrow indicating the direction may be omitted

The electrical drive exerted on the charge carriers is called voltage. The drive phenomenon originally generated in a voltage source is called primary electromotive force E; the loss in voltage caused when current flows through a conductor is called voltage drop U. As unit of the voltage, the volt – V – has been laid down; the most frequently used sub-units are MV, kV and ?V. For the winning of electrical energy, the generation of the primary electromotive force by chemical action and by the action of the magnetic field is of particular importance.

Questions and problems:

1. How many V are

500 mV; 2,5 kV; 350 ?V; 0,6 MV?

2. Give reasons for the fact why in a current passage the sum of all voltage drops must be equal to the entire primary electromotive force!

### 2.3. Resistance and Conductance

Every conductor and every electrical device (electric bulb, heater, electromotor, wireless receiver, etc.) has the property of resisting any current passage. This property is called electrical resistance (formula sign R). Depending on the material used and the design of the conductor or the device, it has a different magnitude.

For a conductor, the geometrical dimensions and the conductor material are decisive for the value of the resistance. The formula for calculating the resistance is called resistance rating formula. It is easily understood and can be checked by experiment that a long thin wire will offer a higher resistance to the current passage than a short thick one. When designating the line length by l and the line cross-sectional area by A, then the resistance R is proportional to l/A, hence,

$$R \sim \frac{l}{A}$$

Finally, the resistance is dependent on the conductor material; for example, iron as a conductor is inferior to copper (iron has a higher resistance). This dependence on material is covered by a material constant which is termed as specific resistance or resistivity (formula sign ?<sup>1</sup>). Hence,

<sup>1)</sup> ? Greek letter rho

$$R = \rho \frac{l}{A} \quad (2.2)$$

where

R resistance

? specific resistance

l length of the conductor

A cross-sectional area of the conductor

The higher the resistance, the poorer the conduction of the current. The permeability to current of a conductor is called conductance (formula sign G) and, hence, is inversely proportional to the resistance.

$$G = \frac{1}{R} \quad (2.3.)$$

where

G conductance

R resistance

Similar relations apply to the material constant. In the place of the specific resistance, the specific

<sup>1)</sup>

$$\kappa = \frac{1}{\rho}$$

conductance (formula sign ? ) can be stated as reciprocal value;  $\kappa = \frac{1}{\rho}$ . From the equations (2.2.) and (2.3.), the rating equation for the electrical conductance is obtained as follows

<sup>1)</sup> ? Greek letter kappa

$$G = \kappa \frac{A}{l}$$

where

G conductance

?  $\kappa = \frac{1}{\rho}$   
specific conductance;

A cross-sectional area of the conductor

l length of the conductor

The unit of the resistance is called ohm in honour of the German physicist Georg Simon Ohm (1789 – 1854) and abbreviated by the Greek letter ?<sup>2)</sup>.

<sup>2)</sup> ? Greek letter omega

$$[R] = ?$$

A conductor has a resistance of 1 ? if a voltage of 1 V drops when a current of 1 A passes this conductor.

The unit of the conductance is called Siemens = S in honour of the German physicist Werner von Siemens (1816 – 1892). (In English-speaking countries, the unit Siemens has not been generally adopted.) The correlation between the units Siemens and ohm is given by equation (2.3).

$$[G] = S = \frac{1}{\Omega}$$

Frequently used sub-units of ohm (?) and Siemens (S) are

$$1 \text{ M?} = 1 \text{ megaohm} = 10^6 ? = 1,000,000 ?$$

$$1 \text{ k?} = 1 \text{ kiloohm} = 10^3 ? = 1,000 ?$$

$$1 \text{ m?} = 1 \text{ milliohm} = 10^{-3} ? = 0.001 ?$$

$$1 \text{ kS} = 1 \text{ kilosiemens} = 10^3 \text{ S} = 1,000 \text{ S}$$

$$1 \text{ mS} = 1 \text{ millisiemens} = 10^{-3} \text{ S} = 0.001 \text{ S}$$

$$1 \text{ ?S} = 1 \text{ microsiemens} = 10^{-6} \text{ S} = 0.000001 \text{ S}$$

Now, units can be given also for the specific resistance and the specific conductance by rearranging the equations (2.2) and (2.4).

For ?, from equation (2.2.) we have

$$\rho = R \frac{A}{l}$$

$$[\rho] = \Omega \cdot \frac{m^2}{m} = \Omega \cdot m$$

$$\Omega \cdot \frac{m^2}{m} = 10^{-6} \Omega \cdot m$$

A frequently used sub-unit is

From equation (2.4), for ? we have

$$\kappa = G \frac{1}{A} \quad [\kappa] = S \frac{m}{m^2} = \frac{S}{m} = \frac{1}{\Omega \cdot m}$$

Table 2.4. shows for a few substances the values of ? and ?.

Example 2.1.

Calculate the resistance and conductance of a copper wire having a length of 175 m and a cross-sectional area of 2,5 mm<sup>2</sup>.

Given: l = 175 mm

To be found: R

$$A = 2,5 \text{ mm}^2 \qquad \qquad \qquad G$$

$$\rho_{Cu} = 0.0178 \frac{\Omega \cdot mm^2}{m}$$

$$((\kappa_{Cu} = \frac{1}{\rho_{Cu}} \approx 56 \cdot 10^6 \frac{S}{m}))$$

$$\text{Solution: } R = \rho \frac{1}{A} \qquad \qquad G = \frac{1}{R}$$

$$R = 0.0178 \frac{\Omega \cdot mm^2}{m} \frac{175 \text{ m}}{2.5 \text{ mm}^2} = 1.246 \Omega$$

$$G = \frac{1}{1.246 \Omega} = 0.804 \text{ S}$$

Example 2.2.

A copper conductor having a cross-sectional area of  $6 \text{ mm}^2$  is to be replaced by an aluminium conductor of the same resistance. What is the size of the cross-sectional area of the aluminium conductor?

Given:  $A_{\text{Cu}} = 6 \text{ mm}^2$  To be found:  $A_{\text{Al}}$

$$\rho_{\text{Cu}} = 0.0178 \frac{\Omega \cdot \text{mm}^2}{\text{m}}$$

$$\rho_{\text{Al}} = 0.0286 \frac{\Omega \cdot \text{mm}^2}{\text{m}}$$

Solution:  $R_{\text{Cu}} = R_{\text{Al}}$

$$R_{\text{Cu}} = \rho_{\text{Cu}} \frac{1}{A_{\text{Cu}}} \quad R_{\text{Al}} = \rho_{\text{Al}} \frac{1}{A_{\text{Al}}}$$

$$\rho_{\text{Cu}} \frac{1}{A} = \rho_{\text{Al}} \frac{1}{A_{\text{Al}}}$$

$$A_{\text{Al}} = \frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}} A_{\text{Cu}}$$

$$A_{\text{Al}} = \frac{0.0286}{0.0178} \cdot 6 \text{ mm}^2 = 9.64 \text{ mm}^2$$

For the aluminium conductor, the standardised cross-sectional area of  $10 \text{ mm}^2$  is selected.

The most striking influence on the resistance of a conductor or device is exerted by the temperature.

The temperature dependence of the electrical resistance can be quantitatively expressed by the temperature coefficient <sup>1)</sup>.

<sup>1)</sup> ? Greek letter alpha

The temperature coefficient states the fraction by which the resistance changes with a change in temperature of 1 K:

$$\alpha = \frac{\Delta R / R}{\Delta \theta} \quad (2.5)$$

where

? temperature coefficient

? <sup>2)</sup> R/R change in resistance related to the initial resistance

<sup>2)</sup> ? Greek letter delta

?  $\Delta \theta$ ; <sup>3)</sup> temperature change

<sup>3)</sup>  $\Delta \theta$ ; Greek letter theta

The unit of the temperature coefficient is

$$[\alpha] = \frac{1}{K} \quad (K = \text{Kelvin})$$

In metallic conductors, the resistance increases with increasing temperature. This is due to the fact that the more intensively oscillating crystal lattices offer a higher resistance to the electron current; hence,  $\alpha$  is positive.

In electrolytes and semiconductors, the resistance diminishes with increasing temperature. This is due to the fact that with rise in temperature more charge carriers are released which then are available as free charge carriers for the transport of electricity; hence,  $\alpha$  is negative.

For practice, the following approximate values of the temperature coefficient will suffice (see also Table 2.4):

- Non-ferromagnetic pure metals (no metal alloys)

$$\alpha \approx + 0.004 \frac{1}{K}$$

The resistance of a copper conductor of 100  $\Omega$ , for example, will increase by 0.4  $\Omega$  to 100.4  $\Omega$  in the event of an increase in temperature of 1 K; in case of a rise in temperature of 80 K (e.g. from 20 °C to 100 °C) it will increase by 32  $\Omega$  to 132  $\Omega$ .

- Ferromagnetic metals (iron, nickel)

$$\alpha \approx + 0.006 \frac{1}{K}$$

- Metal alloys of a special composition (novoconstant, constantan)

$$\alpha \approx 0$$

These special metal alloys are of particular importance to measuring techniques if resistors independent of temperature are required.

- Electrolytes

$$\alpha \approx - 0.02 \frac{1}{K}$$

- Semiconductors

$\alpha$  is negative and largely dependent on temperature; a numerical value cannot be stated; it should be drawn from special Tables for the temperatures involved.

Table 2.4. Specific Resistance  $\rho$ , Conductance  $G$  and Temperature Coefficient  $\alpha$  of a Few Conductor Materials

Conductor Material	$\rho$	$G$	$\alpha$
silver	0.016	62.5	
copper	0.0178	56	
aluminium	0.0286	35	$+ 0.004$
zinc	0.063	16	

lead	0.21	4.8	
nickel	0.10	10	
iron, pure	0.10	10	? + 0.006
Novokonstand <sup>1)</sup>	0.45	2.3	
constantan <sup>2)</sup>	0.5	2	? 0

1) Novokonstant: 82.5 % Cu; 12 % Mn; 4 % Al; 1.5 % Fe

2) constantan: 54 % Cu; 45 % Ni; 1 % Mn

### Example 2.3.

A coil of copper wire has a resistance of 18 ? at room temperature (20 °C). During operation, the temperature rises to 85 °C. Find the resistance of the coil at this temperature.

Given:  $R_{20} = 18 \Omega$  To be found  $R_{85}$

$$\begin{aligned}\Delta R &= 85^\circ C - 20^\circ C \\ &= 65 K\end{aligned}$$

$$\alpha \approx +0.004 \frac{1}{K}$$

Solution: From equation (2.5) we obtain by transposing a value for the change of resistance

$$\Delta R = \alpha R_{20} \Delta T$$

This amount must be added to the resistance  $R_{20}$  in order to determine the final resistance  $R_{85}$ .

$$R_{85} = R_{20} + \Delta R$$

$$R_{85} = R_{20} + \alpha R_{20} \Delta T$$

$$R_{85} = R_{20} (1 + \alpha \Delta T)$$

$$R_{85} = 18 \Omega (1 + 0.004 \frac{1}{K} \cdot 65 K)$$

$$R_{85} = 18 \Omega (1 + 0.26)$$

$$R_{85} = 18 \Omega \cdot 1.26$$

$$R_{85} = 22.68 \Omega$$

=====

Components which are used to limit the current by means of certain resistance values and which are constructed specifically for this purpose are called resistors. Resistor is a component for the realisation of a certain resistance value.

The general graphical symbol of a resistor is shown in Fig. 2.9.



Fig. 2.9. Graphical symbol of a resistor

Resistance and conductance are properties of electrical conductors and devices. The resistance characterises the resistance offered to the passage of current; the conductance indicates how well the conductor or device in question allows the current to pass. The correlation between resistance and conductance results from the relation

$$R = \frac{1}{G}$$

The rating equation of the resistance and of the conductance is

$$R = \rho \frac{A}{L} \quad \text{and} \quad G = \kappa \frac{A}{L}$$

The material constant  $\rho$  is called specific resistance,  $\kappa$  is called specific conductance.

The resistance (and the conductance, too) is primarily depending on temperature. The magnitude of the temperature dependence is covered by the temperature coefficient  $\alpha$  which indicates the relative change in

$$\alpha \approx +0.004 \frac{1}{K}$$

resistance per degree of change in temperature. For non-ferromagnetic metals, this means that the resistance of these materials increases with increasing temperature.

As unit of the resistance, the ohm =  $\Omega$  is specified; the most frequently used sub-units are M  $\Omega$ , K  $\Omega$ , m  $\Omega$ .

$$\text{siemens} = S = \frac{1}{\Omega}$$

The unit of conductance is  $S$ ; the most frequently used sub-units are kS, mS,  $\mu$ S.

A component which is specially built to realise a certain resistance value is called resistor.

Questions and problems:

1. How many  $\Omega$  are

$2 \text{ M } \Omega$ ;  $15 \text{ k } \Omega$ ;  $350 \text{ m } \Omega$ ;  $0.5 \mu\text{S}$ ;  $4 \text{ S}$ ;  $2 \text{ mS}$ ?

2. For the supply of energy to a consumer situated at a distance of 150 m, a 2-core copper line with a cross-sectional area of  $2.5 \text{ mm}^2$  per conductor is used. Calculate the resistance and the conductance of the line (take into consideration the outgoing and the return conductors).

3. Calculate the temperature (related to a reference temperature of  $20^\circ\text{C}$ ) at which the resistance of a copper wire will double.

### 3. Electric Circuits

#### 3.1. Basic Circuit

If an incandescent lamp for illuminating a working place is to be caused to light, the following pre-conditions are required.

- A voltage source whose voltage is available at the point of connection (socket outlet). The voltage source may be far away from the point of connection (e.g. in a power station). A fuse is interposed in each line as overcurrent protection.
- A 2-core line leading to the incandescent lamp which conducts the voltage via a plug to the lamp. For conveniently switching on and off, a switch is interposed. Fig. 3.1. shows the described arrangement, Fig. 3.2 the schematic representation with symbols which is called wiring diagram.

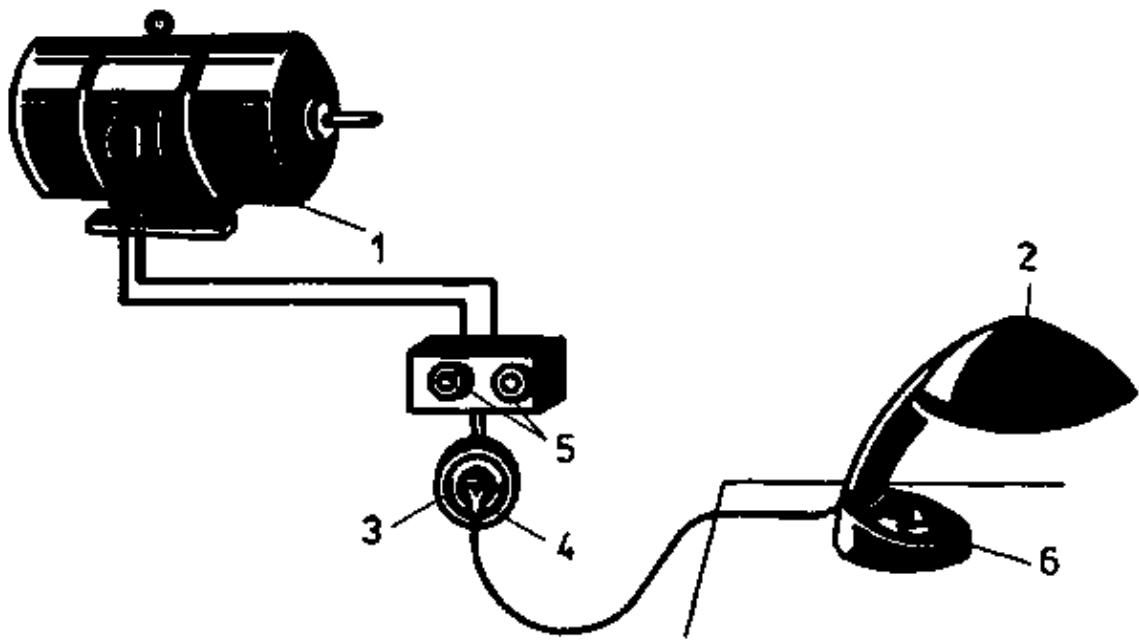


Fig. 3.1. Simplified representation of the arrangement voltage source/table lighting fitting

1 Voltage source; 2 Table lighting fitting; 3 Plug socket; 4 Plug; 5 Fuses; 6 Switch

This shows that a closed connection from the voltage source to the incandescent lamp is essential for operation. The charge carriers driven from the source pass through the conductor, transfer their energy to the lamp and return to the source where they receive again drive energy. This is a circulatory process and, therefore, such an arrangement is called circuit.

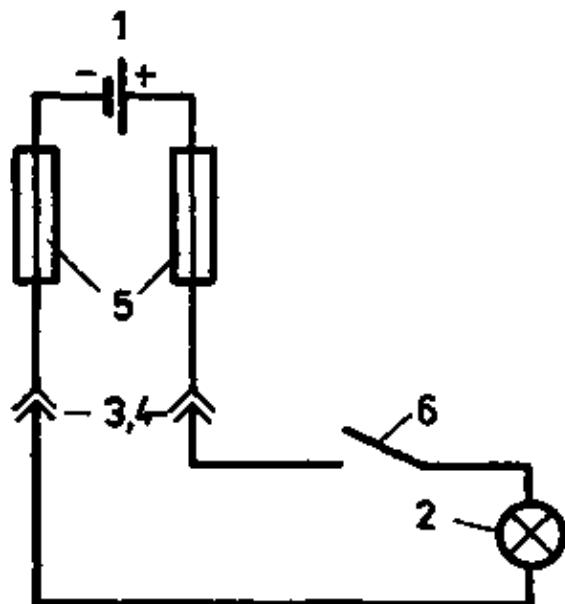


Fig. 3.2. Wiring diagram for Fig. 3.1.

Graphical symbols:

- 1 Voltage source;
- 2 Lighting fitting;
- 3/4 Plugged connection;
- 5 Fuses;
- 6 Switch

Since no charge carriers are lost during the passage, the current is a phenomenon closed in itself, a band without start and without end which has the same intensity at any point.

The above described example is the simplest circuit. Therefore it is called basic circuit. For the principle illustrated here it is of no consequence if in the place of the generator a different voltage source (e.g. an accumulator) is used and a heater, washing machine, motor or another consumer operates in the place of the electric bulb. In the circuit diagram, frequently the fuses, the point of connection and the switch are not represented; the consumer is frequently represented simply by the resistance symbol, see Fig. 3.3.

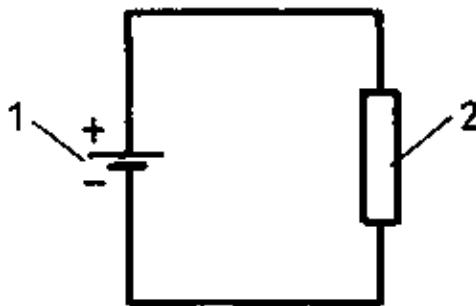


Fig. 3.3. Fundamental circuit

1 Voltage source; 2 Load, general

The basic circuit is a circuit which cannot be further simplified. It consists of a voltage source to which a consumer is connected. The current flowing in this circuit is a phenomenon closed in itself, a band without start and end which has the same intensity at any point.

### 3.2. Ohm's Law

A natural law found by Georg Simon Ohm is called Ohm's law and it has fundamental importance to electrical engineering. It correlates the quantities current, voltage and resistance. When two quantities are known, the third can be found with the help of this law.

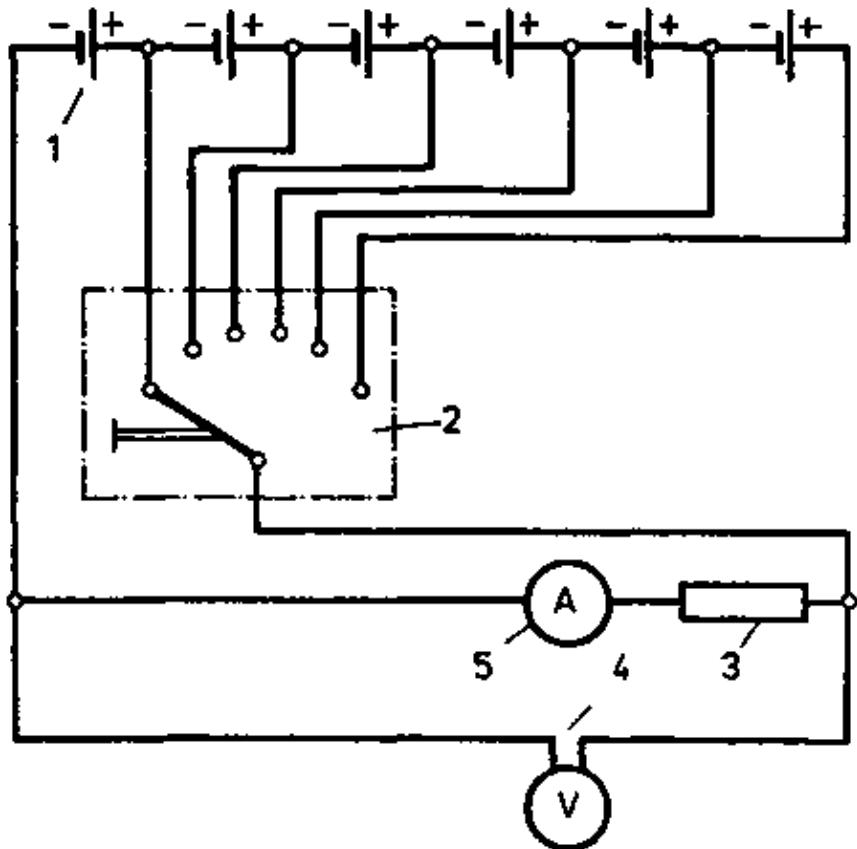


Fig. 3.4. Circuit for the proof of Ohm's law

1 Accumulator with 6 cells of 2 V each; 2 Voltage selector switch; 3 Resistor; 4 Voltmeter; 5 Ammeter

Proof of this fact is given here experimentally by means of an arrangement whose wiring diagram is shown in Fig. 5.4. The following devices and components are required:

- An adjustable voltage source; this may be an accumulator with six cells each of which provides 2 V voltage so that a total of 12 V are available;
- a voltage selector switch with six switch steps;
- two resistors with resistance values of, for example, 10  $\Omega$  and 20  $\Omega$ ;
- two measuring instruments for the measurement of current and voltage.

Measuring instruments are indispensable auxiliaries in electrical engineering; they cannot be dealt with in this book, however. Here, it will suffice to know that the current can be measured by means of ammeters and the voltage by means of voltmeters with adequate accuracy.

At first, one switches on the 10  $\Omega$  resistance and then, using the voltage selector switch, stepwise applies a voltage of 2 V, 4 V, 6 V, 8 V, 10 V, and 12 V. The current flowing at each switch step is measured by means of the ammeter and the quantities are recorded in a table of values.

Table of values 1:  
(for  $R = 10 \Omega$ )

<b>Switch step</b>	<b>Voltage U</b>	<b>Current I</b>
1	2 V	0.2 A
2	4 V	0.4 A
3	6 V	0.6 A
4	8 V	0.8 A
5	10 V	1.0 A
6	12 V	1.2 A

A similar test with the 20  $\Omega$  resistance results in the following table of values:

Table of values 2:  
(for  $R = 20 \Omega$ )

<b>Switch step</b>	<b>Voltage U</b>	<b>Current I</b>
1	2 V	0.1 A
2	4 V	0.2 A
3	6 V	0.3 A
4	8 V	0.4 A
5	10 V	0.5 A
6	12 V	0.6 A

The experiment is indicative of the following facts:

1. With a given voltage U, the current I is the smaller, the greater the resistance R is. (The resistance offers resistance to the passage of current).

E.g. with  $U = 6 \text{ V}$ :

measurement series 1:  $I = 0.6 \text{ A}$  with  $R = 10 \Omega$  ?

measurement series 2:  $I = 0.3 \text{ A}$  with  $R = 20 \Omega$  ?

$$I \sim \frac{1}{R}$$

2. With given current  $I$ , the voltage  $U$  (more exactly: the voltage drop) is the greater, the greater the resistance  $R$  is. (When a higher resistance is offered, the charge carriers must give more energy to it!)

E.g. with  $I = 0.4 \text{ A}$ :

measurement series 1:  $U = 4 \text{ V}$  with  
 $R = 10 \Omega$  ?

measurement series 2:  $U = 8 \text{ V}$  with  
 $R = 20 \Omega$  ?

$$U \sim R$$

$$\frac{U}{I}$$

3. The quotient  $\frac{U}{I}$  yields the same value for all values of a measurement series:

for measurement series 1:  $\frac{U}{I} = 10 \frac{\text{V}}{\text{A}} = 10 \Omega$

for measurement series 2:  $\frac{U}{I} = 20 \frac{\text{V}}{\text{A}} = 20 \Omega$

Obviously, the resistance is constant, its magnitude is thus independent of the current passing through it and of the voltage applied to it.

The result is the important Ohm's law.

It has the following expression in signs

$$R = \frac{U}{I} \quad (3.1.)$$

$$[R] = \frac{\text{V}}{\text{A}} = \Omega$$

where

$R$  resistance

$U$  voltage

$I$  current intensity

When two quantities are known, the third one can be determined; equation must be rearranged in such a way that the unknown is alone, namely,

$$U = RI \quad (3.1.a) \quad I = \frac{U}{R} \quad (3.1.b)$$

$$[U] = \Omega \cdot A = V \quad [I] = \frac{V}{\Omega} = A$$

When the measuring results are interpreted graphically, the  $U$ - $I$  diagram shown in Fig. 3.5 is obtained. The functional correlation between  $U$  and  $I$  is linear. The slope of the straight lines is the steeper, the higher the

resistance R is.

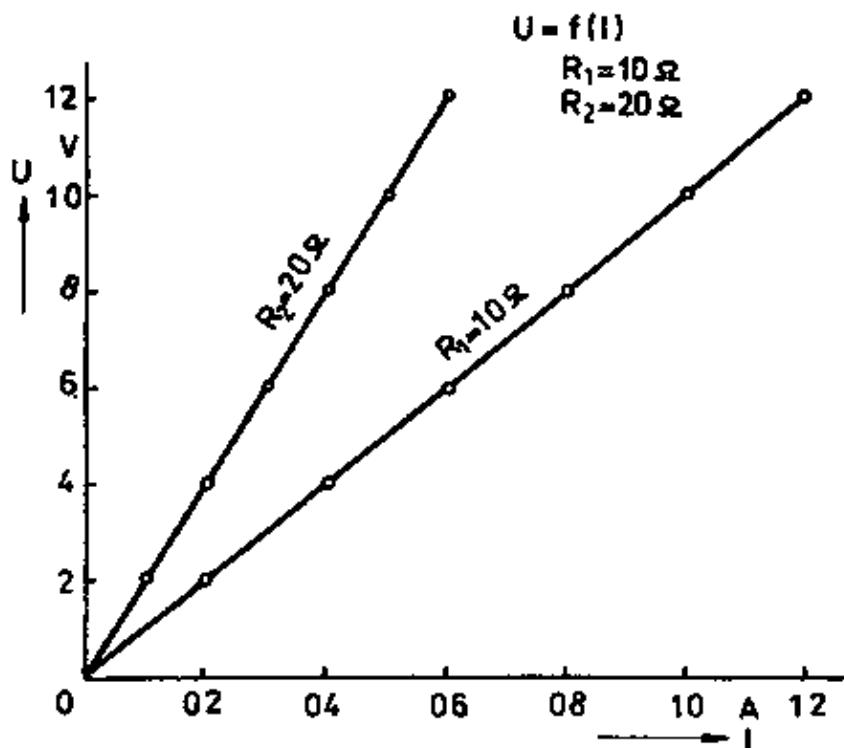


Fig. 3.5. Current/voltage diagram

$$U = (I) R_1 = 10 \Omega; R_2 = 20 \Omega$$

The correlation between the various units of the quantities current, voltage, resistance and conductance is represented in Table 3.1.

$$\frac{V}{A} = \Omega; \frac{A}{V} = S = \frac{1}{\Omega}; \frac{V}{\Omega} = A$$

We know that

	$\mu V$	$mV$	$V$	$kV$	$MV$
$\mu A$	$\Omega$	$k\Omega$	$M\Omega$		
$mA$	$S$	$mS$	$\mu S$		
$A$	$k\Omega$	$\Omega$	$k\Omega$	$M\Omega$	
$kA$		$m\Omega$	$\Omega$	$k\Omega$	

Table 3.1. The Units of the Quantities Current, Voltage, Resistance, and Conductance

When sub-units are used in calculations, the unit of the quantity to be determined may also be a sub-unit.

When taking a closer look at the 3rd column under V (= volt), you read the unit of the resistance or conductance in the first to fourth lines:

$$\frac{V}{\mu A} = M\Omega \quad \frac{V}{mA} = k\Omega \quad \frac{V}{A} = \Omega \quad \frac{V}{kA} = m\Omega$$

$$\frac{\mu A}{V} = \mu S \quad \frac{mA}{V} = mS \quad \frac{A}{V} = S \quad \frac{kA}{V} = kS$$

The other units are handled in a similar manner.

### Example 3.1

Calculate the resistance and conductance of an electrical soldering iron through which a current of 0.4 A passes when connected to a voltage of 220 V.

Given:  $U = 220 \text{ V}$  To be found:  $R$

$$I = 0.4 \text{ A} \qquad G$$

Solution:  $R = \frac{U}{I}$

$$R = \frac{220 \text{ V}}{0.4 \text{ A}} = 550 \frac{\text{V}}{\text{A}}$$

$$R = 550 ?$$

=====

$$G = \frac{1}{R} = \frac{1}{550 \Omega} = 0.00182 \text{ S}$$

$$G = 1.82 \text{ mS}$$

=====

### Example 3.2.

An ammeter has a resistance of  $30 \text{ m}\Omega$ . Calculate the voltage drop across the instrument when a current of 5 A passes.

Given:  $R = 30 \text{ m}\Omega$  To be found:  $U$   
 $I = 5 \text{ A}$

Solution:  $R = \frac{U}{I}$  inverted:  $U$

$$U = R I$$

$$U = 30 \text{ m}\Omega \cdot 5 \text{ A}$$

$$U = 0.03 \frac{\text{V}}{\text{A}} \cdot 5 \text{ A} = 0.15 \text{ V}$$

$$U = 150 \text{ mV}$$

=====

### Example 3.3.

A voltage of 4.5 V is applied to a resistance of  $15 \text{ k}\Omega$ . Calculate the intensity of the current passing through the component.

Given:  $R = 15 \text{ k}\Omega$  To be found:  $I$   
 $U = 4.5 \text{ V}$

Solution:  $R = \frac{U}{I}$  inverted:  $I$

$$I = \frac{U}{R}$$

$$I = \frac{4.5 \text{ V}}{15 \text{ k}\Omega}$$

$$I = \frac{4.5 \text{ V}}{15000\Omega} = 0.0003 \text{ A}$$

$$I = 0.3 \text{ mA}$$

=====

If a current flows through a consumer and a voltage drops across it because of the current passing through it, then this consumer has a resistance.

The correlation between current, voltage and resistance is described by Ohm's law. It has fundamental importance to electrical engineering and is written as

$$R = \frac{U}{I}$$

An inversion of this law for U or I gives

$$I = \frac{U}{R}$$

Questions and problems:

1. A voltage of 220 V is applied to a flat iron while a current of 2.75 A flows through it. Calculate the resistance and conductance of the electrical flat iron.

2. What is the current passing through a resistor of 10 k ? at a voltage of 200 V?

3. The power supply system of a workshop is protected by a fuse of 10 A. The voltage is 220 V. Calculate the resistance which the connected loads must have at least.

4. A small horsepower motor has a resistance of 15 ? during operation. The current intensity is 0.4 A. Calculate the voltage required at the terminals of the motor.

### 3.3. Branched and Unbranched Circuits

#### 3.3.1. Branched Circuits

When connecting several consumers, also known as loads, in parallel in a circuit, then the current can flow at the same time through several consumers. Fig. 3.6. shows such an arrangement.

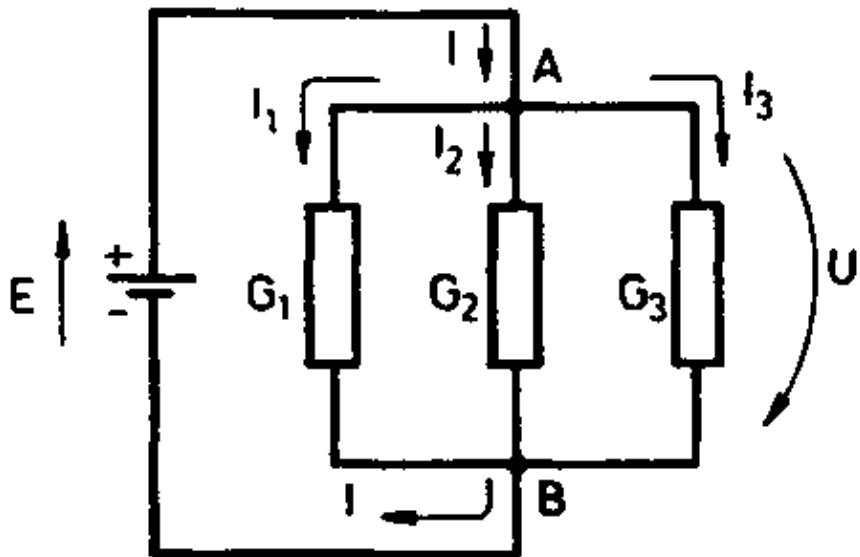


Fig. 3.6. Branched circuit

The current  $I$  driven by the voltage source is divided into three partial currents  $I_1$ ,  $I_2$  and  $I_3$  at branching point A. These currents pass through the three loads, join in point B and return as total current  $I$  to the voltage source. Since no charge carriers are lost, the sum of the partial currents  $I_1 \dots I_3$  branching off from point A must be equal to the incoming total current  $I$ ; in point B the sum of the incoming currents  $I_1 \dots I_3$  must be equal to the total current  $I$  returning to the source.

In general, the statement formulated by Robert Kirchhoff (German physicist 1824 – 1887) and known as the 1st Kirchhoff's law or junction point theorem holds:

In each junction point, the sum of the currents flowing toward the point is equal to the sum of those flowing away from it.

For the example shown in Fig. 3.6. thus, we have

$$I + I_1 + I_2 + I_3 \quad (3.2)$$

Since the voltage through all loads is the same, we have

$$\frac{I}{U} = \frac{I_1}{U} + \frac{I_2}{U} + \frac{I_3}{U}$$

$\frac{I}{U}$  is the conductance of the load; the total conductance or equivalent conductance, which is to be designated  $G_{\text{equ}}$ , is written as

$$G_{\text{equ}} = G_1 + G_2 + G_3 \quad (3.3)$$

In a parallel connection of consumers, the equivalent conductance is equal to the sum of the individual conductances; it is always greater than the greatest individual conductance.

For  $n$ -equal conductances ( $n = 2, 3, 4 \dots$ ) holds

$$G_{\text{equ}} = n G \quad (3.3. a)$$

For two consumers with the conductances  $G_1$  and  $G_2$  we have

$$G_{\text{equ}} = G_1 + G_2 \quad (3.3. b)$$

In practice, resistances are more frequently used for calculating than conductances. In accordance with the

general relation  $R = \frac{1}{G}$ , the following is derived from equation (3.3)

$$\frac{1}{R_{\text{equ}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad (3.4)$$

In a parallel connection of loads, the reciprocal value of the equivalent resistance is equal to the reciprocal values of the individual resistances; the equivalent resistance is always smaller than the smallest individual resistance.

For n-equal resistances ( $n = 2, 3, 4 \dots$ ) holds

$$R_{\text{equ}} = \frac{R}{n} \quad (3.4.a)$$

For two loads with the resistances  $R_1$  and  $R_2$  the following holds:

$$\frac{1}{R_{\text{equ}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$$

$$R_{\text{equ}} = \frac{R_1 R_2}{R_1 + R_2} \quad (3.4.b)$$

The ratio of the partial currents is dependent on the ratio of the partial resistances. It is obvious that the smaller current flows through the larger partial resistance and vice versa. This relation which is known as the current divider rule for two loads connected in parallel is written as

$$\frac{I_1}{I_2} = \frac{G_1}{G_2} \quad (3.5.a)$$

or

$$\frac{I_1}{I} = \frac{G_1}{G_{\text{equ}}} \quad (3.5.b)$$

In a current divider, the ratio of the partial currents is like that of the partial conductances.

Expressed in terms of resistances, we obtain from equation (3.5.)

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} \quad (3.6.a)$$

or

$$\frac{I_1}{I} = \frac{R_{\text{equ}}}{R_1} \quad (3.6.b)$$

In a current divider, the ratio of the partial currents is inverse to that of the partial resistances.

Example 3.4.

Two loads with the conductances  $G_1 = 12.2 \text{ mS}$  and  $G_2 = 8.7 \text{ mS}$  are connected in parallel while a voltage of 24 V is applied to them. Draw a sketch of the circuit. Calculate the equivalent conductance and the equivalent resistance as well as the total current and the partial currents.

Given:  $G_1 = 12.2 \text{ mS}$  To be found:  $G_{\text{equ}}$   
 $G_2 = 8.7 \text{ mS}$   $R_{\text{equ}}$   
 $U = 23 \text{ V}$   $I$   
 $I_1; I_2$

Solution:

Circuit:

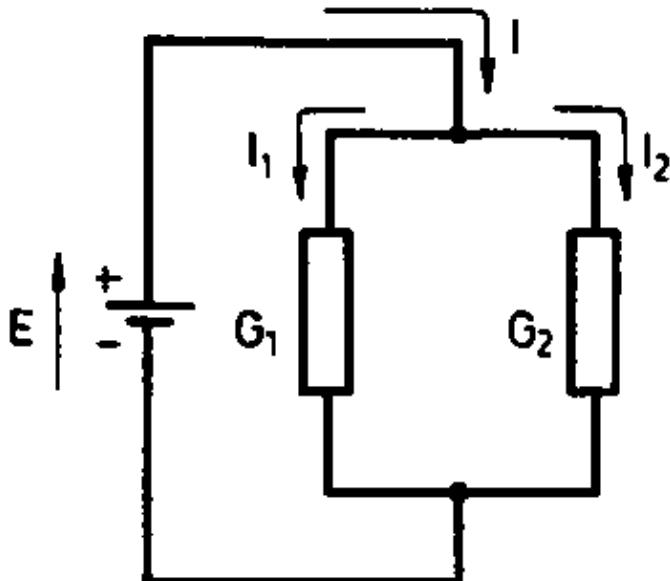


Fig. 3.7. Circuit with two loads connected in parallel

According to equation (3.3.b):

$$G_{\text{equ}} = G_1 + G_2$$

$$G_{\text{equ}} = 12.2 \text{ mS} + 8.7 \text{ mS}$$

$$G_{\text{equ}} = 20.9 \text{ mS};$$

the equivalent conductance is greater than the greatest individual conductance

According to equation (2.3.):

$$R_{\text{equ}} = \frac{1}{G_{\text{equ}}}$$

$$R_{\text{equ}} = \frac{1}{0.0209 \text{ S}} = 48 \frac{1}{\text{S}}$$

$$R_{\text{equ}} = 48 ?$$

=====

or according to equation (3.4.b) where for

$$R_1 = \frac{1}{G_1} = \frac{1}{0.0122 \text{ S}} = 82 \Omega$$

and for

$$R_2 = \frac{1}{G_2} = \frac{1}{0.0087 \text{ S}} = 115 \Omega$$

we have to write

$$R_{\text{equ}} = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_{\text{equ}} = \frac{82 \Omega \cdot 115 \Omega}{82 \Omega + 115 \Omega} = \frac{9430 \Omega}{197 \Omega} = 47.9 \Omega$$

$R_{\text{equ}}$  ? 48 ?;

the equivalent resistance is smaller than the smallest individual resistance

According to equation (3.1.b):

$$I = \frac{U}{R_{\text{equ}}}$$

$$I = \frac{24 \text{ V}}{48 \Omega}$$

$$I = 0.5 \text{ A} = 500 \text{ mA}$$

=====

$$I_1 = \frac{U}{R_1} = \frac{24 \text{ V}}{82 \Omega}$$

the greater partial current flows through the smaller individual resistance

$$I_1 = 0.292 \text{ A} = 292 \text{ mA}$$

=====

$$I_2 = \frac{U}{R_2} = \frac{24 \text{ V}}{115 \Omega}$$

$$I_2 = 0.208 \text{ A} = 208 \text{ mA}$$

=====

$I_1$  and  $I_2$  can also be found with the help of the current divider rule, i.e. equations (3.5. and 3.6.), namely,

$$\frac{I_1}{I} = \frac{G_1}{G_{\text{equ}}} \quad \text{or} \quad \frac{I_1}{I} = \frac{R_{\text{equ}}}{R_1}$$

$$I_1 = I \frac{G_1}{G_{\text{equ}}} \quad I_1 = I \frac{R_{\text{equ}}}{R_1}$$

$$I_1 = 500 \text{ mA} \frac{12.2}{20.9} \quad I_1 = 500 \text{ mA} \frac{48}{82}$$

$$I_1 = 292 \text{ mA} \quad I_1 = 292 \text{ mA}$$

The calculation for  $I_2$  has to be performed analogously.

Check with the help of equation (3.2.):

$$I = I_1 + I_2$$

$$I = 292 \text{ mA} + 208 \text{ mA}$$

$$I = 500 \text{ mA}; \quad \text{the total current is equal to the sum of all partial currents}$$

=====

### 3.3.2. Unbranched Circuits

When connecting several load in one circuit in such a manner that the current passes through all loads one after the other, then this circuit is called series connection or series circuit.

Fig. 3.8. shows such an arrangement.

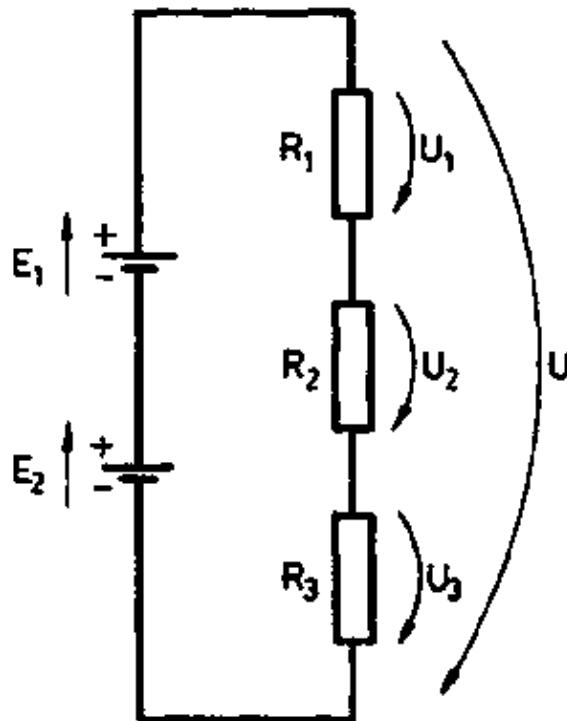


Fig. 3.8. Unbranched circuit

The two voltage sources connected in series produce a total voltage of

$$E = E_1 + E_2.$$

The current driven by the total voltage has the same intensity at any point in the circuit and causes an adequate voltage drop at any load. Since the charge carriers give their entire energy to the circuit, the statement in the form of the 2nd Kirchhoff's law or the mesh-network theorem holds:

In each unbranched circuit, the sum of all voltage drops is equal to the total voltage.

For the example shown in Fig. 3.8., thus we have

$$E = E_1 + E_2 = U = U_1 + U_2 + U_3 \quad (3.7.)$$

As the current through all loads is the same, we have

$$\frac{U}{I} = \frac{U_1}{I} + \frac{U_2}{I} + \frac{U_3}{I}$$

$\frac{U}{I}$ , however, is the resistance of the load; the total resistance or equivalent resistance will be called  $R_{\text{equ}}$  and obtained in the form of

$$R_{\text{equ}} = R_1 + R_2 + R_3 \quad (3.8.)$$

In a series connection of loads, the equivalent resistance is equal to the sum of the individual resistances; it is always greater than the greatest individual resistance.

For  $n$ -equal resistances ( $n = 2, 3, 4, \dots$ )

$$R_{\text{equ}} = n R \quad \text{holds.} \quad (3.8.a)$$

If conductances have to be calculated, the required equations can easily be derived from the relation  $G = \frac{1}{R}$ . In practice, calculations of conductances are required very seldom for unbranched circuits.

The ratio of the partial voltages is dependent on the ratio of the partial resistances. It is obvious that the greater voltage drop occurs via the greater partial resistance and vice versa. This relation called voltage divider rule is expressed for two loads connected in series in the following way:

$$\frac{U_1}{U_2} = \frac{R_1}{R_2} \quad (3.9.a)$$

or

$$\frac{U_1}{U} = \frac{R_1}{R_{\text{equ}}} \quad (3.9.b)$$

In a voltage divider, the ratio of the partial voltages is the same as that of the partial resistances.

#### Example 3.5.

Two loads with the resistances  $R_1 = 18 \Omega$  and  $R_2 = 72 \Omega$  are connected in series and a voltage of 60 V is applied to them. Draw a sketch of this circuit. Calculate the equivalent resistance, the current and the partial voltages.

Given: $R_1 = 18 \Omega$	To be found: $R_{\text{equ}}$
$R_2 = 72 \Omega$	$I$
$U = 60 \text{ V}$	$U_1; U_2$

Solution:

Circuit:

According to equation (3.8.):

$$\begin{aligned} R_{\text{equ}} &= R_1 + R_2 \\ R_{\text{equ}} &= 18 \Omega + 72 \Omega \\ R_{\text{equ}} &= 90 \Omega; \\ \hline \end{aligned}$$

the equivalent resistance is greater than the greatest individual resistance

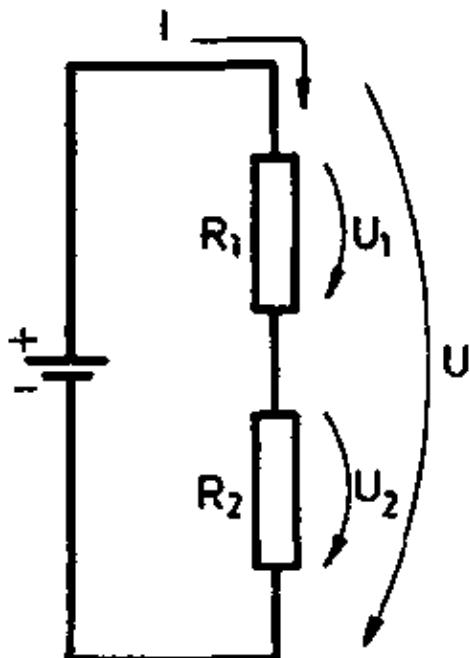


Fig. 3.9. Circuit with two loads connected in series

According to equation (3.1.b):

$$I = \frac{U}{R_{\text{equ}}}$$

$$I = \frac{60 \text{ V}}{90 \Omega}$$

$$I = 0.667 \text{ A} = 667 \text{ mA}$$

=====

According to equation (3.1.a):

$$\begin{aligned} U_1 &= R_1 I \\ U_1 &= 18 \Omega \cdot 0.667 \text{ A} \\ U_1 &= 12 \text{ V}; \\ \hline \end{aligned}$$

the smaller partial voltage drops via the smaller individual resistance

$$\begin{aligned} U_2 &= R_2 I \\ U_2 &= 72 \Omega \cdot 0.667 \text{ A} \\ U_2 &= 48 \text{ V} \\ \hline \end{aligned}$$

$U_1$  and  $U_2$  can also be calculated according to the voltage divider rule, equation (3.9.b), namely,

$$\frac{U_1}{U} = \frac{R_1}{R_{\text{equ}}} \quad \text{or} \quad \frac{U_2}{U} = \frac{R_2}{R_{\text{equ}}}$$

$$U_1 = U \frac{R_1}{R_{\text{equ}}} \quad U_2 = U \frac{R_2}{R_{\text{equ}}}$$

$$U_1 = 60 \text{ V} \frac{18}{90} \quad U_2 = 60 \text{ V} \frac{72}{90}$$

$$U_1 = 12 \text{ V} \quad U_2 = 48 \text{ V}$$

Verification by means of equation (3.7.):

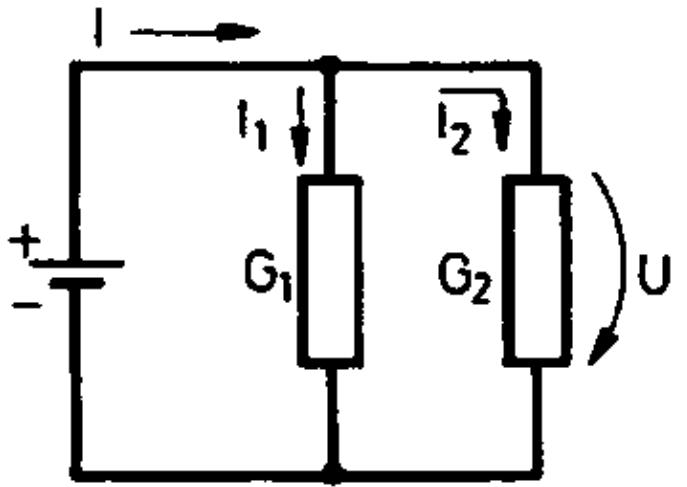
$$\begin{aligned} U &= U_1 + U_2 \\ U &= 12 \text{ V} + 48 \text{ V} \\ U &= 60 \text{ V}; \text{ the sum of all voltage drops is equal to the total voltage.} \\ \hline \end{aligned}$$

In the following Table 3.2., an unbranched circuit is compared with a branched circuit. In each of the two cases, two loads are included. It is obvious that the formulas of one circuit have the same structure as those of the other circuit. The only difference is that, as compared to the branched circuit, for the unbranched circuit  $U$  is put instead of  $I$ ,  $I$  instead of  $U$ ,  $R$  instead of  $G$  and  $G$  instead of  $R$ .

Table 3.2. Relations in Branched and Unbranched Circuits Having two Loads Each

**Branched Circuit**

**Unbranched Circuit**



$U$  is equal across all loads

$$I = I_1 + I_2$$

$$G_{\text{equ}} = G_1 + G_2$$

$$\frac{1}{R_{\text{equ}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\rightarrow R_{\text{equ}} = \frac{R_1 R_2}{R_1 + R_2}$$

for  $G_1 = G_2 = G = \frac{1}{R}$  is

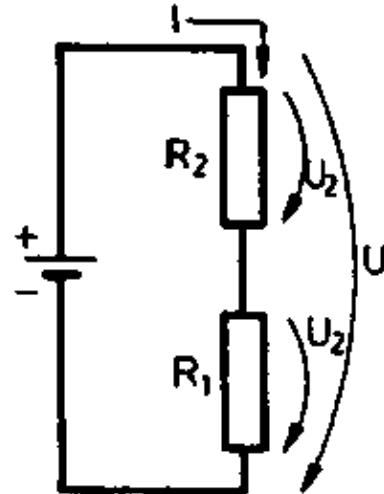
$$G_{\text{equ}} = 2G, \text{ since } n = 2$$

$$\text{or } R_{\text{equ}} = \frac{R}{2}$$

$$\frac{I_1}{I_2} = \frac{G_1}{G_2} = \frac{R_2}{R_1}$$

$$\frac{I_1}{I} = \frac{G_1}{G_{\text{equ}}} = \frac{G_1}{G_1 + G_2}$$

$$\frac{I_1}{I} = \frac{R_{\text{equ}}}{R_1} = \frac{R_2}{R_1 + R_2}$$



$I$  is equal through all loads

$$U = I_1 + U_2$$

$$R_{\text{equ}} = R_1 + R_2$$

$$\frac{1}{G_{\text{equ}}} = \frac{1}{G_1} + \frac{1}{G_2}$$

$$\rightarrow G_{\text{equ}} = \frac{G_1 G_2}{G_1 + G_2}$$

for  $R_1 = R_2 = R = \frac{1}{G}$  is

$$R_{\text{equ}} = 2R, \text{ since } n = n$$

$$\text{or } G_{\text{equ}} = \frac{G}{2}$$

$$\frac{U_1}{U_2} = \frac{R_1}{R_2} = \frac{G_2}{G_1}$$

$$\frac{U_1}{U} = \frac{R_1}{R_{\text{equ}}} = \frac{R_1}{R_1 + R_2}$$

$$\frac{U_1}{U} = \frac{G_{\text{equ}}}{G_1} = \frac{G_2}{G_1 + G_2}$$

### 3.3.3. Meshed Circuits

Meshed Circuits, also known as interconnected circuits, are such circuits where loads are connected in parallel and other loads in series. In addition, the various branches can contain voltage sources. Computations of these circuits are possible with the help of the two Kirchhoff's laws. Here, we confine ourselves to the loaded voltage divider as the simplest case which is of great importance to practical electrical engineering.

By means of a voltage divider, one can draw from an available voltage any desired smaller partial voltage which is used to supply a certain load  $R_v$ . Through the latter, an adequate current will then flow. Besides a voltage division, a current division is also involved; Fig. 3.12. shows these conditions.

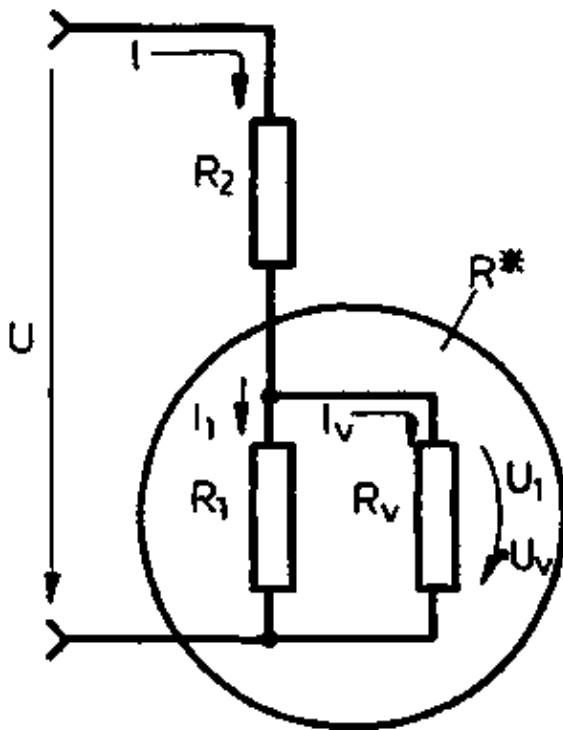


Fig. 3.12. Loaded voltage divider

$$R = R_{\text{equ}_1}$$

Circuits of this type where resistances are connected in parallel and other resistances in series are called series-parallel connections. According to Fig. 3.12., the following considerations should be made:

Through the load  $R_v$  a current passes which is part of the total current. As a consequence, the current flowing through  $R_1$  only has the intensity of  $I_1 = I - I_v$ , i.e. it is smaller than in the case of a non-connected load. A lower current causes a smaller voltage drop according to Ohm's law. This fact is described by the following equation.

$$\frac{U_1}{U} = \frac{R_{\text{equ}_1}}{R_{\text{equ}}} \quad (3.10.)$$

$$R_{\text{equ}_1} = \frac{R_1 R_v}{R_1 + R_v} R_{\text{equ}_1}$$

is smaller than the smaller resistance of  $R_1$  and  $R_v$

$$R_{\text{equ}} = R_{\text{equ}_1} + R_2$$

#### Example 3.6.

From a voltage source of 24 V, a voltage of 6 V is to be supplied to a load  $R_v = 30 \Omega$  via a voltage divider. The total current is 0.5 A. Calculate the partial resistances  $R_1$  and  $R_2$ .

Given:  $U = 24 \text{ V}$   
 $I = 0.5 \text{ A}$   
 $U_1 = 6 \text{ V}$   
 $R_v = 30 \Omega$

To be found:  $R_1$  and  $R_2$

$$U_2 = U - U_1 = 24 \text{ V} - 6 \text{ V} = 18 \text{ V}$$

Solution:

$$R_2 = \frac{U_2}{I}$$

$$R_2 = \frac{18 \text{ V}}{0.5 \text{ A}}$$

$$R_2 = 36 \Omega$$

=====

$$R_{\text{equ}_1} = \frac{U_1}{I}$$

$$R_{\text{equ}_1} = \frac{6 \text{ V}}{0.5 \text{ A}} = 12 \Omega \quad R_{\text{equ}_1} = \frac{1}{R_1} + \frac{1}{R_V} \quad \text{inverted for } R_1$$

$$\frac{1}{R_1} = \frac{1}{R_{\text{equ}_1}} - \frac{1}{R_V} = \frac{R_V - R_{\text{equ}_1}}{R_V R_{\text{equ}_1}}$$

$$R_1 = \frac{R_V R_{\text{equ}_1}}{R_V - R_{\text{equ}_1}}$$

$$R_1 = \frac{38 \Omega \cdot 12 \Omega}{30 \Omega - 12 \Omega} = \frac{360}{18} \Omega$$

$$R_1 = 20 \Omega$$

Verification by means of equation (3.10.):

$$U_1 = U \frac{R_{\text{equ}_1}}{R_{\text{equ}}}$$

$$U_1 = 25 \text{ V} \frac{12 \Omega}{12 \Omega + 36 \Omega} = 24 \text{ V} \frac{12}{48}$$

$$U_1 = 6 \text{ V}$$

=====

If the load had not been connected, then for the selected divider resistances the voltage dropping via  $R_1$  would have been greater and, according to equation (3.9.b) would be

$$U_1 = U \frac{R_1}{R_{\text{equ}}}$$

$$U_1 = 24 \text{ V} \frac{20}{36} = \frac{40}{3} \text{ V}$$

$$U_1 = 13.3 \text{ V}$$

=====

According to their circuit structures, a distinction is made between branched, unbranched and meshed circuits. In branched circuits all loads are connected in parallel. Their equivalent conductance is equal to the sum of the individual conductances, hence, always greater than the greatest individual conductance. The equivalent resistance, however, is always smaller than the smallest individual resistance.

$$G_{\text{equ}} = G_1 + G_2 + G_3 + \dots$$

$$\frac{1}{R_{\text{equ}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

The voltage across all loads is equal. At each junction point, the sum of the currents flowing toward the point is equal to the sum of those flowing away from it.

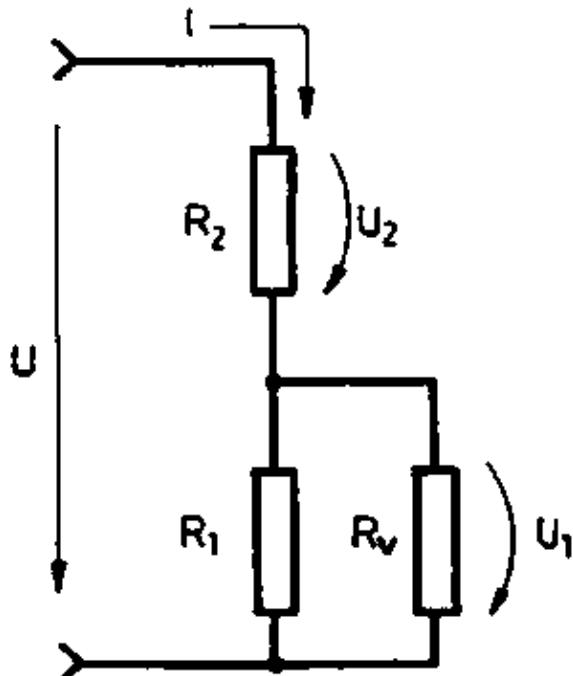


Fig. 3.13. Circuit for example 3.6.

$$I = I_1 + I_2 + I_3 + \dots$$

The division of the voltage by the loads is effected in the same ratio as that of their conductances.

$$\frac{I_1}{I} = \frac{G_1}{G_{\text{equ}}}, \quad \frac{I_2}{I} = \frac{G_2}{G_{\text{equ}}}, \quad \frac{I_3}{I} = \frac{G_3}{G_{\text{equ}}} \quad \text{etc.}$$

In unbranched circuits, all loads are connected in series. Their equivalent resistance is equal to the sum of the individual resistances, hence always greater than the greatest individual resistance.

$$R_{\text{equ}} = R_1 + R_2 + R_3 + \dots$$

The current is equal through all loads. The total voltage is equal to the sum of all individual voltage drops.

$$U = U_1 + U_2 + U_3 + \dots$$

Voltage division by the loads is effected in the same ratio as that of their resistances. etc.

$$\frac{U_1}{U} = \frac{R_1}{R_{\text{equ}}}, \quad \frac{U_2}{U} = \frac{R_2}{R_{\text{equ}}}, \quad \frac{U_3}{U} = \frac{R_3}{R_{\text{equ}}} \quad \text{etc.}$$

In meshed circuits, some of the loads are connected in parallel and some of them in series. Such a circuit is called series-parallel connection. For computing or designing a series-parallel connection, the laws at the bottom of both branched and unbranched circuits have to be taken into account. A series-parallel connection of particular importance to electrical engineering is the loaded voltage divider.

Questions and problems:

1. The following loads are connected in parallel to a voltage of 110 V: 1 soldering iron having 200 ?, 1 incandescent lamp having 120 ?, 1 heater having 24 ?. Calculate the partial currents and, with the help of the 1st Kirchhoff's law, the total current. Check the result according to Ohm's law after having determined the equivalent resistance of the three loads connected in parallel.

2. By means of a certain ammeter, a current of maximum  $I_i = 10 \text{ mA}$  can be measured. Assume the resistance of the instrument is  $R_i = 10 \Omega$ . In order to measure a higher current, for example, a current of  $I = 100 \text{ mA}$ , a resistor of a suitable resistance value must be connected in parallel through which (that is to say, past the instrument) a current of  $90 \text{ mA}$  (in the case of full-scale deflection) or, in general, of  $I - I_i$  can flow. Calculate the value of the parallel resistance for the conditions given above.

3. In an electronic device, three resistors are connected in series to a voltage of  $12 \text{ V}$ :  $R_1 = 360 \Omega$ ,  $R_2 = 240 \Omega$ ,  $R_3 = 120 \Omega$ . Calculate the equivalent resistance, the current and the partial voltage dropping via  $R_1$  to  $R_3$ .

4. By means of a certain voltmeter, a maximum voltage of  $U_i = 15 \text{ mV}$  can only be measured. Assume the resistance of the instrument is  $R_i = 5 \Omega$ . In order to be in a position to measure a higher voltage, for example, a voltage of  $U = 3 \text{ V}$ , a resistor of a suitable resistance value must be connected in series with the voltmeter. Via the latter, the excessive voltage of  $2.985 \text{ V}$  (in the case of full-scale deflection) or, in general, of  $U - U_i$  can drop. Calculate the value of the series resistor for the conditions specified above.

5. Two resistors  $R_1 = 18 \text{ k}\Omega$  and  $R_2 = 36 \text{ k}\Omega$  are connected in series and to a voltage of  $U = 110 \text{ V}$ . A load of  $R_v = 10 \text{ k}\Omega$  is connected in parallel to  $R_1$ . Calculate the voltage at  $R_v$ , the total current and the partial currents.

## 4. Electrical Energy

### 4.1. Energy and Power

The electrical energy is used advantageously for the drive of machines, for lighting and other purposes. Here, a study is made to find out how the quantities current, voltage and resistance discussed above can be used to determine the converted electrical energy or the available electrical power. For this purpose we should remember the comparison of the electrical circuit and the circulation of water. When a pump is used to pump water from a vessel arranged at a lower level into a vessel at a higher level, energy must be expended. When the water is allowed to flow from the upper to the lower vessel, the energy expended originally, actually fed into the water, will be released. The magnitude of the energy of the water is dependent on the weight of the water and the difference between the two levels. The following holds

$$W = G h \quad (4.1.)$$

where:

G weight  
h difference in height

When comparing the weight G with the quantity of electricity Q and the height with the voltage U, we have for the electrical energy

$$W = Q U$$

Since  $Q = I t$ , we have for the electrical energy  $W = U I t$  (4.2.)

The energy stored in the upper water basin can flow down in different times. The energy conversion related to time is called power.

$$P = \frac{W}{t} = \frac{U I t}{t} \quad (4.3.)$$

$$P = U I$$

This is illustrated by an example. When I fill the upper water basin by means of a hand pump, so I can do this during a long period of time without particular effort or in a very short time exerting myself. The shorter the time, the greater the energy I have to expend or, in other words, the higher the power attained. The unit of power is expressed as

$$[P] = [U] [I]$$

$$[P] = V \cdot A \quad \text{and from } 1 \text{ V} \cdot \text{A} = 1 \text{ W follows}$$

$$[P] = W$$

The product  $V \cdot A$  is called watt in honour of the English physicist James Watt (1736 – 1819).

Thus, for the unit of the electrical energy we have

$$[W] = [U] [I] [t]$$

$$[W] = V \cdot A \cdot b$$

$$[W] = W \cdot s$$

The energy unit Nm used in mechanics and the energy unit J used in heat engineering are of the same magnitude as the Ws.

$$1 \text{ Ws} =; \text{Nm} = 1 \text{ J}$$

where:

W watt

s second

N Newton

m metre

J joule

Since a watt-second is a very small energy unit and in most cases the operating times of electrical equipment amounts to many hours, the kWh (kilowatt-hour) is also used as energy unit. Thus, we have

$$1 \text{ kWh} = 3,600,000 \text{ Ws} = 3.6 \text{ MWs}$$

With the help of Ohm's law, the equations (4.2.) and (4.3.) can be written in the following form

$$W = UIt = I^2 R t_2 = \frac{U^2}{R} t$$

$$P = UI = I^2 R = \frac{U^2}{R}$$

In accordance with the great variety of electrical devices used in practice, the magnitude of the power input ranges from very small values to very great values. Table 4.1. shows some examples.

Table 4.1. Power Input to Selected Electrical Devices

Device	mean power input
electronic pocket computer	?W
headphone	mW
loudspeaker	5 W
incandescent lamp	60 W
ventilator	100 W

motors at machines	1 kW
motor of electric locomotive	1 MW
power station generator	100 MW

Example 4.1.

In the supply line for a soldering iron connected to 220 V, a current of 0.2 A is measured. What is the power input to the soldering iron? What is the amount of energy converted within 8 hours?

Given:  $U = 220 \text{ V}$  To be found:  $P$  in W

$$I = 0.2 \text{ A} \quad W \text{ in kWh}$$

$$t = 8 \text{ h}$$

Solution:  $P = U I$

$$P = 220 \text{ V} \cdot 0.2 \text{ A}$$

$$P = 44 \text{ W}$$

=====

$$W = U I t$$

$$W = 220 \text{ V} \cdot 0.2 \text{ A} \cdot 8 \text{ h}$$

$$W = 352 \text{ Wh}$$

$$W = 0.352 \text{ kWh}$$

=====

The energy input to the soldering iron is 44 W. Within 8 hours, an energy of 0.352 kWh is converted.

Example 4.2.

An electrical hardening furnace having a resistance of  $20 \Omega$  requires a current input of 5 A. Calculate the electrical energy consumed within a period of 24 h.

Given:  $R = 20 \Omega$  To be found:  $W$  in kWh

$$I = 5 \text{ A}$$

$$t = 24 \text{ h}$$

Solution:  $W = I^2 R t$

$$W = 5\text{A} \cdot 5\text{A} \cdot 20 \Omega \cdot 24 \text{ h}$$

$$W = 12 \text{ kWh}$$

=====

Within 24 hours, the hardening furnace consumes 12 kWh of electrical energy.

Example 4.3.

The circuit of a washing machine with the rating-plate markings  $P = 2 \text{ kW}$  and  $U = 220 \text{ V}$  is to be provided with fuses. Find out whether a fuse with the rated current of 6 A will be sufficient.

Given:  $P = 2 \text{ kW}$  To be found:  $I$  in A

$$U = 220 \text{ V}$$

Solution:  $P = U I$

$$I = \frac{P}{U}$$

$$I = \frac{2000W}{220V}$$

$$I = 9.09 A$$

The current input to the washing machine is 9.09 A; therefore, a fuse with the rating of 6 A will not suffice. A fuse having a rated current of 10 A must be used.

The electrical energy is calculated according to  $W = U I t$  and the electrical power according to  $P = U I$ . As unit for the energy, the Ws has been laid down, while the greater unit kWh may be used when required. The unit of power is W. The relation with other energy units is  $1 Ws = 1 Nm = 1 J$ .

Questions and problems:

1. Derive from the relations  $P = U I$  and  $W = U I t$  further formulas, taking the resistance into account.
2. Gather information about the price of 1 kWh.
3. How great is the energy in Nm corresponding to one kWh?
4. Using the markings on the rating-plate of various technical devices (P and U), determine the resistance of these devices.
5. What is the current input to a washing machine connected to 220 V having a power input of 2.2 kW?
6. What is the time for which the washing machine mentioned in problem 5. has been operated when the electric meter indicates a consumption of 5 kWh?

## 4.2. Efficiency

A conversion of energy without loss is not possible. For example, the electrical energy fed to a motor is converted not only into mechanical energy but also in heat due to the rise in temperature of the motor. Since this heating is not desired, this portion of the fed electrical energy which is converted into heat energy is called energy loss or lost energy. The efficiency is defined as the ratio of the energy delivered by the device to the energy supplied to it.

$$\eta = \frac{W_e}{W_i} = \frac{P_e t}{P_i t} = \frac{P_e}{P_i} \quad (4.4.)$$

where:

?<sup>1)</sup> efficiency

$W_e$  effective energy (energy delivered)

$W_i$  indicated energy (energy supplied)

$P_e$  effective power

$P_i$  indicated power

<sup>1)</sup> ? Greek letter eta

Since the delivered energy is always smaller by the lost energy than the supplied energy, the efficiency is always smaller than 1. According to equation (4.4.), the same statement applies to power. When the losses are small, the efficiency will have a high value. The developmental level of a device is substantially determined by the magnitude of the efficiency. Great efforts are made to further improve the efficiency in order to convert the supplied energy into the desired energy with losses as small as possible.

Table 4.2. shows a few typical examples of the values involved.

Table 4.2. Efficiency of Selected Technical Equipment

Equipment	mean efficiency
incandescent lamp	0.05
steam locomotive	0.2
small electric motor	0.5
large electric motor	0.85
transformer	0.95
power station generator	0.98

In many cases, several devices having a certain efficiency each are connected together and then the total efficiency of the arrangement is of interest. Fig. 4.1. shows an example. The arrangement shown may be, for example, a motor generator where device A is the electric motor, which takes up  $P_{\text{supplied}1}$  as electrical power and delivers  $P_{\text{delivered}1}$  as mechanical power. At the same time  $P_{\text{delivered}1}$  is the drive power supplied to the generator (device B) designated as  $P_{\text{supplied}2}$ . The power delivered by the generator is designated as  $P_{\text{delivered}2}$ . The motor has the efficiency  $\eta_1$  and the generator the efficiency  $\eta_2$ . The total efficiency is expressed as

$$\eta = \frac{P_{\text{delivered}2}}{P_{\text{supplied}1}}$$

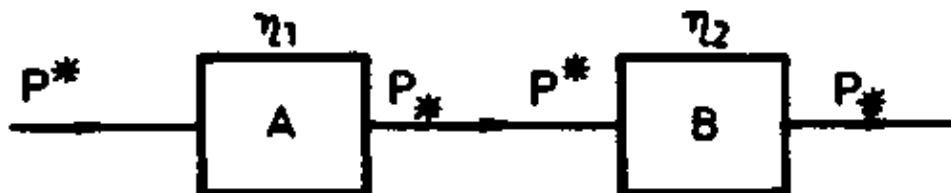


Fig. 4.1. Interaction of two technical devices

$$P = P_{zu}; P = P_{ab}$$

$$\eta_2 = \frac{P_{\text{delivered}2}}{P_{\text{supplied}2}} \quad \text{for } P_{\text{supplied}2} \quad \text{and} \quad \eta_1 = \frac{P_{\text{delivered}1}}{P_{\text{supplied}1}} \quad \text{for } P_{\text{supplied}1}$$

When inverting the relation  $P_{\text{supplied}1} = \frac{P_{\text{delivered}1}}{\eta_1}$  we obtain  $P_{\text{supplied}2} = \eta_2 P_{\text{delivered}2}$

Substituted into the initial equation we obtain

$$\eta = \frac{\eta_2 P_{\text{supplied}2} \eta_1}{P_{\text{delivered}1}}$$

Since, however,  $P_{\text{delivered}1} = P_{\text{supplied}2}$ , it follows that

$$\eta = \eta_1 \eta_2 \quad (4.5.)$$

This shows that the total efficiency is equal to the product of the individual efficiencies and, thus, always smaller than the smallest individual efficiency.

#### Example 4.4.

The motor of a motor generator has an efficiency of 0.8 and the generator an efficiency of 0.75. What is the total efficiency?

Given:  $\eta_1 = 0.8$  To be found:  $\eta$

$$\eta_2 = 0.75$$

Solution:  $\eta = \eta_1 \cdot \eta_2$   
 $\eta = 0.8 \cdot 0.75$   
 $\eta = 0.6$

=====

The total efficiency is 0.6.

In any energy conversion process, losses occur. This fact is described by the efficiency. The conversion losses should be as small as possible; this is expressed by a value of the efficiency near 1. The efficiency is always smaller than 1. The total efficiency is the product of the individual efficiencies.

Questions and problems:

1. Why calls technical progress for an increase in the efficiency?
2. Give proof of the fact that for three devices connected together the total efficiency is:  $\eta = \eta_1 \cdot \eta_2 \cdot \eta_3$ .
3. A motor delivers a mechanical power of 650 W. What is its efficiency when the current input is 3.5 A at a voltage of 220 V?
4. A motor generator has an input of 3 A while connected to a voltage of 220 V and delivers a voltage of 48 V to the generator. The motor has an efficiency of 0.8 and the generator of 0.78. What is the current drawn from the generator?

### 4.3. Conversion of Electrical Energy into Heat

The electrical energy in a resistor through which current passes ( $W = I^2 R t$ ) is converted into heat energy. According to the law of conservation of energy, the consumed electrical energy is equal to the generated heat energy. This correlation is also clearly described by the energy units of the international system of units SI. For the heat energy, the unit J (joule)<sup>1)</sup> is used.

<sup>1)</sup> Joule, English physicist 1818 – 1889

The following holds

$$1 \text{ Ws} = 1 \text{ J}$$
$$1 \text{ kWh} = 3.6 \text{ MJ}$$

The formerly used unit for heat energy, cal (calorie), is no longer permitted;  $1 \text{ Ws} = 0.239 \text{ cal}$ .

Since any conductor (wire) has a resistance, in every conductor through which current passes electrical energy is converted in heat energy and, consequently, the conductor is heated. This phenomenon is a disturbing factor in supply lines to electrical equipment and in windings of motors and transformers. Therefore, lines included in an electrical installation should not be subjected to high currents or any value. When the current is too high, the rise in temperature will become excessive so that the insulation may be destroyed or the line catch fire. Depending on the method of the installation of the line, the maximum permissible current intensity for a certain cross-sectional area of the conductor can be drawn from special Tables. Table 4.3. gives some information in this respect.

Table 4.3. Power Rating of Moisture-proof Cables

Cross-sectional area of conductor in mm <sup>2</sup>	1.5	2.5	4	6
maximum continuous current in A				
copper	20	27	36	47
aluminium	—	21	29	37
rated safety current in A				
copper	20	25	35	50
aluminium	—	20	25	35
voltage drop per 100 m of distance in V				
copper	47.6	38.5	32.2	28
aluminium	—	47.3	41	34.8

For example, a multi-core moisture-proof cable of copper whose individual conductors have a cross-sectional area of 1.5 mm<sup>2</sup> may be loaded with a current of up to 20 A. Considerable voltage drops occur which are dependent on the length of the conductor and which may be so detrimental that in many cases the choice of a greater cross-sectional area is necessary. For the above line, a length between feed point and consumer of 100 m would cause the extremely high voltage drop of 47.6 V when loaded with the maximum permanent current.

#### Example 4.5.

A load having a current input of 15 A is to be connected to a moisture-proof cable of copper of a length of 70 m. The voltage drop should not exceed 20 V. Determine the required cross-sectional area of the line.

Given: S = 70 m

To be found: A in mm<sup>2</sup>

$$I = 15 \text{ A}$$

$$U = 20 \text{ V}$$

$$\rho_{\text{Cu}} = 0.0178 \frac{\text{mm}^2 \cdot \Omega}{\text{m}}$$

Solution:

From the Table, the minimum permissible cross-sectional area of the conductor can be drawn which is A = 1.5 mm<sup>2</sup>. Check that the voltage drop in the line will not become too great.

$$R = \frac{\rho l}{A}$$

$$A = \frac{\rho l}{R} \quad \text{with } R = U / I$$

$$R = \frac{\rho l I}{U}$$

For the length, twice the distance  $s$  must be used taking the outgoing line and the return line into consideration.

$$A = \frac{0.0178 \Omega \text{ mm}^2 \cdot 140\text{m} \cdot 15\text{A}}{\text{m} \cdot 20\text{V}}$$

$$A = 1.87 \text{ mm}^2$$

=====

A cross-sectional area of  $2.5 \text{ mm}^2$  of the conductor must be chosen.

For windings of motors and transformers, the heat dissipation from the winding wires to the environment is worse than in installation lines. Therefore, the individual wire cross-sections should be loaded with an adequately lower current than installation lines. For this purpose, there are Tables, too, from which the required conductor cross-sectional area can be drawn and in which frequently the maximum permissible current density  $S$  is stated.

$$S = \frac{I}{A} \quad [S] = \frac{A}{\text{mm}^2} \quad (4.6.)$$

where:

$S$  current density

$I$  current intensity

$A$  cross-sectional area of the conductor

In screwed, plugged or clamped connections high transition resistances occur due to insufficient contact pressure or dirty surfaces, these connections will be subjected to particularly high rises in temperature which may lead to the destruction of the connection or the line may catch fire.

Therefore, great care must be taken when preparing such connections and one must see that the contact resistance is kept low. All connections have to be checked at regular intervals and when an impermissible rise in temperature is observed, the cause must be removed immediately.

Besides the given examples of an undesired development of heat, in many cases the heating effect of the electrical current is ingeniously used. In order to protect lines and equipment from currents of impermissible high values, various types of safety devices are used. As to the safety fuse (Fig. 4.2.), a particularly thin wire is used inside the fuse which is heated up to the melting point when the rated current of the fuse is reached.

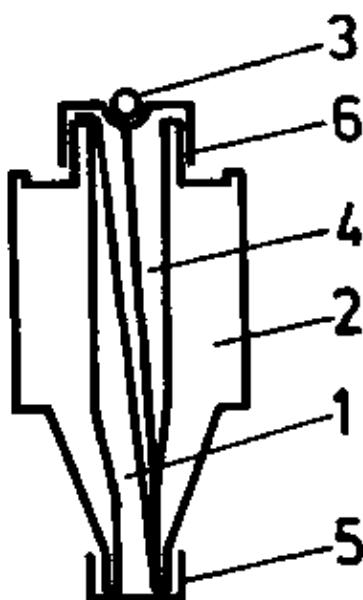


Fig. 4.2. Sectional view of a fuse

1 Fusible wire; 2 Ceramic body; 3 Springy clamping plate; 4 Quartz sand filling; 5 Foot contact; 6 Head contact

As a consequence, the circuit is interrupted. The springy small identification plate is detached and thus shows that the fuse is blown. As to the bimetal safety device, a strip of two different metals on which a resistance winding is arranged, through which the current to be controlled flows, is used. In the case of heating, the bimetallic strip shown in Fig. 4.3. is bent and, consequently, the circuit is interrupted by means of a switching mechanism not shown in this illustration. The heated strip is bent because the metals used expand to different degrees when subjected to heat. When, for example, the lower metal in Fig. 4.3. will bend to a higher degree than the upper one, the strip will bend upwards.

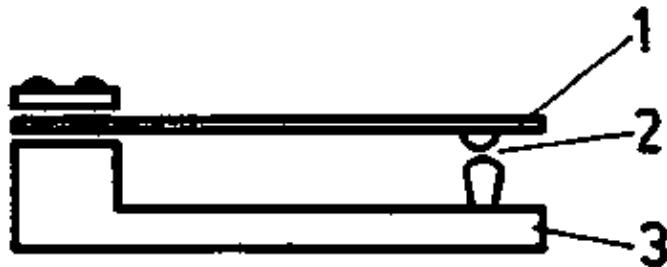


Fig. 4.3. Bimetal safety device

1 Bimetallic strip; 2 Contact; 3 Carrying body

In heat-generating electrical devices usually a spiral of resistance wire is embedded in a ceramic carrier body. In this way, heating cartridges of different shapes are made which are used for electrical soldering irons, boiling plates, immersion heaters and heating inserts for other heat-generating electrical appliances.

Another important use of electric heat is in the process of spot welding (Fig. 4.4.). Two electrodes of copper properly press the parts of sheet metal together. Then current is switched on which heats the joint area to such an extend that the sheet metal parts are welded together. The great heat involved in an electric arc is utilised for technical purposes. Thus, in properly closed vessels a high-grade melt of a metal can be prepared (Fig. 4.5.) or arc welding can be carried out, a process which is widely used today.

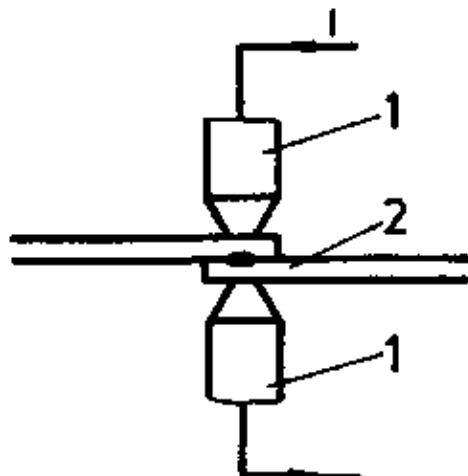


Fig. 4.4. Spot welding

1 Copper electrodes; 2 Sheet-metal parts to be welded

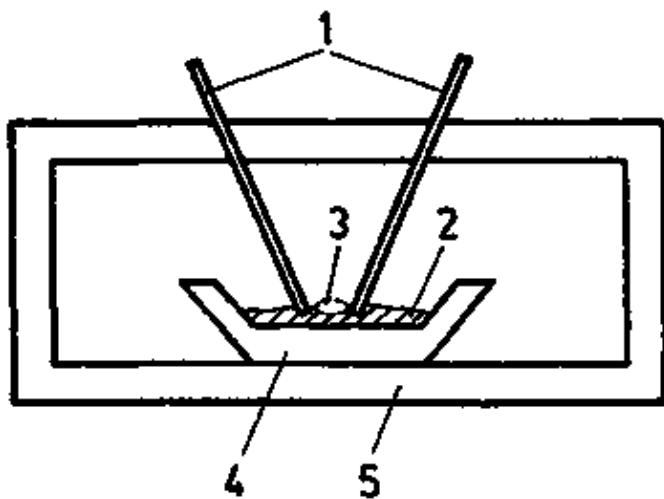


Fig. 4.5. Arc furnace

1 Electrodes; 2 Melt; 3 Electric arc; 4 Melting crucible; 5 Vessel

#### Example 4.6.

In a thermal storage water heater, 5 l of water (this equal to a mass  $m$  of about 5 kg) with a temperature of 20 °C is to be heated up to the boiling point. The heating coil has a power of 1000 W. For energy conversion, an efficiency of 0.9 is assumed. The specific heat of the water  $c$  is drawn from a table. After which time will the water boil after switching on?

Given:  $V = 5 \text{ l}$  To be found:  $t$  in min

$$m = 5 \text{ kg}$$

$$\vartheta_1 = 20 \text{ } ^\circ\text{C}$$

$$\vartheta_2 = 100 \text{ } ^\circ\text{C}$$

$$P = 1000 \text{ W}$$

$$\eta = 0.9$$

$$c = 4.19 \frac{\text{kJ}}{\text{K} \cdot \text{kg}}$$

Solution:

Starting point of the solution is the fact that the required heat energy must be equal to the electrical energy converted into heat which corresponds to the supplied electrical energy multiplied by the efficiency.

$$W_Q = W_{\text{el}} \cdot \eta$$

With  $W_Q = m c (\vartheta_2 - \vartheta_1)$  and  $W_{\text{el}} = P t$  we have  
 $m c (\vartheta_2 - \vartheta_1) = P t \cdot \eta$

Now the equation is inverted with respect to the quantity to be found, namely,

$$t = \frac{m c (\vartheta_2 - \vartheta_1)}{P \eta}$$

$$t = \frac{5 \text{ kg} \cdot 4.19 \text{ kJ} \cdot (100 \text{ } ^\circ\text{C} - 20 \text{ } ^\circ\text{C})}{\text{K} \cdot \text{kg} \cdot 1 \text{ kW} \cdot 0.9}$$

$$t = \frac{5 \cdot 4.19 \text{ Ws} \cdot 80 \text{ K}}{\text{K} \cdot 1 \text{ W} \cdot 0.9}$$

$$t = 1862 \text{ s}$$

$$t = 31 \text{ min}$$

=====

After 31 minutes the water will boil in the thermal storage water heater.

In each resistor through which current passes, electrical energy is converted into heat energy. The rise in temperature involved is not desired in all electrical installations and windings of motors and transformers. Advantage is taken of this heat in all heat-generating devices (soldering iron, boiling plate, hardening furnace, melting furnace) and in the process of electric welding. The action of safety devices frequently is based on the heating when current passes through them.

Questions and problems:

1. Describe further examples of the utilisation of the heat effect of electrical current!
2. Why are fuses or other safety devices required in circuits?
3. Describe and substantiate the consequence of excessively high contact resistances!
4. Explain the mode of action of the fuse and the bimetallic safety device!
5. Calculate the voltage drop in a 50 m long lead of copper cores having a cross-sectional area of  $1.5^2$  mm to a load with a current input of 15 A!
6. By means of an immersion heater of 1000 W, 5 l of water are heated from an initial temperature of 20°C. What is the temperature of the water after 15 min when an efficiency of 0.9 is assumed for the energy conversion?

#### 4.4. Conversion of Electrical Energy into Mechanical Energy

The conversion of electrical energy into mechanical energy is a frequent utilisation of electrical energy. Thus, electric motors are used for moving air in air-conditioning plants, in water pumps, for the drive of machines and vehicles, and many other electrical equipment. The fact that electrical energy is easily convertible into mechanical energy and the possibility of adapting the motor to be used to the given technical task have led to the preferred use of electric motors in practice. In most cases, electric motors are considerably cheaper and require less maintenance than engines. For the various applications, types of electromotors are available which are optimally adapted to the purpose.

Let us consider at first a few important physical quantities of rotary motion. The shaft of a motor rotates at a certain rotational speed  $n$  which is usually measured in revolutions per minute (rpm). Fig. 4.6. shows a belt drive. The smaller belt pulley is mounted on the shaft of the motor and the larger pulley on the machine to be driven. The V-belt transmits the force from the motor to the machine. The force transmitted by the V-belt is an essential quantity for computation. The force multiplied by the radius  $r$  is called torque  $M$ .

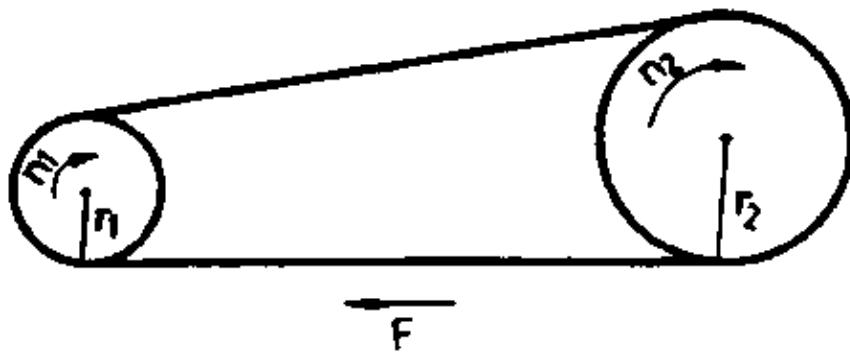


Fig. 4.6. Belt drive

$$M = F \cdot r \quad [M] = N \cdot m \quad (4.7.)$$

where:

M torque  
F force  
r radius

The sketch shows that the torque acting at the belt pulley with the radius  $r_1$  is smaller than the torque at the belt pulley with the larger radius  $r_2$ . For the rotational speeds of the belt pulleys the relation

$$n_1 : n_2 = r_2 : r_1 \text{ holds.} \quad (4.8.)$$

where:

n speed of rotation  
r radius

The mechanical power delivered by the motor is expressed by the following formula:

$$P = 2 \cdot \pi^1 M n \quad [P] = \frac{\text{Nm}}{\text{s}} = W \quad (4.9.)$$

<sup>1)</sup>  $\pi$  Greek letter pi

where:

P power  
M torque  
n speed of rotation

Example 4.7.

On the belt pulley of a motor having a radius of 50 mm, a force of 20 N is acting. The rotational speed is 1000 rpm. What is the power delivered by the motor?

Given:  $r = 50 \text{ mm}$

To be found: P in W

$F = 20 \text{ N}$

$n = 1000 \text{ rpm}$

Solution:  $P = 2 M n$

$$P = 2 F r n$$

$$P = 2 \cdot 3.14 \cdot 20 \text{ N} \cdot 50 \text{ mm} \cdot 1000 \text{ rpm}$$

$$P = 2 \cdot 3.14 \cdot 20 \text{ N} \cdot 0.05 \text{ m} \cdot \frac{1000}{60} \frac{1}{\text{s}}$$

$$P = 105 \text{ W}$$

=====

The power delivered by the motor is 105 W.

An important item for judging the performance of a motor is the dependence of the rotational speed on the torque (and thus on the load) and the current path at the instant of switching on. The dependence of the efficiency on the torque should also be taken into consideration for an appropriate use. In Fig. 4.7. these dependencies are represented for a three-phase squirrel-cage induction motor. This motor, which is the most widely used type, is shown in Fig. 4.8. The characteristics show that the rotational speed is reduced only slightly under load. This property is desired in many drives. When, however, the stalling torque – also known as pull-out torque –  $M_k$  is reached, which is above the rated torque  $M_n$ , the motor will stall suddenly.

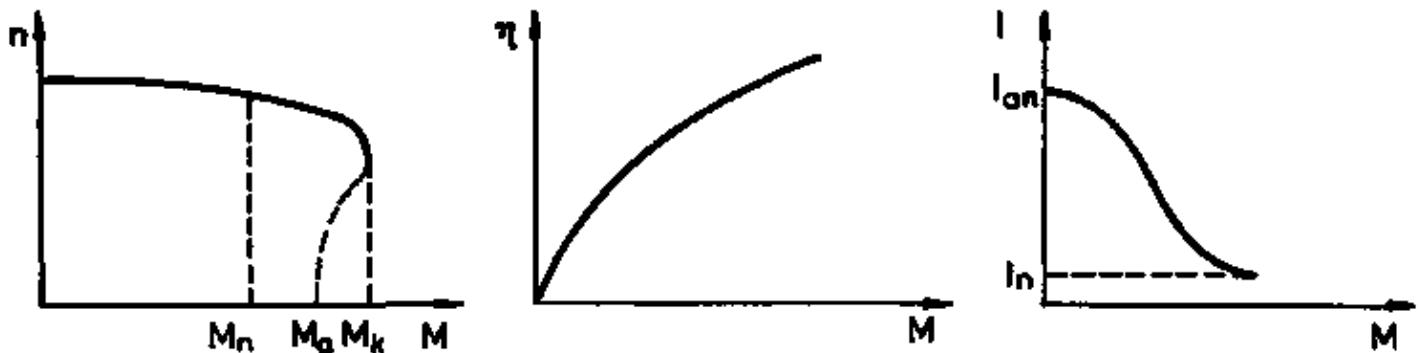


Fig. 4.7. Characteristics of the three-phase asynchronous motor

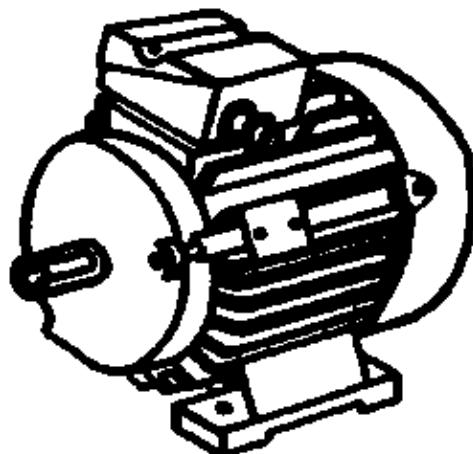


Fig. 4.8. Three-phase squirrel cage induction motor

At the instant of switching on, only the starting torque  $M_q$  is available which is frequently insufficient for heavy drives to cause the motor to start up. Additional measures are then required in order to allow the motor to start up without load at first. This motor is not suited as a drive motor for vehicles because it cannot produce the required starting moment. The efficiency  $\eta$  increases with increasing load. In order to operate the motor with as high an efficiency as possible, a load with the rated moment should be effected. Therefore, the motor power for drives should be chosen according to the required power. It should be noted that at the instant of switching on the input of current  $I_{an}$  is several times higher than for rated operation  $I_n$ . The fuses in series with the motor must comply with these conditions. Frequently switching on and off is unfavourable except for special motors.

As a drive motor for vehicles, the series-wound motor is particularly suitable. The dependence of the rotational speed on the torque is shown in Fig. 4.9. It is evident that the motor produces a very high torque at a low rotational speed; this is conducive to the starting of electric vehicles.

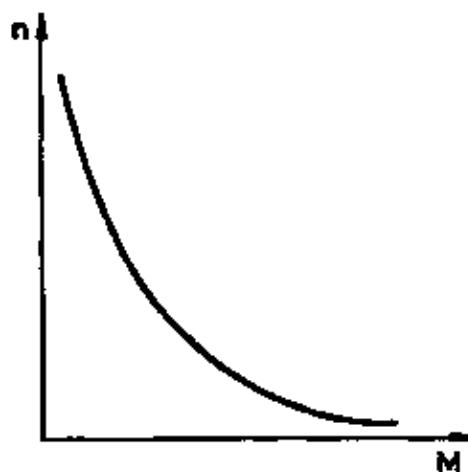


Fig. 4.9. Characteristic of the series motor

If, for example, a squirrel-cage induction motor is to be used for driving a centrifugal pump which is frequently used as a water pump, a motor having a suitable power must be selected. For this purpose, first we have to know the dependence of the required torque on the rotational speed of a centrifugal pump (Fig. 4.10.). The point of intersection of the pump characteristics and the motor characteristics should coincide with the rated torque of the motor.

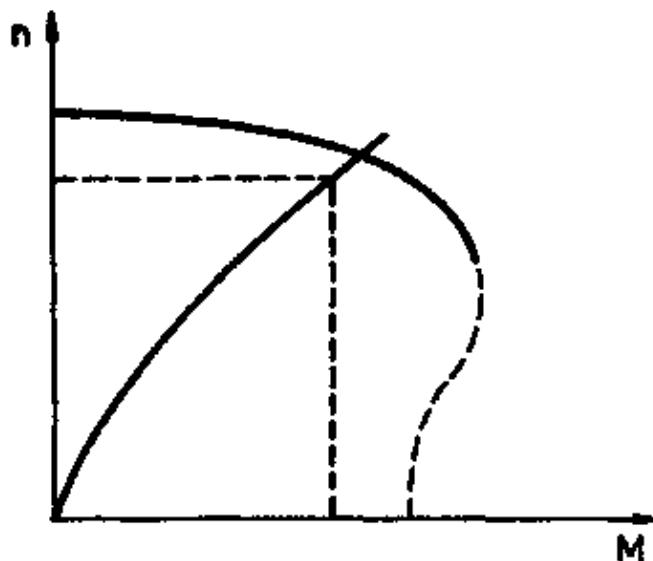


Fig. 4.10. Interaction of a centrifugal pump and a three-phase squirrel-cage induction motor

For many drives, the electromotor is an ideal solution. Its construction is sturdy and it calls for limited maintenance only. Various designs enable an optimum adaptation to the different drive requirements in the various fields of application. The mechanical power delivered by the motor is determined by the rotational speed and the torque. Every motor has its typical characteristics from which conclusions can be drawn as to the practical use.

Questions and problems:

1. Quote some examples of devices which are operated by means of electric motors!
2. Calculate the power delivered by a motor when its belt pulley having a diameter of 100 mm rotates at a speed of 1350 rpm and a force of 50 N acts on the pulley!
3. Why is a three-phase squirrel-cage induction motor not suitable for driving electric vehicles!
4. Why is a series-wound motor not suitable for driving a drilling machine?

## 4.5. Conversion of Electrical Energy into Light

### 4.5.1. Fundamentals of Illumination Engineering

The greater part of our duties can only be performed with the help of our eyes. Adequate light is also required. The hours in the daytime alone are no longer sufficient for production; in large factory halls, the daylight is insufficient anyway. Artificial lighting is indispensable. Zest for work and labour productivity are largely depending on the quality of a lighting system. The influence of light faultlessness and quality of work is considerably. When the lighting is good, the danger of accidents is diminished. Precision work can only be performed with sufficient light.

Illumination engineering is a comprehensive special field. Here, the most essential fundamentals will be explained which are indispensable for understanding technical data of light sources and a few principles of lighting. Light in the physical sense is a form of energy of matter. In the sense of illumination engineering, the

exact amount of energy is less interesting than the brightness perceived by our eyes. Below, all statements are related to perception or sensation – the physical quantities are provided with an index in the form o a v<sup>1)</sup>.

<sup>1)</sup> v: visual – relating to vision

The known white daylight consists of many light colours, the light spectrum which becomes visible in a rainbow or prismatic ground glasses. The following spectral colours are included: violet, blue, green, yellow, orange, red. The uniform mixture of these colours produces the sensation of white light in the eyes – without optical means, the individual constituents of light cannot be discerned. In the form of energy radiation, which is not perceivable by our eyes, the ultraviolet radiation is adjacent to violet and the heat radiation (infrared) adjacent to red. Since all light colours are contained in daylight, we speak of a continuous spectrum. The artificial light sources produced by man do not emit the individual spectral colours in the same composition as in the daylight and sometimes a few spectral colours are even missing.

We perceive an object as coloured only because it reflects of the colours present in the spectrum only that part which corresponds to its colour and absorbs all of the other spectral colours. From this also follows that the object can only look red when red is contained in the light. In contrast to daylight, some of the artificial sources of light have a smaller proportion of red in their spectrum and that is why the red object illuminated by such light will not appear as red but grey to black. Therefore, the colour edition is an important factor in the evaluation of light sources.

The most important quantity to be measured of light is the luminous flux  $\Phi_v$ . It comprises the whole light power radiated from a light source to all directions of space. Judged by the perception by our eyes, this light power is measured in 1m (lumen). This light power radiated by a lamp and stated in 1m should not be confused with the electrical power (stated in V) taken up by the lamp. On the other hand, the ratio of the emitted light power to the electrical power consumed – the luminous efficiency – is of particular interest; though it physically

$$\frac{\text{lm}}{\text{W}}$$

corresponds to the efficiency, it is here stated in terms of ; this unit is used because the sense–organ eye takes part in the evaluation.

$$\eta = \frac{\Phi_v}{P_{el}} \quad [\eta] = \frac{\text{lm}}{\text{W}} \quad (4.10.)$$

where:

? luminous efficiency

$\Phi_v$  light flux

$P_{el}$  electrical power

The development of light sources is oriented toward an increase in the luminous efficiency. When the first

incandescent lamps only offered about  $2 \frac{\text{lm}}{\text{W}}$ , sodium-vapour high-pressure lamps attain about today.  $100 \frac{\text{lm}}{\text{W}}$

For the evaluation of the brightness, the illumination intensity E is used as an approximate quantity. It indicates the part of the luminous flux incident on a certain area and is measured in lux (lx).

$$E = \frac{\Phi_v}{A} \quad [E] = \text{lx} \quad (4.11.)$$

where:

E illumination intensity

$\Phi_v$  light flux

A Area

The following examples will enable an imagination of the magnitudes involved:

full moon      0.1      lx

working room 300 lx

midday sunlight 100,000 lx

Within this huge range of illumination intensity, the human eye enables optical sensation. Depending on the problem of vision, certain values of illumination intensity are required which are laid down in the relevant legal regulations. As an example, Table 4.4. is given. In order to attain optimum conditions for the solution of the problem of vision and to ensure the necessary expenditure of energy for the illumination involved, great care must be taken for the determination of the required illumination intensity.

Table 4.4. Values of Intensities of Illumination for Various Tasks

<b>Demands of work on illumination</b>	<b>mean intensity of illumination in lx</b>
extraordinarily fine work	1500 to 3000
very fine work	500 to 1500
fine work	200 to 500
medium-fine work	100 to 200
rough work	100
very rough work	50

#### 4.5.2. Light Sources

The triumphant advance of electrical light sources began with the first carbon-filament lamp made by Edison in 1879. Today, a great variety of lamps with different properties is available for the most diversified fields of application. Table 4.5. gives a selection including some important technical data.

Table 4.5. Important Technical Data of Electrical Sources of Light

<b>Light source</b>	<b>Power steps in W manufactured</b>	<b>Luminous efficiency <math>\frac{\text{lm}}{\text{W}}</math></b>	<b>Service life in hours of operation</b>
incandescent lamp	25 to 1,000	10 to 15	1,000
halogen incandescent lamp	10 to 5,000	16.7 to 22	25 to 2,000
fluorescent lamp	8 to 65	30 to 55	4,000 to 10,000
mercury-vapour high-pressure lamp	50 to 1,000	32 to 54	5,000 to 12,000
sodium-vapour high-pressure lamp	175 to 400	70 to 100	8,000
halogen metal-vapour lamp	175 to 2,000	62 to 87	1,000 to 6,000

The incandescent lamp is the oldest source of light which is still in frequent use today. Fig. 4.11. shows the basic design of this lamp. The lamp cap for general-purpose incandescent lamps is provided with an Edison screw having a diameter of 27 mm or 14 mm (designation of the cap: E27 or E14). In addition, there is a great variety of other cap shapes and diameters for quite different applications. Likewise, the glass bulb is made in many shapes. The glass may be clear, frosted, opal (white enameled) or coloured; ornamental glass is also used.

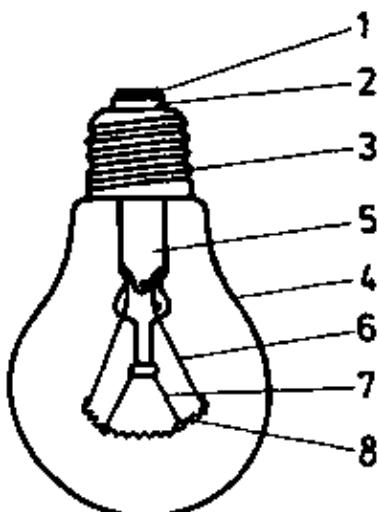


Fig. 4.11. Design of an incandescent lamp

1 Contact; 2 Insulating body; 3 Cap with Edison thread; 4 Glass bulb; 5 Pump tube end; 6 Electrode; 7 Supporting wire; 8 Glow wire (filament)

A disadvantage of incandescent lamps is the low luminous efficiency. About 95 % of the input of electrical power are delivered in the form of heat energy; and this is also associated with problems of heat dissipation in lighting fittings. The short service life does not meet our wishes. It is due to the fact that the incandescent filament is reduced to powder (evaporates) at its surface to some extent. Due to the deposition of the filament material, the glass bulb is blackened. Slight differences in the cross-section of the incandescent filament due to manufacture lead to an increase in the resistance at points of reduced cross – section and thus to a local increase in temperature which, in turn, causes a more intensive evaporation and further reduction of the cross–section until the incandescent filament is destroyed at this point. Fig. 4.12. shows the remarkable dependence of light flux and service life on the mains voltage.

Since operation of an incandescent lamp does not call for ballasts and ignition devices, the lighting fitting is of a simple design.

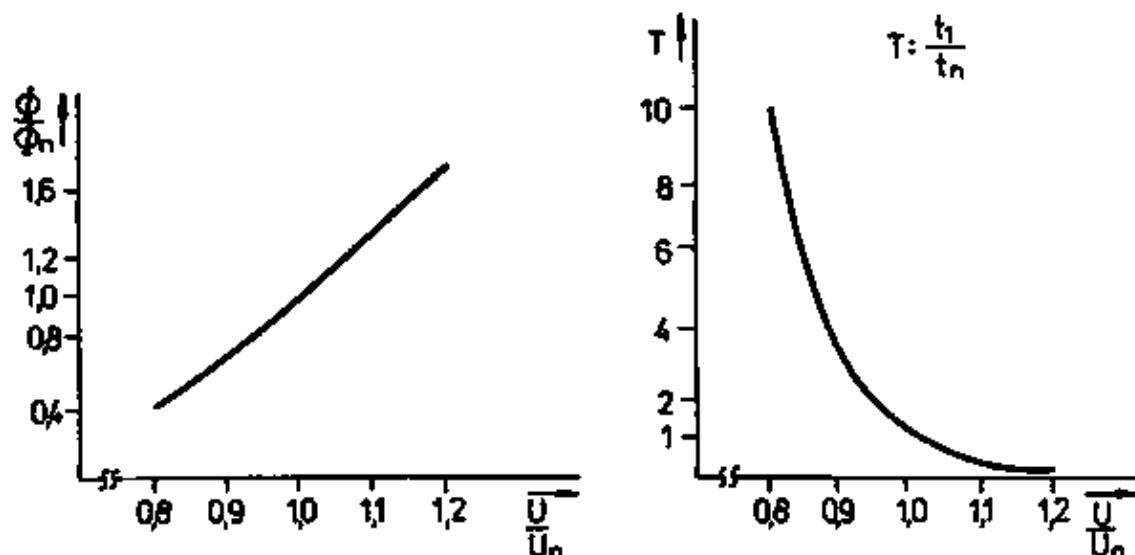


Fig. 4.12. Luminous flux and service life of an incandescent lamp in dependence on the operating voltage

$\Phi_n$  = rated flux;

$U_n$  = rated voltage

$t_1$  = service life

$t_n$  = rated service life

The colour of the light of this bulb is agreeable in the case of low illumination intensities ( $< 100$  lx). When high illumination intensities are required ( $> 200$  lx), incandescent lamps cannot be used. Because of these properties, the incandescent lamp is today primarily used for the illumination of sitting–rooms, restaurants,

theatres and other rooms where frequent switching is necessary (staircases, service-rooms).

Since about 1960, halogen lamps have been produced as a special form of incandescent lamps. In contrast to the conventional incandescent lamp, the glass bulb of halogen lamps contains a small amount of a halogen, usually bromine, in addition to the filling gas. The incandescent filament particles evaporating during lamp operation form a cycle with the halogen and are deposited on the incandescent filament again and again. Thus, blackening of the bulb is avoided. The cycle calls for a high temperature at the outer wall; that is why quartz glass bulbs have to be used in relatively small designs.

In many cases, additional cooling is required. On no account should the glass bulb be touched with bare hands. It should always be gripped at the point of compression or a clean piece of cloth has to be used. If the glass bulb has been touched inadvertently, it must be cleaned by means of a rag soaked in spirit. For some types, a special position of burning has to be observed.

For the dependence of the service life and the light flux on the voltage, the same relations as for the conventional incandescent lamp apply. The mean service life is stated to be anything between 20 and 2000 hours depending on the type. An advantage is the higher luminous efficiency as compared with the conventional incandescent lamp and the good spectral composition of the light. For incorporation in optical systems, the small dimensions of incandescent filament and lamp (spot light) are of particular advantage.

Halogen incandescent lamps are used for taking photographs, as projector lamps and special motor-vehicle lamps. For an illumination of sports fields and in other floodlight installations, halogen incandescent lamps of the sizes 1000 and 5000 W are used. Illumination systems in cinematographic film and television studios and in theatres can also be equipped with halogen incandescent lamps.

Already in 1936, the fluorescent lamp was produced for the first time; today it is primarily used for interior lighting in industry and administration. Large quantities of these lamps have been used since 1950, after the incorporation in the lamps of decisive technical improvements.

The fluorescent lamp is a low-pressure gas discharge lamp. Fig. 4.13. shows the basic design. In a tube two electrodes are fused in place which have to enable the electron emission.

As filling in the tube a small amount of mercury and of argon, an inert gas, is provided under the very low pressure of a few hundred pascal (the normal air pressure is about 100,000 pascal). The interior wall of the glass tube is covered with a luminescent material. When voltage is applied and ignition effected, a gas discharge occurs. In this event, the current passage is effected by ions and electrons flowing through the gas. The discharge in the mercury vapour filling produces normally ultraviolet light which is not perceived by our eyes. The portion of argon in the filling produces only a very weak bluish light. The luminescent material is caused to light by the ultraviolet rays. Depending on the type of the luminescent material used, a visible light of different spectral composition is produced.

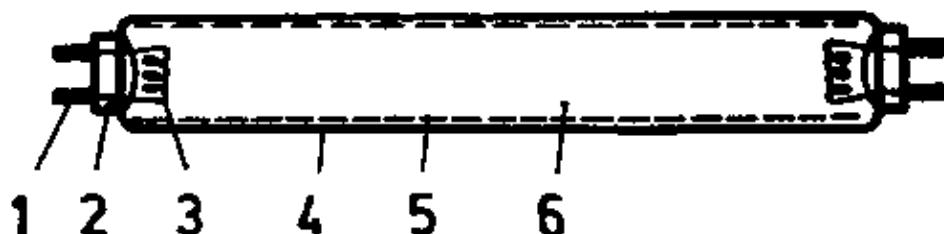


Fig. 4.13. Design of a fluorescent lamp

1 Cap in; 2 Cap; 3 Electrode (filament); 4 Glass tube; 5 Layer of luminescent material; 6 Noble gas filling and mercury vapour

Fig. 4.14. shows the complete circuit of a fluorescent lamp.

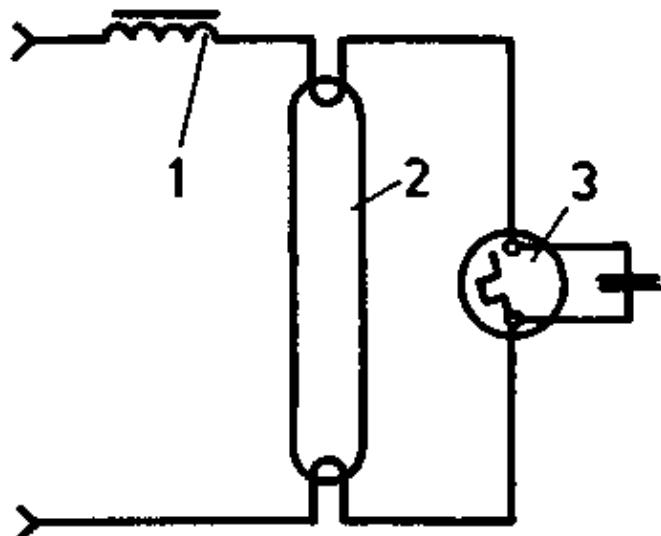


Fig. 4.14. Circuit of a fluorescent lamp

1 Ballast; 2 Fluorescent lamp; 3 Starter with anti-interference capacitor

For operation, a ballast and a starter are required. When connecting the circuit to 220 V a.c., no current will flow through the fluorescent lamp at first. Almost the full mains voltage is applied via the ballast to the starter in which a glow discharge is developing. At the same time, the bimetallic strip in the starter is heated closing the circuit via ballast, heating spiral, starter, heating spiral. The heating spirals start glowing. At the same time, the bimetallic strip is cooling down, opening the circuit. The high voltage peak now generated by the ballast of more than 400 V leads to the ignition of the discharge in the fluorescent lamp. Due to the voltage drop across the ballast, a burning voltage of 50 V to 110 V remains at the fluorescent lamp depending on the length of the lamp while, with this voltage, a glow discharge cannot be repeated in the starter. The ignition process may occur several times in exceptional cases. For a reliable ignition and a long service life of the fluorescent lamp it is absolutely necessary to use only such ballasts and starters which are appropriate for the type of lamp.

Frequently, a capacitor for power-factor compensation is added according to Fig. 4.15. whose function and mode of operation will be discussed in Chapter 7.

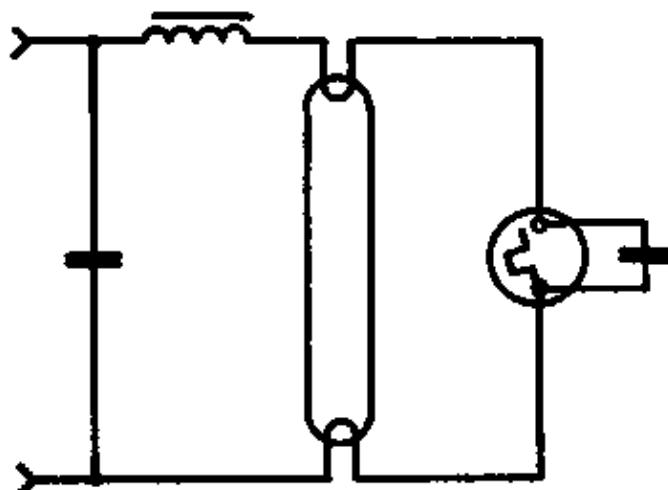


Fig. 4.15. Circuit of a fluorescent lamp with capacitor for power-factor compensation

In fluorescent lamps the luminous flux is largely depending on the operating voltage, as is shown in Fig. 4.16. The dependence of the luminous flux on the ambient temperature must also be taken into consideration; at high outdoor temperatures, a good ventilation of the lighting fittings is required in this event (Fig. 4.17.).

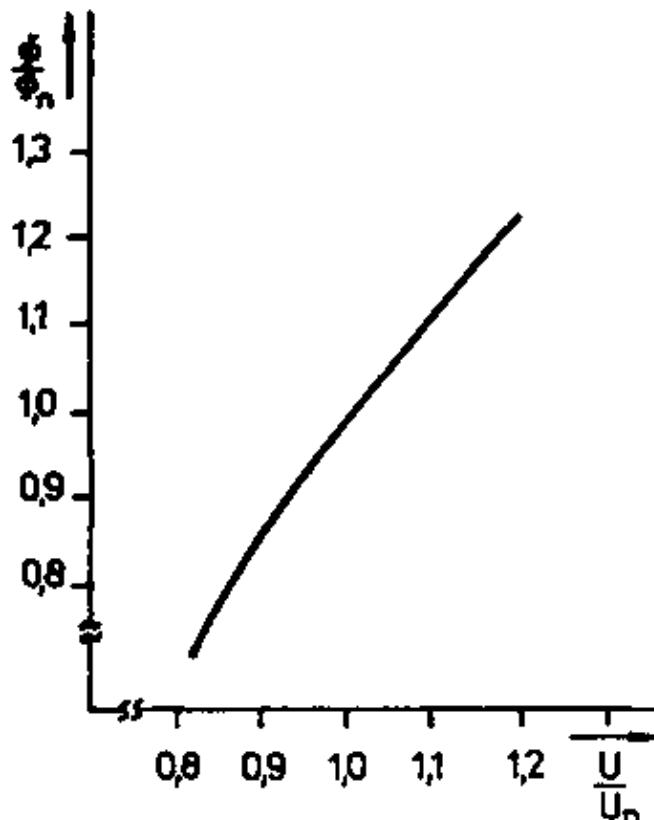


Fig. 4.16. Dependence of the luminous flux on the mains voltage for fluorescent lamps

$\Phi_n$  = rated flux;  
 $U_n$  = rated voltage

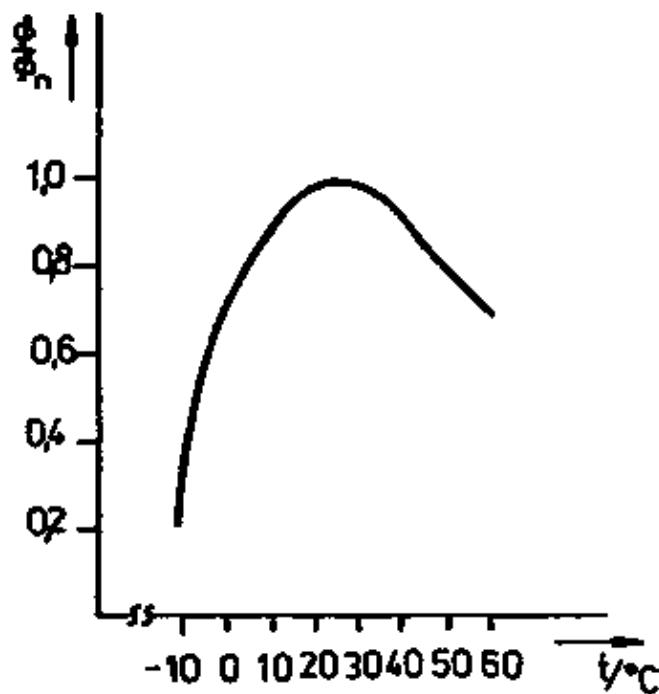


Fig. 4.17. Dependence of the luminous flux on the ambient temperature for fluorescent lamps

$\Phi_n$  = rated flux

Fluorescent lamps are made for a power input of 8 to 65 W. For room illumination, especially the power steps 40 and 65 W are used. For sitting-room lighting and direct place illumination, the lower power steps may be used. Primarily fluorescent lamps in rod form are on offer. For decorative and sitting-room lighting, lamps in U-shape and circular shape are included in the offer.

An advantage of the fluorescent lamp in case of rooms of small height is the large light-emitting surface. Even in case of direct sight, no or insignificant dazzling will occur. Different light colours can be achieved by various compositions of luminescent materials. The light colour "daylight white" most closely resembles the daylight but it is felt to be agreeable only in case of very high illumination intensities ( $> 1000 \text{ lx}$ ). For working place lighting, the light colour "natural white" is an agreeable colour because of its good colour rendition properties between 200 lx and 1000 lx. In the event of high demands on colour rendition, the brilliant light of the light colour "natural white de luxe" should be used; its disadvantage, however, is a lower luminous efficiency. For low values of brightness, the light colour "warm white" has been developed, and in the event of fastidious demands "warm white de luxe" should be used. In Table 4.6. light colours suitable for various tasks of illumination are given.

Table 4.6. Lamp Light Colours Suitable for Various Tasks of Illumination

Task of illumination	Suitable lamp light colour				
	daylight white	neutral white	neutral white de luxe	warm white	warm white de luxe
administration room		x			
conference room		x	x		x
lecture room		x			
restaurants, hotels		x	x		x
butcher's shop			x		x
dwelling				x	x
industry		x			
dye-house	x		x		
manufacture of ready-made articles of dress	x	x	x		
drawing office		x			
medical facilities					

For special purposes, the light colours "Lumoflor", "blue", "green" and "red" are on offer.

For outdoor lighting and for the illumination of high working rooms, small light sources with a high luminous flux (point sources of light) are required. For this purpose, the mercury vapour high-pressure lamps, which have been produced since 1936, the halogen metal-vapour lamps, which have been developed only in recent years, and the sodium-vapour high-pressure lamps are used. These three types of lamps have in common that the light is produced in a burner in a high-pressure gas discharge; the burner is provided with a further glass bulb. Ballasts are necessary. A running-in period of a few minutes is required until the full luminous efficiency is attained. After a short current interruption, the lamp will go out and can only be ignited again after a cooling time of some minutes.

Today, the mercury-vapour lamp frequently is additionally coated with luminescent material in order to obtain a better spectral composition of the light. Nevertheless, an unnatural colour rendering is inevitable. These lamps are made in power steps from about 50 W to 2000 W. Fig. 4.18. shows the circuit with ballast. The external bulb prevents the high portion of ultraviolet light to get outside. When the external bulb is damaged, the lamp remains serviceable but for man there is the danger of burns due to ultraviolet light. These lamps are primarily used for street lighting.

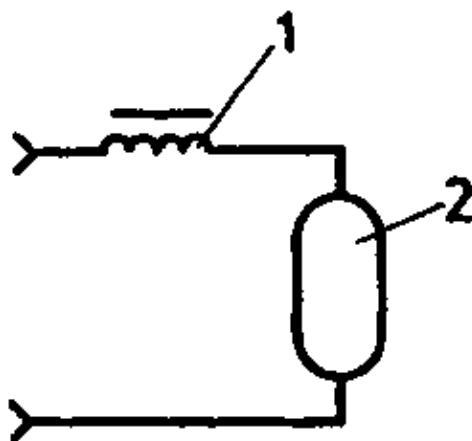


Fig. 4.18. Circuit of a mercury vapour high-pressure lamp

1 Ballast; 2 Mercury vapour high-pressure lamp

The sodium-vapour high-pressure lamps, which have been made only recently, offers not only a higher luminous efficiency but also an improved spectral composition of the light and, consequently a more natural colour rendition. For operation, both a ballast and an igniting device are required (see Fig. 4.19.). The lamps are made in power steps from 175 W to 400 W; a further extension of the manufacturing programme is to be expected. Sodium-vapour high-pressure lamps are used to advantage in street lighting, illumination of large open-air facilities (ports, marshalling yards, construction sites) and in high halls without particular demands on colour rendition (foundries, assembling halls, storage halls). Because of their high luminous efficiency, they will replace the mercury-vapour high-pressure lamps in many fields.

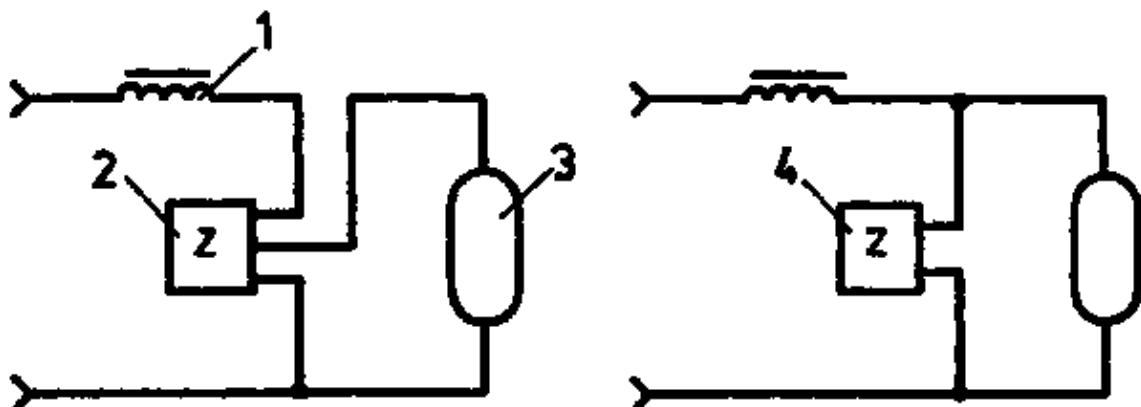


Fig. 4.19. Circuit of a sodium vapour high-pressure lamp with different igniting devices

1 Ballast; 2 Thyristor ignitor; 3 Sodium vapour high-pressure lamp; 4 Starter ignitor

A better colour rendition is provided by the halogen metal-vapour lamp though the luminous efficiency is slightly reduced. In its design it resembles the mercury-vapour high-pressure lamp but contains in the burner an additional substance, i.e. an addition of halogen. By means of various types of additives and the luminescent material used, the light colours of daylight white, neutral white or warm white can be achieved. The lamps are made in power steps from 175 W to 2000 W. For operation, an igniting device is required in addition to the ballast (also see Fig. 4.19.).

The lamp is used in outdoor illumination and in high halls where demands on colour rendering are more fastidious. A few types have been specifically developed for use in colour photography studios where mixing with daylight or with the light emitted by the halogen incandescent lamp is possible.

#### 4.5.3. Illuminating Engineering

Besides the demand on the illumination intensity specified in Section 4.5.1., a few further principles have to be taken into account for the installation of illumination systems. The difference in brightness between working field and surround is of great importance. The best visual efficiency is ensured when working field and

surround have the same brightness (see Fig. 4.20.). On no account should the surround be brighter than the working field. This can be explained best when one tries to investigate the texture of a black strip of fabric on a black or on a white ground.

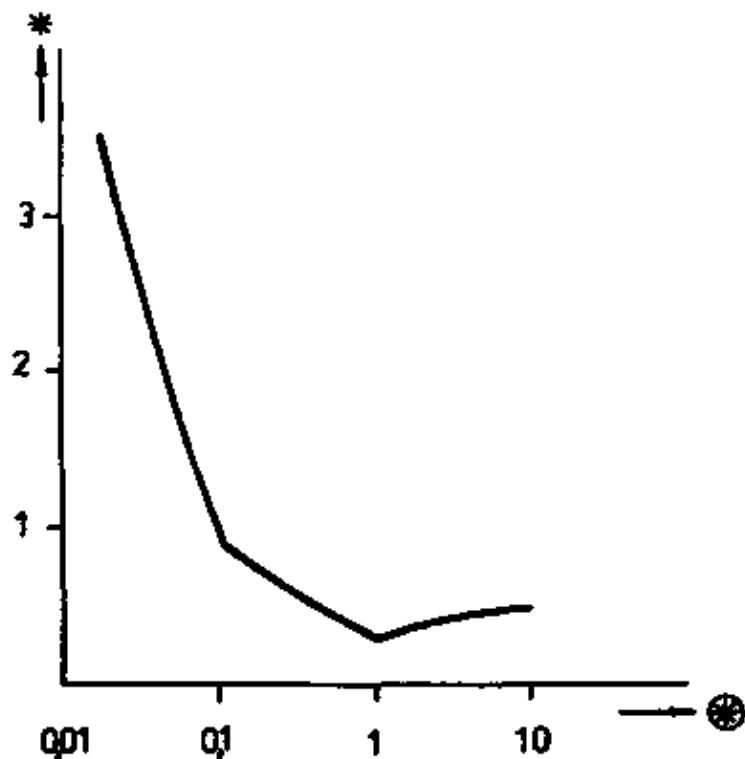


Fig. 4.20. Visual power in dependence on the brightness difference between working field and surround

$$\frac{\text{brightness of working field}}{\text{brightness of surround}} = \text{smallest perceptible contrast}$$

It is quite difficult to avoid dazzling. Direct dazzling or glare occurs when the source of light is in the line of vision. It can be avoided by an appropriate sheathing of the light source and arrangement of the latter outside of the angle of view. Fig. 4.21. shows that within an angle of  $20^\circ$  with respect to the line of view no light source should be arranged. Indirect glare occurs when the ray of light emitted from the source of light is reflected by an object of work into the eye. Fig. 4.22. shows how indirect glare is brought about. Remedy can be provided by an appropriate arrangement of the lighting fitting. Whenever possible, objects of work should be mat (diffusely reflecting) and not bright on the surface in order to avoid highlights. In the case of objects of work having an intensely directed reflection, mainly diffuse light should be used for work.

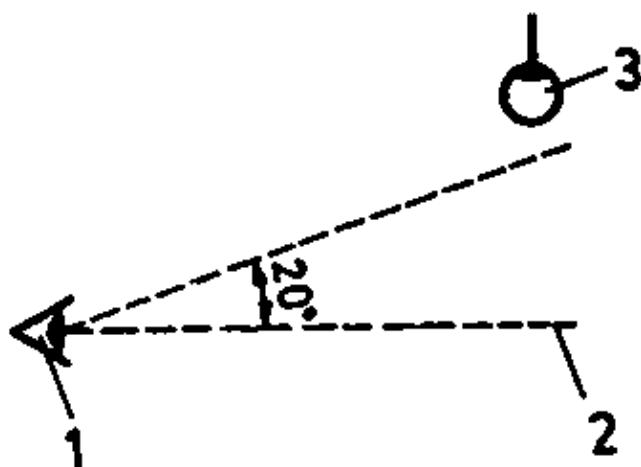


Fig. 4.21. Avoiding direct glare

1 Eye; 2 Direction of sight; 3 Source of light

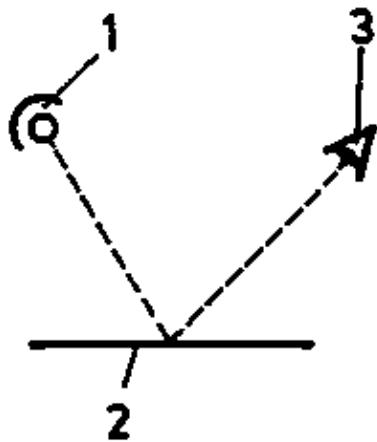
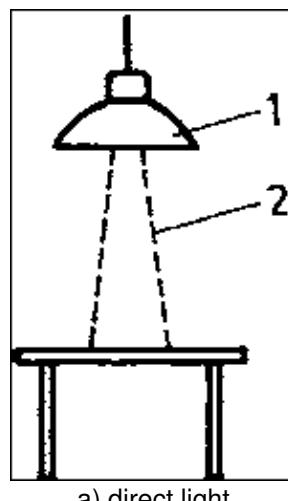


Fig. 4.22. Origin of an indirect glare

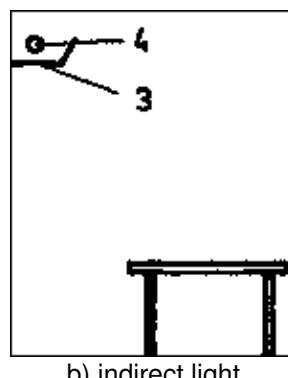
1 Source of light; 2 Reflecting object of work; 3 Eye

Light which is incident on the object of work when being emitted in a directed manner from the source of light is called direct light. When, however, the light of the source is directed to large diffusing screens or to the ceiling of the room and then to the working place, we speak of indirect or diffuse light (see Fig. 4.23.). For most of the problems of vision, a correct mixture of direct and indirect light should be provided. Spatial sensation is dependent on the formation of shadow and, thus, on a portion of direct light. Cast shadows which impair the perceptibility of objects are due to intense direct light.

Fig. 4.23. Origin of primarily



a) direct light



b) indirect light

1 Lighting fitting with screen impervious to light; 2 Directed light; 3 Opaque fillet; 4 Source of light

The angle of light incidence is also of importance to an avoidance of fatigue in work. One should take care that daylight and artificial light have the same angle of incidence to avoid double shadows. The illumination

intensity should take the same course inside a room; this can be ensured by suspending the lighting fittings asymmetrically and closer by the windows.

Well-being and efficiency of man largely depend on type and intensity of light. Good illumination leads to an increase in labour productivity, reduction of rejects and of the number of accidents at work. When daylight is missing or insufficient, illumination with artificial light is necessary. The light sources used differ with respect to luminous efficiency, spectral composition of the light, size and service life. There are types of light sources which have to be operated by means of special ballast and to be ignited by means of ignition devices. High-pressure discharge lamps reach their full brightness only after a few minutes and, after an interruption of voltage, require a few minutes rest until repeated ignition will be possible.

All light sources and their properties are largely dependent on the mains voltage. For indoor lighting, the fluorescent lamp is widely used today, for high rooms and outdoor facilities the high-pressure discharge lamps which, as point light sources, require only small lighting fitting dimensions though the luminous flux is high. The light colours should be adapted to the illumination intensity involved and to the desired colour perceptibility.

The lighting installation must be designed in such a way that glare is avoided, the problem of vision, can be solved without fatigue, sufficient shade without cast shadow is available and an appropriate light distribution in the space under consideration is attained.

#### Questions

1. Which factors of production are influenced by illumination?
2. What are the factors on which the different colour rendition in case of different light sources is dependent?
3. Why is the luminous efficiency an important factor in the evaluation of light sources?
4. What are the properties and fields of application of the various light sources?
5. Why is the fluorescent lamp particularly suitable for rooms of small height?
6. For which tasks of illumination are fluorescent lamps of different light colours used?
7. By means of which measures can direct and indirect glare be avoided?
8. Why should the working field be not darker than the surround?

#### 4.6. Conversion of Electrical Energy into Chemical Energy and Chemical Energy into Electrical Energy

For many electrically operated devices it is desired to operate them independently of a central power supply system. For this purpose, small and light-weight electrical energy sources (batteries) are required. Batteries which can be recharged after discharge are called primary elements. Batteries which can be recharged several times after discharge are called secondary elements. Below, primary elements are discussed first.

The Italian physicist Galvani (1737 – 1798) was the first to find that a primary electromotive force is produced between two different metals or between metal and carbon in an electrolyte (aqueous solution of an acid, a lye or a salt). The magnitude of the voltage and the polarity are dependent on the metals used. The Italian physicist Volta (1745 – 1827) arranged the individual metals in an electromotive series, named after him, in such a way that magnitude and polarity of the primary electromotive force can be determined. The greater the distance between two elements in the electromotive series, the greater the primary electromotive force that will be brought about.

Of particular importance to engineering is the carbon-zinc element with a primary electromotive force of 1.5 V with carbon forming the positive pole and zinc the negative pole. It is offered in different shapes and sizes and serves for the supply of portable wireless sets, torch lamps, pocket calculators, and many other devices. Fig. 4.24. shows the most frequently used designs; their technical data are given in Table 4.7. The inner structure of such an element (design of the Leclanché element) is shown in Fig. 4. 25.

Table 4.7. Technical Data of Some Carbon–zinc Cells

Specification	Designation				
	R6	R14	6F22	R20	3R12
diameter in mm	14	26		34	
height in mm	50.5	50		61.5	
length in mm			26.5		62
width in mm			17.5		22
height in mm			48.5		67
voltage in V	1.5	1.5	9.0	1.5	4.5
medium capacity in Ah	1.0	2.6	0.38	5.8	1.6

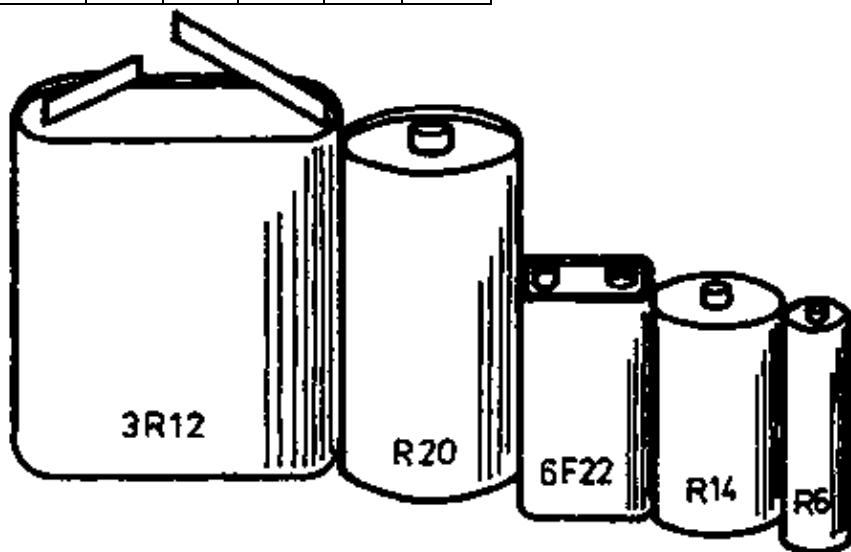


Fig. 4.24. Frequently used primary elements

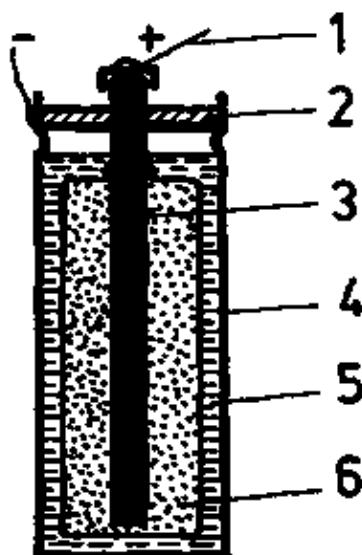


Fig. 4.25. Design of the carbon–zinc element

1 Metal cap; 2 Sealing compound; 3 Carbon; 4 Zinc cup; 5 Electrolyte (thickened ammonium chloride solution); 6 Linen bag with pyrolytic graphite filling

When current is drawn from the element, the zinc sheath is disintegrated. The chemical energy liberated in this way is the equivalent of the produced electrical energy. It is disadvantageous that a disintegration of the

zinc also takes place during storage although no current is drawn. The service life of such elements therefore is only about 6 months. Another disadvantage is the risk of leakage of the thickened ammonium chloride solution when the disintegration of the zinc sheath begins. As ammonium chloride solution is chemically aggressive, devices and equipment in which such elements are incorporated can be damaged. For sensitive device, the expensive tightly enclosed elements are made. In this case, a carefully sealed steel-sheet coat is the enclosure of the element proper; in this way, leakage is effectively prevented.

An improved design is the alkali-manganese cell which can deliver a considerably higher quantity of energy with the same dimensions as the above element. Because of the higher price, which is due to manufacture, it was not yet in a position to supersede the above described Leclanché element. For practical use it is advisable to use in a device always batteries of the same manufacturer and of the same type designation and to replace the batteries in the device when they are in the same state of discharge. At the end of discharge, the voltage drops to about 0.9 V per cell. Recharging is not possible. Consumed batteries must be removed immediately from the device.

In contrast to primary elements, secondary elements show the advantage that they can be recharged after discharge. It is disadvantage that the quantity of energy that can be stored is smaller in an element of the same size as the primary element. In practice, two designs of the secondary element have gained particular importance, namely, the lead accumulator and the nickel-iron or nickel-cadmium accumulator.

In a lead accumulator, there are lead plates as electrodes and sulphuric acid as electrolyte. In accordance with the various applications, the lead plates – consisting of a frame and pressed-in lead powder – are made in different shapes. Because of this plate construction, lead accumulators are sensitive to vibrations. Lead powder dissolved out of the plates is deposited under the plates as lead sludge in the course of time. When the accumulator is overloaded, a formation of large amounts of lead sludge will occur. When the lead sludge can touch the lower edge of the lead plates, self-discharge will occur and the accumulator become useless.

In the charging process, lead oxide is formed at the positive plate and lead at the negative plate. Due to discharge, the two plates are converted into lead sulphate. This shows that the sulphuric acid takes directly part in the process of conversion of chemical energy into electrical energy. Since during the process of charging the concentration of the sulphuric acid increases, the state of charge can be determined by measuring the acid density by means of a hydrometer. There are specifically made hydrometers where the state of charge can be read directly.

The voltage of each cell shows during charging and discharging a typical behaviour (Fig. 4.26.). When charging, the voltage will at first rapidly rise from 2.0 V to about 2.15 V, and another rapid increase in voltage will only take place a short time before the end of the charging process, namely, to 2.7 V per cell. If, after this voltage rise, the process of charging is continued, a further chemical change cannot take place in the plates. The supplied electrical energy will cause a decomposition of the electrolyte and, consequently, an intense evolution of gas.

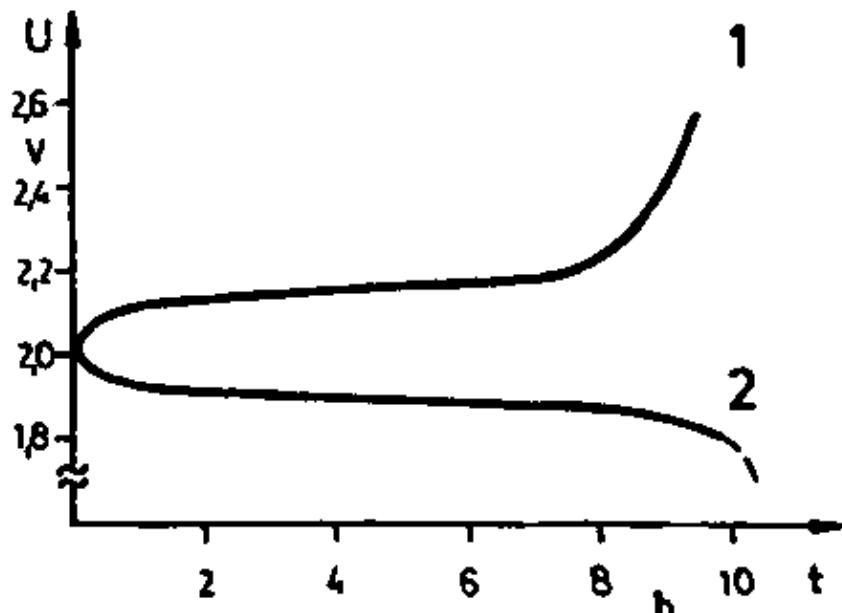


Fig. 4.26. Course of charging and discharging voltage at the lead accumulator

1 Charging; 2 Discharging

The gas produced is highly explosive (oxyhydrogen). Therefore, in accumulator rooms any use of open fire or smoking is strictly forbidden. Overcharging should be avoided in any case. The rise of the cell voltage to 2.7 V is used in automatic charging equipment for switching off the charging process. In discharging, the mean cell voltage first drops to about 1.95 V and then it again rapidly drops near the end of discharging. A discharging voltage should not fall short of 1.8 V per cell. Depending on the magnitude of the discharging current, this voltage will be reached after different periods of discharging.

The product of discharging time times discharging current is termed as capacity of the accumulator in ampere-hours. A certain discharging time is always used. The capacity can be used for classifying accumulators and it is stated in Ah (ampere-hours).

The internal resistance of lead accumulators is low. A high current can, therefore, be drawn for a short time (e.g. starter battery in a motor-car); it should be noted, however, that any short circuit must be avoided in any case because of the extremely high short-circuit current.

The lead accumulator is used as starter battery in motor-cars, for emergency current supply in plants which have to be serviceable even when the mains voltage fails (e.g. telephone exchanges, emergency lighting) and, in special designs, it is used for power supply to portable electronic devices.

In maintaining accumulators care should be taken to see to it that the gas escape valves are clean so that the gas evolved in charging and discharging can escape. The poles must be kept clean and protected by means of a special grease. The plates must always be covered by electrolyte; for topping up only distilled water (no sulphuric acid) has to be used. An excessive discharge must be avoided.

The plates of the nickel-iron accumulator consist of nickel hydroxide and iron hydroxide, aqueous solution of potassium hydroxide or caustic potash is used as electrolyte. In contrast to lead accumulators, the density of the electrolyte does not change in charging and discharging. A measurement of the state of charge by means of a hydrometer is not possible therefore.

The voltage behaviour during charging and discharging of the nickel-iron accumulator is shown in Fig. 4.27. Since no distinct voltage rise takes place at the end of the charging process, the state of charge cannot be determined on the basis of the charging voltage. During discharge, the cell voltage first rapidly drops from 1.4 V to 1.25 V and only at the end of the possible discharge again rapidly drops to 1.1 V. The state of charge can therefore be conveniently derived from the discharging curve.

The mean cell voltage of the nickel-iron accumulator is for about 0.5 V lower than that of the lead accumulator while the internal resistance is higher. The nickel-iron accumulator requires less maintenance, it

has a longer service life and a lower weight. For some applications, the higher internal resistance and the lower cell voltage are disturbing factors.

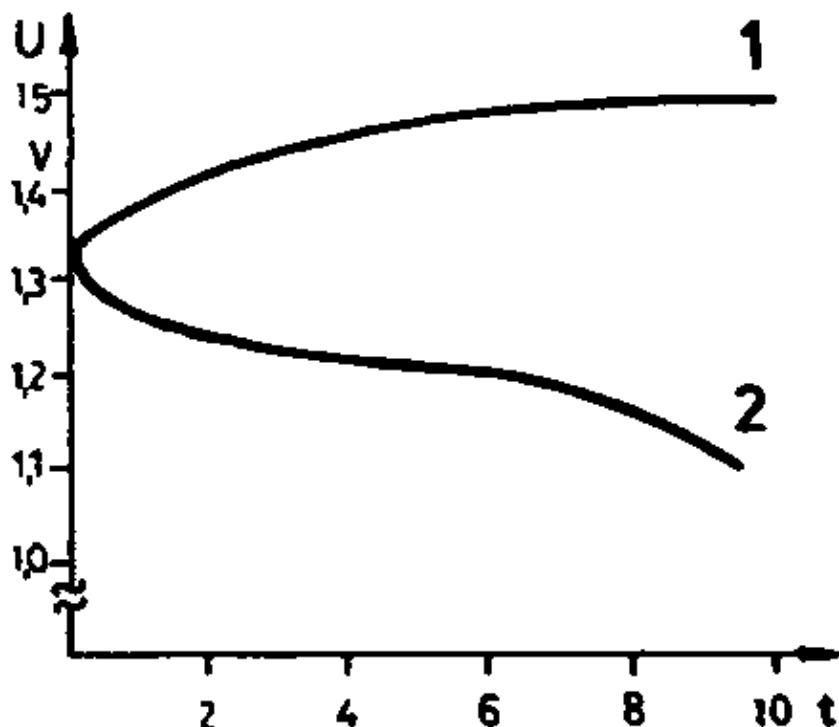


Fig. 4.27. Course of charging and discharging voltage at the nickel–iron accumulator

1 Charging; 2 Discharging

The nickel–cadmium accumulator has gained great importance to electronic devices. It can be made gas-tight and thus be installed in pieces of equipment in any desired mounting position. The service life is stated to be about 5 years or about 3000 charging cycles. The charging specifications must be strictly observed. In contrast to nickel–cadmium accumulators, carbon–zinc accumulators have the three-fold to ten-fold energy content so that the accumulator of the former type must be charged at least three times during the same operating time and when of the same size as the latter. Nevertheless, the use of the gas-tight nickel–cadmium accumulator is of advantage to frequently employed electronic devices even after a short time of operation.

For the supply of power to mains-independent devices and equipment, primary elements and secondary elements can be employed. Primary elements are useless after discharge while secondary elements can be recharged after discharge. In primary elements, the carbon–zinc element is primarily used. It has a cell voltage of 1.5 V. Manufacture is in a great variety of shapes and sizes; the battery voltage ranges from 1.5 V to 9 V depending on the number of cells connected in series, in exceptional cases the battery voltage may be even higher.

Lead accumulators are mainly used for large current consumers. Careful maintenance will considerably increase the service life. Due to the low internal resistance, high currents can be drawn for short periods. When a long service life and limited maintenance are required, the nickel–iron accumulator is employed. In portable electronic devices, the gas – tight nickel–cadmium accumulator is used which is on offer with capacities from about 10 mAh (button cell) to 1 Ah and it does require practically no maintenance.

#### Questions and problems

1. What is the difference between primary elements and secondary elements?
2. What are the mean cell voltages of the described voltage sources?
3. How can the state of charge be measured at the lead accumulator?
4. Quote examples of application and designs of electrochemical sources of voltage!

## 5. Magnetic Field

### 5.1. Magnetic Phenomena

Magnetic phenomena are caused by magnets. These are bodies which exert forces of attraction on iron and steel.

A distinction is made between

- permanent magnets; these magnets are made of steel or special materials for permanent magnets and they maintain their magnetic properties in almost unchanged strength for a long period.
- electromagnets; these are coils which usually have an iron core. Magnetic actions only occur when a current passes through the coils.

The ends of the magnets exert the greatest force of attraction and are called poles. Under the influence of the terrestrial magnetic field, a rod-shaped magnet is adjusted in north-south direction. The magnetic end which points to north is called north pole, the opposite end is called south pole. The connecting line between the two poles is called magnetic axis.

When we place two magnets opposite to each other (see Fig. 5.1.), then we will find:

Non-homonymous poles attract each other, homonymous poles repel each other.

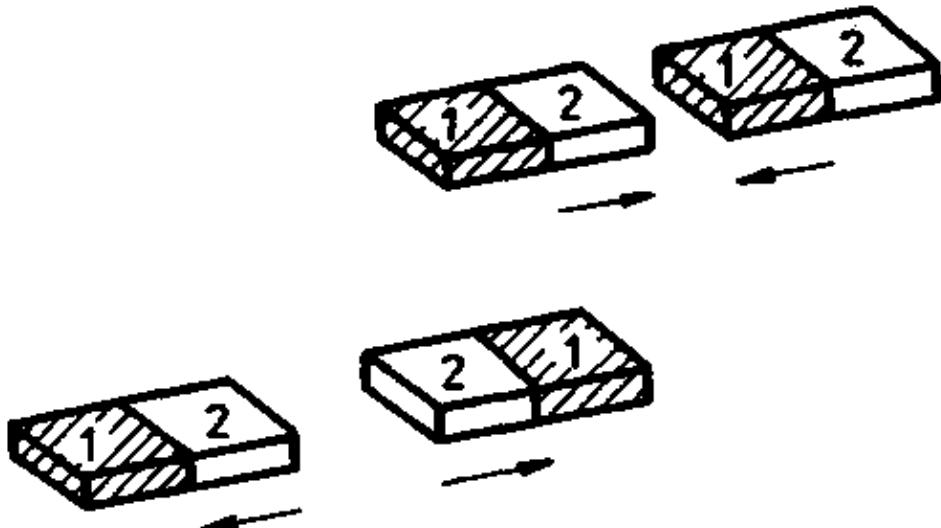


Fig. 5.1. Force actions between magnets opposing each other

1 North pole; 2 south pole

The effects of the force of magnets can be represented by force action lines which are called magnetic field lines. Fig. 5.2. shows the course taken by the magnetic field lines of a rod magnet. Their direction has been fixed arbitrarily.

Obvious is the following:

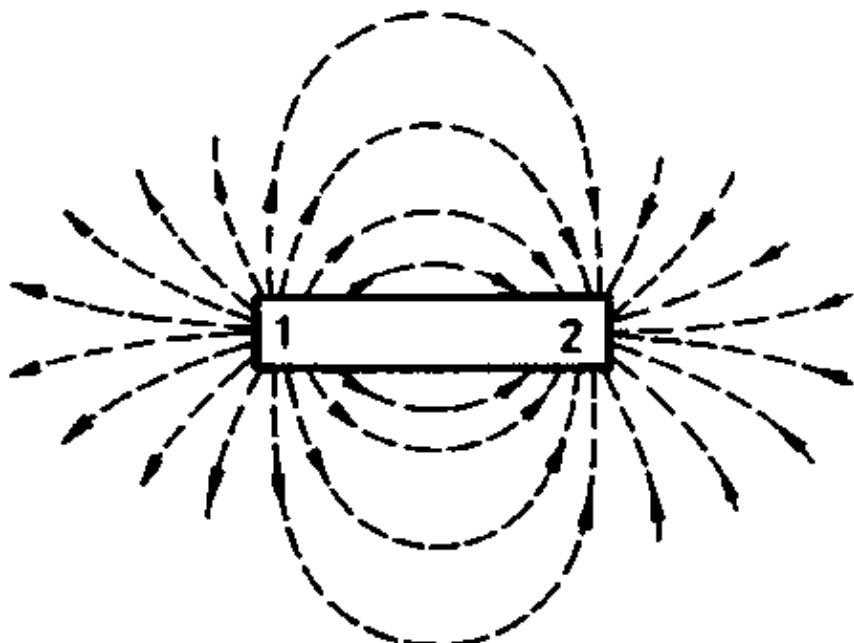


Fig. 5.2. Magnetic field of a rod magnet

1 North pole; south pole

The magnetic field lines are closed in themselves; outside of the magnet they run from the north pole to the south pole and inside the magnet from the south pole to the north pole.

In Section 2.1. we have already said that every electrical current is associated with a magnetic field. This fact is described by the so-called circulation law, which is also known as Biot–Savart's law. To give proof of it, one passes a straight conductor through a sheet of paper or a glass plate, in accordance with Fig. 5.3., and then allows a current to flow through the conductor. Then fine iron powder is sprinkled on the paper or plate of glass and immediately the iron particles will adjust themselves in the form of concentric circles around the conductor, thus, forming a typical field line pattern. The direction of the field lines can be determined by means of a magnetic needle and has been entered in Fig. 5.3.

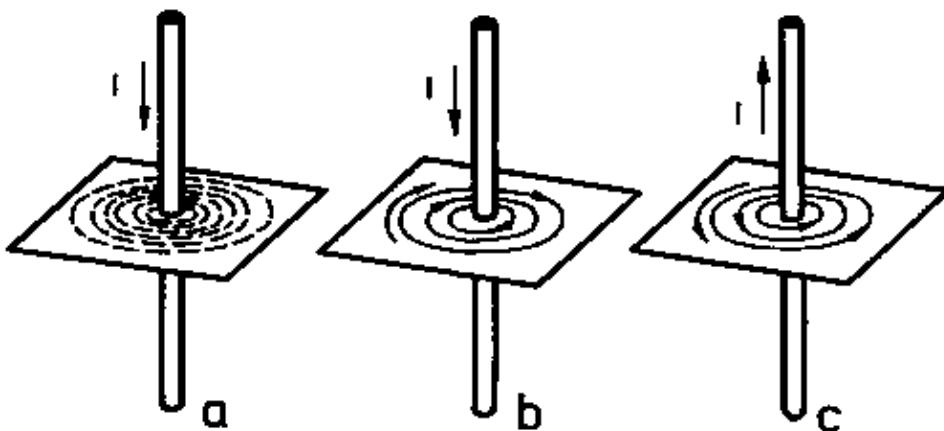


Fig. 5.3. Magnetic field of a current carrying conductor

- Pattern of field lines
- Direction of field lines
- Direction of field lines after reversal of current direction

The correlation between current direction and field line direction is defined by the screw rule:

When one imagines a right-handed screw to be moved in a conductor in the direction of the current, the sense of rotation required for this longitudinal motion indicates the direction of the field lines.

When one intends to mark the electrical current direction in a conductor, the symbols shown in Fig. 5.4. have to be used. Basically, the direction of current is indicated by an arrow. When looking at the cross-section of the conductor, one sees the arrow-head as a point when the current flows in the direction toward the

observer. When the current flows away from the observer while he continues to look in the same direction, he sees the feathers of the arrow – head in the form of a cross on the cross-section of the conductor.

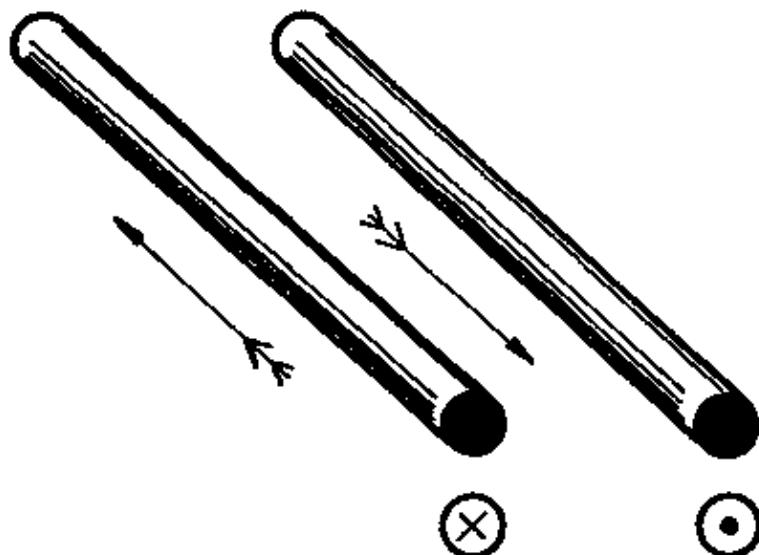


Fig. 5.4. Symbolising the current direction in conductors

When a conductor is wound on a cylinder in the form of a spiral, one obtains a coil. When current passes through it, a magnetic field is brought about which is made up of the fields of the individual turns of the coil. A long, cylindrically wound coil has a field which resembles that of a rod magnet (Fig. 5.5.). Which end of the coil forms the north pole and which the south pole is dependent on the direction of current and can easily be found with the help of the pole determination rule:

When the current flows clockwise through the coil, one looks at a south pole; when the current flows anti-clockwise through the coil, one looks at a north pole.

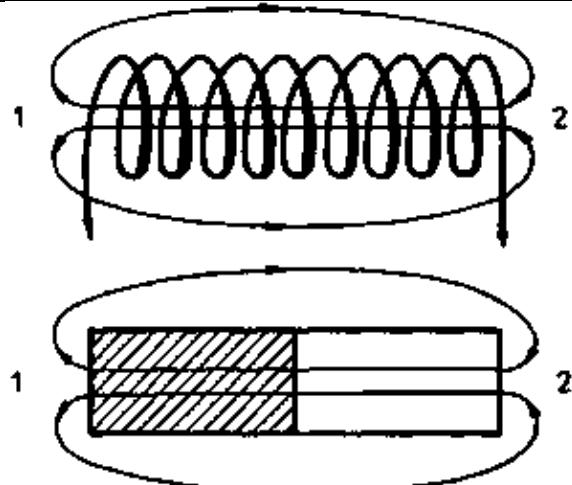


Fig. 5.5. Magnetic fields of a current-carrying coil and a rod magnet

1 North pole; south pole

When placing a piece of iron in the field of a magnet, the field line distribution will change considerably (B.5.6.). Like nickel, cobalt and their alloys, iron also is a very good conductor for magnetic field lines. Therefore, these materials are called ferromagneticum. Fig. 5.6. shows how the parallel course of the field lines (homogeneous, i.e. uniformly built up field) is changed by a ferromagneticum. In general, the following holds:

A ferromagneticum bunches magnetic field lines together.

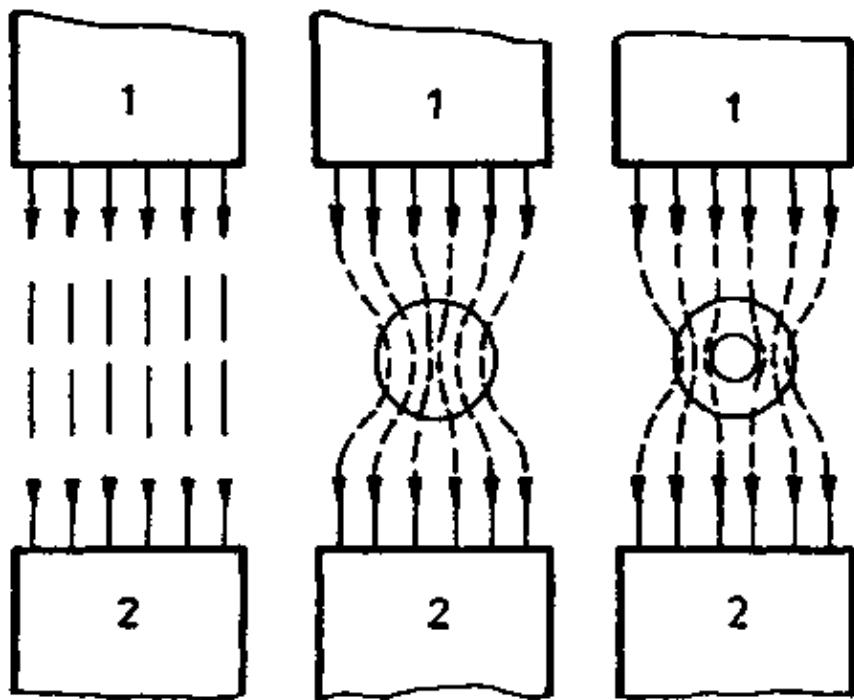


Fig. 5.6. Changing the course of the field by ferromagnetic bodies

1 North pole; 2 south pole

When the ferromagnetic body has the shape of a ring, inside the ring the space is field-free because the field lines run in the wall of the ring. This fact is used for magnetic screening. Components which must be protected from disturbing magnetic fields are surrounded by sheaths of ferromagnetic material.

As has already been said above, every current is associated with a magnetic field. A coil through which current flows is also surrounded by a magnetic field. In the coil, electrical energy is converted into magnetic energy. Since the field lines are closed in themselves, we speak of a circle in which the magnetic field is caused by a magnetomotive force. The coil through which current flows is the seat of this magnetomotive force <sup>1)</sup>. It is the greater, the greater the current is and the more turns the coil includes. Fig. 5.7. shows a magnetic basic circuit. This circuit consists of a coil with  $N$  turns through which a current  $I$  passes. The ferromagnetic conductor is not closed but separated by an air gap.

1) ? Greek letter theta

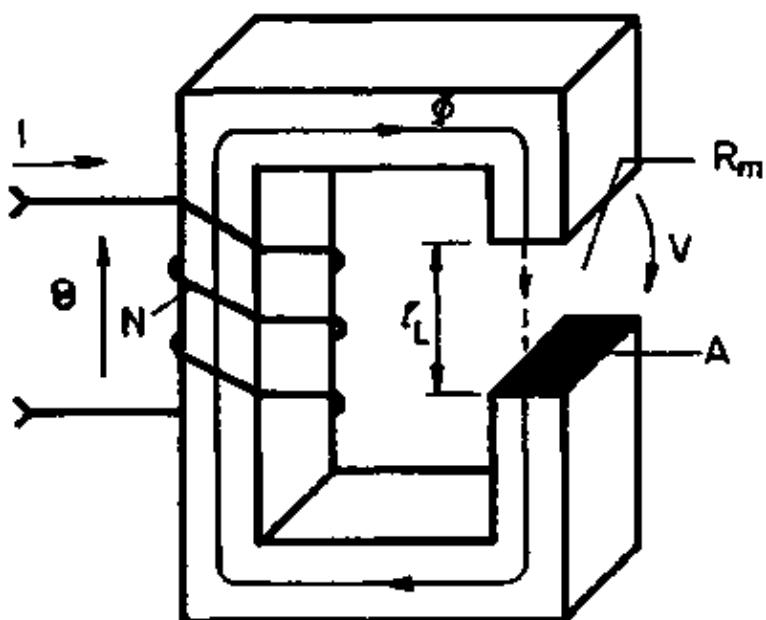


Fig. 5.7. Magnetic fundamental circuit

Analogously to the electric basic circuit, the following occurs: When an electrical current flows through the coil, a magnetomotive force  $\mathcal{M}$  is produced. It drives a magnetic flux  $\Phi^2)$  whose direction can be found with the help of the pole determination rule. Since air conducts the magnetic flux much worse than a ferromagnetic material, the air gap represents the decisive magnetic resistance or reluctance  $R_m$  in the magnetic circuit. The magnitude of the magnetic resistance is dependent on the geometric dimensions of the air gap. The magnetic flux now causes a magnetic potential drop  $V$  at the magnetic reluctance  $R_m$ ; this potential drop is – with a given  $\Phi$  – the greater, the greater  $R_m$  is. The magnetic potential difference, i.e. the magnetic potential related to the length of a magnetic resistance or reluctance, is called magnetic field-strength  $H$ . The latter is closely related with the magnetic flux density  $B$  (also known as magnetic induction) which indicates the magnetic flux which vertically penetrates a certain area. The material-specific conductivity for the magnetic flux is called permeability  $\mu$ . Mostly, the permeability is given as the product of the absolute permeability  $\mu_0$  times relative permeability  $\mu_r$ .

<sup>2)</sup>  $\Phi$  Greek letter phi

$$\mu = \mu_0 \mu_r$$

where:

- $\mu$  permeability
- $\mu_0$  absolute permeability (induction constant)
- $\mu_r$  relative permeability

The absolute permeability applies to vacuum and amounts to

$$\mu_0 = 1.256 \cdot 10^{-6} \frac{\text{Vs}}{\text{Am}}$$

Hence, the relative permeability  $\mu_r$  is a numerical factor without a unit and indicates how many times the magnetic flux is conducted better by a certain material than by a vacuum. For electrical engineering, a classification of materials with respect to permeability into two types will provide sufficient accuracy:

1. non-ferromagnetic materials = vacuum, air, copper, wood, water

For these materials we have  $\mu_r = \text{const. } \approx 1$

2. ferromagnetic materials = iron, nickel, cobalt and certain alloys

For these materials we have  $\mu_r = \text{const. } \gg 1$  (100 to 10,000)

Below, the above mentioned magnetic quantities are presented in the form of a list; further their mathematical correlations and their units are given. The shown fundamental equations can be applied easily. In analogy to the electrical circuits one has to observe that, for example, the magnetic flux in a branched magnetic circuit is divided into individual fluxes and the total magnetic resistance  $R_{\text{mers}}$  is smaller than the smallest individual resistance. In an unbranched magnetic circuit with several magnetic resistances connected in series, the total resistance  $R_{\text{mers}}$  is equal to the sum of the individual resistances; the sum of all magnetic potential drops is equal to the total magnetomotive force. This shows that, analogously, Ohm's law and the 1st and 2nd Kirchhoff's laws are applicable. Example 5.1. deals with an unbranched magnetic circuit with two magnetic resistances  $R_{mL}$  and  $R_{mFe}$  connected in series.

### Survey of important magnetic quantities

#### Formula signs and notations

$\mathcal{M}$	magnetomotive force
$V$	magnetic potential drop
$\Phi$	magnetic flux
$R_m$	magnetic resistance, also known as

	reluctance
? $\mu$	permeability
? <sub>0</sub>	absolute permeability
? <sub>r</sub>	relative permeability
H	magnetic field-strength
B	magnetic flux density

Equations      Units

$$? = NI \quad (5.1) \quad A$$

$$V = R_m \emptyset \quad (5.2) \quad A$$

$$\emptyset = \frac{V}{R_m} \quad (5.3) \quad Wb = V \cdot s$$

Wb = Weber in honour of the German physicist Wilhelm Eduard Weber (1804 – 1891)

$$R_m = \frac{V}{\emptyset} \quad (5.4)$$

$$R_m = \frac{1 \cdot 1}{\mu A} \quad (5.5) \quad \frac{1}{H} = \frac{A}{Wb} = \frac{A}{V \cdot s} \quad \frac{V \cdot s}{A} = H$$

= Henry in honour of the American physicist Josef Henry (1797 – 1878)

$$? = ?_0 ?_r \quad (5.6) \quad \frac{H}{m} = \frac{Wb}{A \cdot m} = \frac{V \cdot s}{A \cdot m}$$

$$H = \frac{V}{I} \quad (5.7) \quad \frac{A}{m}$$

$$B = \frac{\emptyset}{A} \quad (5.8)$$

$$B = ?H \quad (5.9) \quad T = \frac{Wb}{m^2} = \frac{V \cdot s}{m^2}$$

T = tesla in honour of the Yugoslavian physicist Nicola Tesla (1856 – 1943)

Example 5.1.

For the magnetic circuit shown in Fig. 5.7., the magnetic flux  $\emptyset$ , the magnetic flux density B, and the magnetic field-strengths in the ferromagnetic conductor  $H_{Fe}$  and in the air gap HL have to be found. The 250 turns of the coil carry 2 A; the length of the ferromagnetic conductor  $l_{Fe} = 80$  cm and its cross-sectional area  $A = 4 \cdot 4$  cm<sup>2</sup>. The relative permeability is assumed to be  $?_r = 600$ . The air gap length is  $l_L = 2$  mm.

Given:    N = 250                          To be found:     $\emptyset$

I = 2 A    B

$l_{Fe} = 80$  cm                                   $H_{Fe}$

$l_L = 2$  mm

$A = 4 \cdot 4$  cm<sup>2</sup> = 16 cm<sup>2</sup>

$?_T = 600$

$$\mu_0 = 1.256 \cdot 10^{-6} \frac{V \cdot s}{A \cdot m}$$

Solution:

$$\emptyset = \frac{\Theta}{R_{mers}}$$

According to equation (5.3)

According to equation (5.1)  $\emptyset = N I = 250 \cdot 2 A = 500 A$

$$R_{mers} = R_{mL} + R_{mFe}$$

According to equation (5.5)

$$R_{mL} = \frac{I_L}{\mu_0 A} = \frac{2 \cdot 10^{-3} m}{1.256 \cdot 10^{-6} \frac{V \cdot s}{A \cdot m} \cdot 16 \cdot 10^{-4} m^2}$$

$$= \frac{20 \cdot 10^6}{1.256 \cdot 16} \frac{A}{V \cdot s} \approx 10^6 \frac{A}{V \cdot s}$$

$$R_{mFe} = \frac{I_{Fe}}{\mu_0 \mu_r A} = \frac{80 \cdot 10^{-2} m}{1.256 \cdot 10^{-6} \frac{V \cdot s}{A \cdot m} \cdot 600 \cdot 16 \cdot 10^{-4} m^2}$$

$$\approx 0.665 \cdot 10^6 \frac{A}{V \cdot s}$$

=====

Pay attention to the fact that the ferromagnetic conductor has a smaller magnetic resistance than the air gap although the length of the former is 400 times that of the latter.

$$R_{mers} = 10^6 \frac{A}{V \cdot s} + 0.665 \cdot 10^6 \frac{A}{V \cdot s}$$

$$R_{mers} = 1.665 \cdot 10^6 \frac{A}{V \cdot s}$$

$$\emptyset = \frac{500 A}{1.665 \cdot 10^6 \frac{A}{V \cdot s}} = \frac{500 \cdot 10^{-6}}{1.665} V \cdot s = 300 \cdot 10^{-6} V \cdot s$$

$$\emptyset = 300 \text{ ?Wb}$$

=====

$B$  is in the air gap as great as in the ferromagnetic conductor because  $\emptyset$  (and  $A$ ) is equal everywhere.

According to equation (5.8)

$$B = \frac{\emptyset}{A}$$

$$B = \frac{300 \cdot 10^{-6} \text{ Wb}}{16 \cdot 10^{-4} \text{ m}^2} = 18.8 \cdot 10^{-2} \frac{\text{Wb}}{\text{m}^2}$$

$$B \approx 190 \text{ mT}$$

=====

According to equation (5.9)

$$H_{Fe} = \frac{B}{\mu_0 \mu_r} \quad (\mu_r H_{Fe} = \frac{B}{\mu_0})$$

$$H_{Fe} = \frac{190 \cdot 10^{-3} \frac{\text{Vs}}{\text{m}^2}}{1.256 \cdot 10^{-6} \frac{\text{V}\cdot\text{s}}{\text{A}\cdot\text{m}} \cdot 600} = \frac{1900 \text{ A}}{1.256 \cdot 6 \text{ m}} = 250 \frac{\text{A}}{\text{m}}$$

$$H_{Fe} \approx 250 \frac{\text{A}}{\text{m}}$$

=====

and

$$H_L = \frac{B}{\mu_0} = \mu_r H_{Fe}$$

$$H_L = 600 \cdot 250 \frac{\text{A}}{\text{m}} = 150000 \frac{\text{A}}{\text{m}}$$

$$H_L = 150,000 \frac{\text{A}}{\text{m}}$$

=====

Magnetic phenomena are caused by magnets. A distinction is made between permanent magnets and electromagnets. They exert forces of attraction on iron and steel.

The lines of force action are called magnetic field lines; they are closed in themselves, take their course from the north pole to the south pole outside of the magnet and run from the south pole to the north pole inside the magnet.

Every electrical current is associated with a magnetic field. It whirls around the current-carrying conductor. Its direction can be determined with the help of the screw rule.

A conductor wound in several turns is called coil. When it carries an electrical current, a magnetic field is also built up. Its direction can be found with the help of the pole determination rule.

A ferromagnetic body bunches up the field lines of a magnetic field. When this body has the shape of a ring, the space inside the ring is field-free. Consequently, magnetic screening is possible.

A coil energised by an electrical current in connection with a magnetic conductor and the magnetic resistances forms a magnetic circuit. By means of the defined quantities, this circuit can be dealt with mathematically like an electrical circuit.

Questions and problems:

1. Quote examples of magnetic phenomena in electrical components and devices!
2. Explain why in the air gap of the ferromagnetic circuit the field strength increases while the magnetic flux density remains constant!
3. What is the formal analogy between the electrical circuit and the magnetic circuit?  
Compare wiring diagrams and circuit diagrams, characteristics and fundamental equations!

## 5.2. Force Actions in a Magnetic Field

The forces occurring in a magnetic field are utilised in many ways for the construction and operation of machines and devices. There are three different types of forces:

- force on boundary areas of substances having different permeabilities (e.g. iron – air)
- force on currents in a magnetic field

- force between two currents

On the basis of the field line pattern, one can always draw conclusions regarding the force actions when one imagines that the field lines are subject to two forces:

The field lines try hard to shorten themselves; this means that they are subjected to a longitudinal force. The field lines in addition try hard to get away from each other as far as possible; this means that they produce a lateral force exerted upon each other.

#### Force in a magnetic field at boundary areas

Boundary areas are surfaces at which the permeability changes by leaps and bounds, e.g. the boundary surfaces of a piece of iron. A piece of iron in air is attracted by a magnet due to the shortening tendency of the field lines (Fig. 5.8). This can be effected by a permanent magnet and also by an electromagnet.

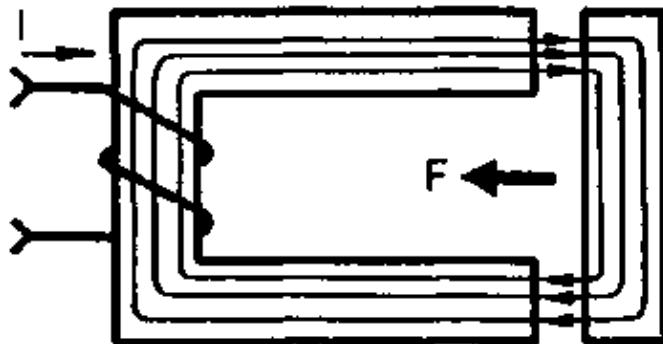


Fig. 5.8. Force in a magnetic field at boundary areas

Electromagnets are preferred because they attain greater magnet flux densities. The force with which a ferromagnetic surface is drawn towards the non-ferromagneticum is

$$F/N = 40 (B/T)^2 \text{ A/cm}^2$$

where:

F pull of one magnetic pole in N = newton

B magnetic flux density in T

A pole area in cm<sup>2</sup>

In electromagnets, the magnetic field is produced by the electric current. This shows the advantage that the great forces can be controlled conveniently and from a place remote from the magnet by changing the current.

The basic form of all electromagnets is the coil with iron core and air gap (Fig. 5.9.). The part carrying the winding is called core (1), the connecting pieces in the ferromagnetic circuit yoke (2), and the movable ferromagnetic piece is termed as armature (3).

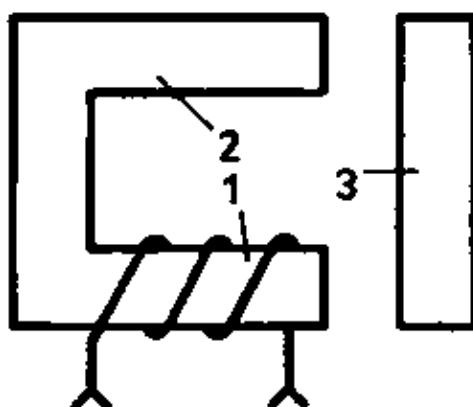


Fig. 5.9. Basic form of an electromagnet

1 Core; 2 Yoke; 3 Armature

Electromagnets are used in large numbers in all fields of engineering. Here are a few characteristic examples:

- lifting magnets (see Fig. 5.10.); they are suitable for lifting iron parts having a large mass
- chucks for clamping and holding workpieces on machine tools
- electromagnetic clutches. They can be operated conveniently; their power of coupling is adjustable even from larger distances without steps
- relays (Fig. 5.11.) for the electromagnetic control of switching actions in telecommunication and telecontrol engineering
- headphones in telephony
- electromagnetic loudspeakers

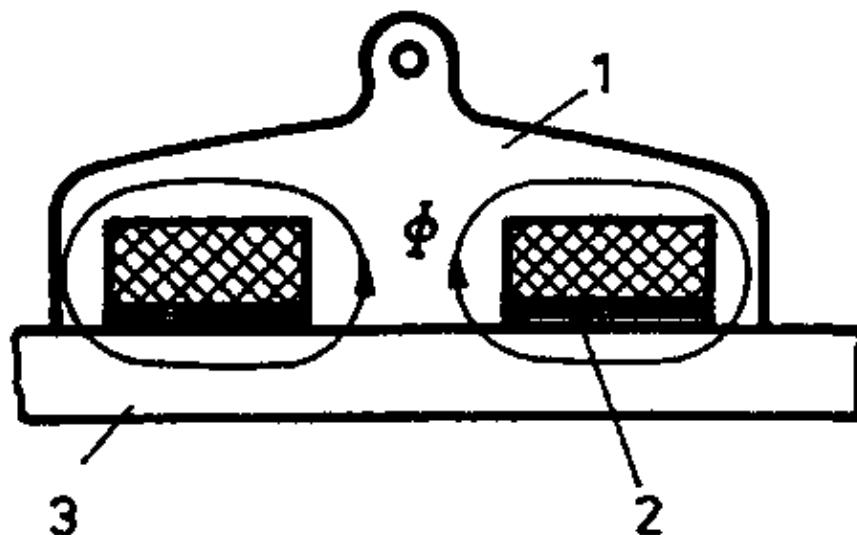


Fig. 5.10. Lifting magnet

1 Steel casting; 2 Manganese steel (non-magnetic) 3 Workpiece of iron

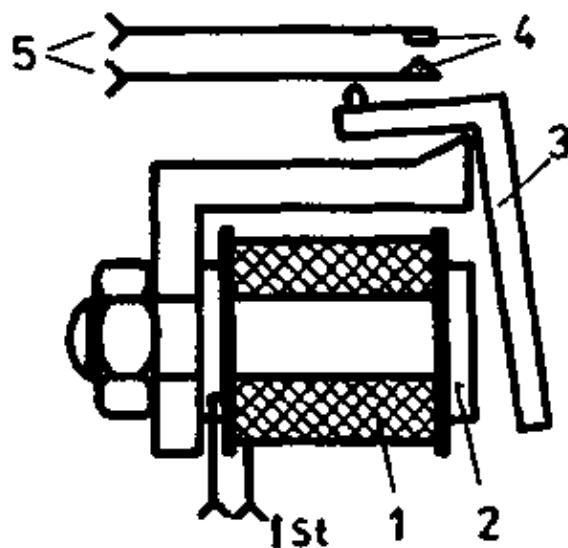


Fig. 5.11. Relay

1 Winding; 2 Core; 3 Armature; 4 Working contacts; 5 Connecting receptacles for the working contacts

Example 5.2.

Calculate the force with which a relay armature is attracted when the magnetic flux density in the air gap is 0.5 T and the pole area 0.25 cm<sup>2</sup>!

Given: B = 0.5 T

To be found: F in N

$$A = 0.25 \text{ cm}^2$$

Solution:  $F/N = 40 (B/T)^2 \text{ A/cm}^2$

$$F/N = 40 \cdot 0.5^2 \cdot 0.25$$

$$F/N = 40 \cdot 0.25 \cdot 0.25$$

$$F/N = 40 \cdot 0.0625 = 2.5$$

$$F = 25 \text{ N}$$

=====

#### Force exerted upon currents in a magnetic field (electro-dynamic force)

When in a homogeneous magnetic field, a current-carrying conductor is placed according to Fig. 5.12., a force is exerted on this conductor in the direction indicated by an arrow.

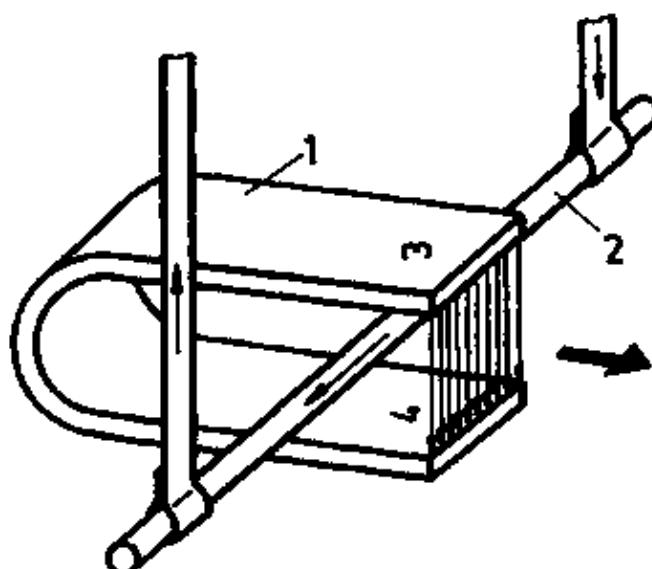
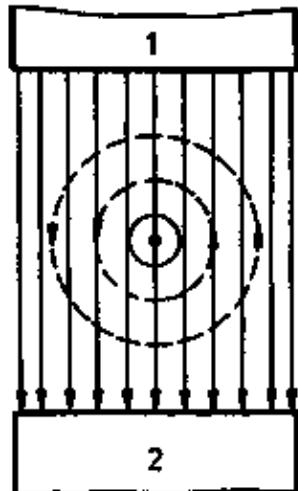


Fig. 5.12. Current-carrying conductor in a magnetic field

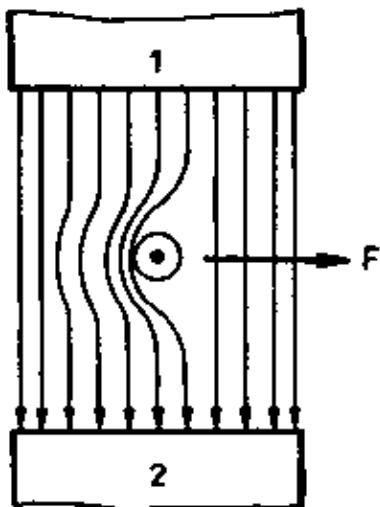
1 Horseshoe magnet; 2 Movable current-carrying conductor; 3 North pole; 4 South pole

This phenomenon can be explained easily. The vortex field of the current-carrying conductor (hatched area in Fig. 5.13.) is superimposed on the present field (thin lines). In the directional conditions selected in the Figs. 5.12. and 5.13., a field weakening is obtained to the left of the conductor whereas a field strengthening is obtained to the right. The resultant field is represented by broad lines. From the force actions of the field lines (longitudinal force – lateria force), a force F perpendicular to the conductor in the direction of the lowest magnetic flux density is obtained. This force is has the greatest value when the conductor is perpendicular to the field because then the most intense densification of field line takes place. When reversing the current direction or the direction of the field, the force brought about will act to the opposite direction. When current and field direction are reversed at the same time, the force action remains in the same direction. For the direction of the action of force, the left-hand rule (motor rule) holds.

Fig. 5.13. Electrodynanic force



a) Superposition of the present homogeneous field on the field produced by the conductor



b) Resulting magnetic field

1 North pole; 2 South pole

When extending the opened left hand into the magnetic field in such a way that the field lines enter the inner palm and the extended fingers point in the direction of the current flow, the thumb spread out indicates the force exerted on the conductor (Fig. 5.14.).

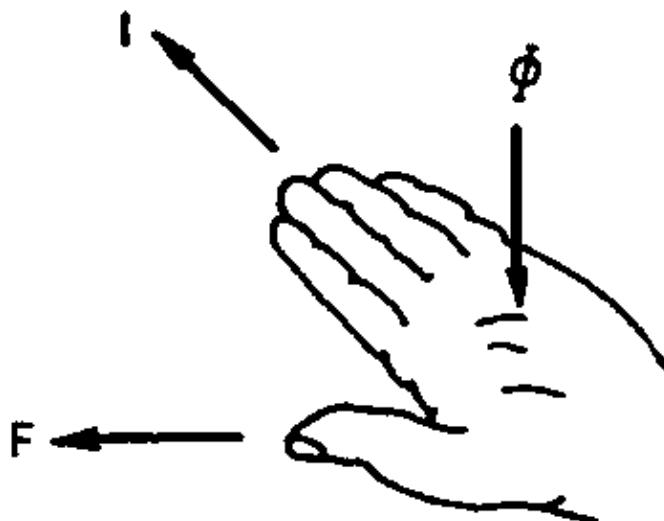


Fig. 5.14. Left-hand rule (motor rule)

The magnitude of force is derived from the electrodynamic law of force:

$$F = B I l \quad (5.11.)$$

where:

- F force exerted on the current-carrying conductor
- B magnetic flux density of the homogeneous magnetic field
- I current intensity in the conductor
- l length of the conductor added to the field

$$M = [B] [I] [l]$$

$$[F] = T \cdot A \cdot m = \frac{V \cdot s}{m^2} \cdot A \cdot m$$

$$[F] = \frac{V \cdot A \cdot s}{m} = \frac{W \cdot s}{m} \quad \text{and with } \frac{W \cdot s}{m} = N \text{ we have}$$

$$[F] = N$$

Example 5.3.

Between the 20 cm wide poles of a magnet there is a magnetic flux density of 0.5 T. A conductor arranged vertically with respect to the field carries a current of 5 A. Calculate the force exerted on the conductor!

Given:  $B = 0.5 \text{ T}$  To be found:  $F$

$$I = 5 \text{ A}$$

$$l = 20 \text{ cm}$$

Solution:  $F = B I l$

$$F = 0.5 \frac{V \cdot s}{m^2} \cdot 5A \cdot 0.2m$$

$$F = 0.5 \frac{V \cdot A \cdot s}{m} = 0.5 \frac{W \cdot s}{m}$$

$$F = 0.5 \text{ N}$$

The electrodynamic force is remarkable and technically of greatest importance.

On the basis of the force action described above, the motor principle can be explained. A conductor loop or a coil is arranged in a homogeneous magnetic field (Fig. 5.15.).

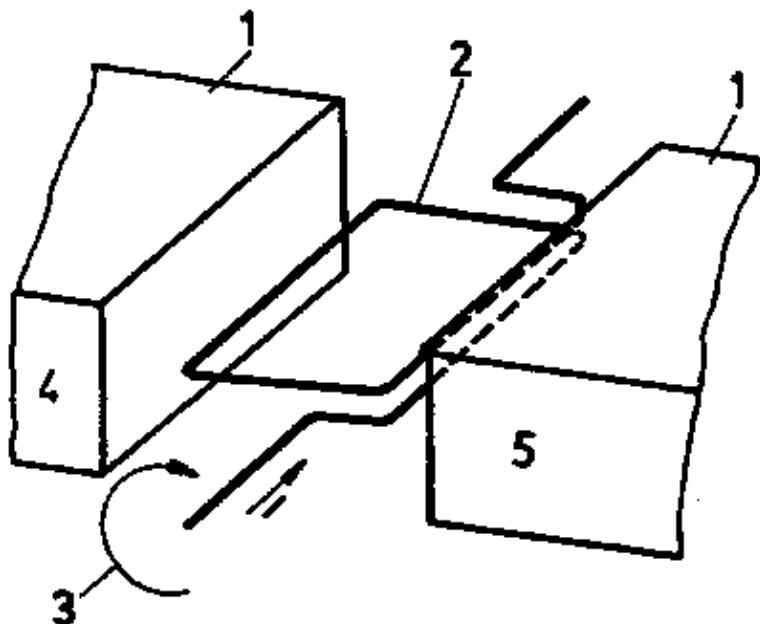


Fig. 5.15. Current-carrying conductor loop in a magnetic field

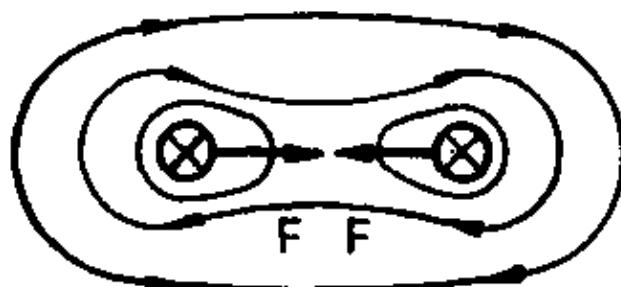
1 Magnet poles; 2 Pivoted conductor loop; 3 Sense of rotation; 4 North pole; 5 South pole

It is pivoted so that it can be rotated about its central axis which is perpendicular to the direction of the field. When a current of sufficient intensity passes through this coil, it will be subjected to a rotary motion the direction of which can be determined with the help of the left-hand rule. Since the force acting according to equation (5.11.) is exerted on adequate force arms (radius of the conductor loop), the torque according to equation (4.7.) is obtained which has been dealt with in Section 4.4. The motor principle is the basis of a series of electrical measuring instruments and electric motors.

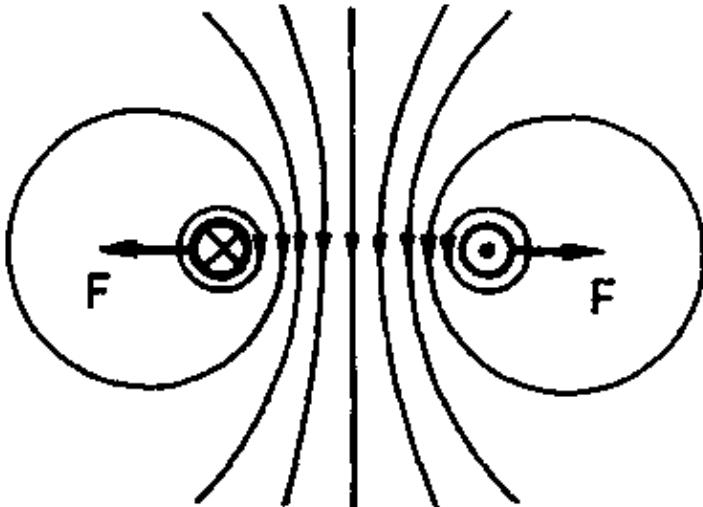
#### Force between two currents

Between two parallel current-carrying conductors force actions are attained by the superposition of magnetic fields. The field patterns (Fig. 5.16.) show:

Fig. 5.16. Force between two currents



a) Currents flowing in the same direction



b) Currents flowing in opposite directions

Equidirectional currents attract themselves due to the shortening tendency of the field lines; currents of opposite direction repel each other due to the widening tendency of the field lines.

The forces occurring are small; they should be taken into account in case of high currents (e.g. short-circuits). Therefore, bus-bar systems, heavy-duty windings in transformers and current-limiting reactors must be mechanically stiffened and reliably fastened, taking a high safety factor into account.

In a magnetic field, different force actions occur. All of them are due to the shortening and widening tendencies of the field lines.

Force actions in a magnetic field at boundary surfaces are primarily caused by means of electromagnets. Decisive is the fact that the current, which passes through a coil, exerts an attractive force on ferromagnetic bodies. This force is directed in such a way that the magnetic resistance is reduced. The value of the force is  $F \sim B^2$  and can be determined with the help of equation (5.10.).

Force actions on current-carrying conductors in a magnetic field result from the superposition of two magnetic fields. The direction of force can be determined with the help of the left-hand rule; its magnitude can be derived from equation (5.11.) (it can be considerable). The motor principle which can be derived from these actions is the basis of the designing of electric motors and, thus, technically of greatest importance.

Forces also occur between current-carrying conductors, namely, an attractive force with equidirectional currents and a repelling force with currents flowing opposite to each other. But only with very intensive currents (e.g. short-circuit) these forces reach noticeable magnitudes which then have to be taken into consideration.

Questions and problems:

1. At the two poles of  $15 \text{ mm} \times 20 \text{ mm}$  each of an electromagnet, a magnetic flux density of  $645 \text{ mT}$  is present. Calculate the force with which the armature attached to the two poles is attracted!
2. Calculate the induction in front of the armature of a relay which shows a pull of  $2.5 \text{ N}$  with a pole area of  $0.25 \text{ cm}^2$ !
3. Explain why a magnetic field exerts a force on a current-carrying conductor and in which direction this force acts!
4. Calculate the force with which a wire carrying a current of  $20 \text{ A}$  will turn aside; the wire runs through the  $5 \text{ cm}$  wide homogeneous magnetic field of a magnetic flux density of  $850 \text{ mT}$  at right angles!
5. Under which conditions occurs a) a repellent and b) an attracting force action exerted by two parallel current-carrying conductors? (substantiation!)

## 5.3. Electromagnetic Induction

### 5.3.1. The General Law of Induction

The phenomenon of electromagnetic induction was discovered by the English physicist Michael Faraday (1791 – 1867) in 1831. The law of induction, which was named after him, is of paramount importance to electrical engineering. Together with Biot–Savart's law (see Section 5.1.), it forms the theoretical basis of all electromagnetic phenomena and numerous technical applications such as generators, motors, transformers, measuring instruments, etc.

In Section 2.2. (see Fig. 2.7.) we have already pointed out that a primary electromotive force is produced in a conductor loop encircling a magnetic flux when this flux changes. A distinction is made between two types:

- induction of rest
- induction of motion

The two processes can also take place at the same time; then the phenomena of induction, which will be dealt with separately below, are superimposed.

In the process of induction of rest, a conductor at rest (a conductor loop, a coil) is penetrated by a magnetic field changing as to time. This may be effected, for example, by approaching a permanent magnet to a coil and then withdrawing it in the manner shown in Fig. 5.17.

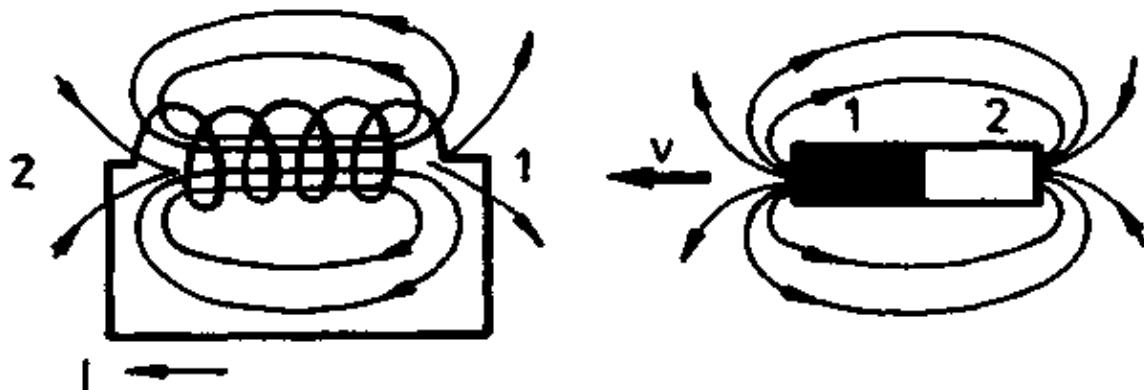


Fig. 5.17. Generation of primary electromotive force by magnetic flux variation

1 North pole; 2 South pole

It is obvious that during the approaching of the permanent magnet the magnetic flux penetrating the coil becomes greater; during withdrawing, this flux diminishes again. It is found that, in the coil, a primary electromotive force is produced as long as the magnetic flux penetrating the coil changes. The electromotive force is the greater, the quicker the magnetic flux changes, in other words, the higher the speed of approach and the greater the number of turns in the coil. The direction of the electromotive force is derived from the law of conservation of energy. When, in the manner demonstrated by Fig. 5.17., a permanent magnet is approached to a coil, then at the side facing the magnet, a homonymous magnetic pole is formed. It exerts a repellent force on the magnet to be approached. Hence, a work has to be done against the repellent force exerted by the coil through which the induced current flows. This work is the equivalent of the electrical energy obtained. When withdrawing the magnet from the coil, the conditions are inverted. These facts were described by the Russian physicist Heinrich Friedrich Emil Lenz (1804 – 1865) and, called Lenz law, run as follows:

The induced electromotive force is always in such direction that, if it produces a current, the magnetic effect of that current opposes the flux variation responsible for both electromotive force and current.

A positive induced voltage is caused by a negative flux variation (flux reduction). Mathematically, this is expressed by a negative sign (“-”).

Consequently, as the general law of induction we have:

$$E = -N \frac{\Delta \emptyset}{\Delta t} \quad (5.12.)$$

where:

E induced electromotive force in the coil  
N number of turns

$\frac{\Delta \emptyset}{\Delta t}$  speed of variation of the magnetic flux

$$[E] = \frac{Wb}{s} = \frac{V \cdot s}{s} = V$$

Example 5.4.

In a coil having 1200 turns, the magnetic flux varies within 50 ms from 7.5 ?Wb to 70 ?Wb. Calculate the value of the electromotive force induced in this coil!

Given: N = 1200 To be found: |E|

$$\frac{\Delta \emptyset}{\Delta t} = \frac{(70 - 7.5)\mu Wb}{50ms}$$

Solution:  $E = -N \frac{\Delta \emptyset}{\Delta t}$

$$|E| = N \frac{\Delta \emptyset}{\Delta t}$$

$$|E| = 1200 \frac{62.5 \cdot 10^{-6} V \cdot s}{50 \cdot 10^{-3} s} = 1200 \cdot 1.25 \cdot 10^{-3} V$$

$$|E| = 1.5 V$$

=====

An induction of rest also takes place when – according to the demonstration given in Fig. 5.18. – two coils are magnetically coupled and one is energised. Coil 1 (primary coil) has a number of turns expressed as N<sub>1</sub>; the current I<sub>1</sub> passing through them can be interrupted by a switch. Coil 2 (secondary coil) has a number of turns expressed as N<sub>2</sub> and is connected with a load.

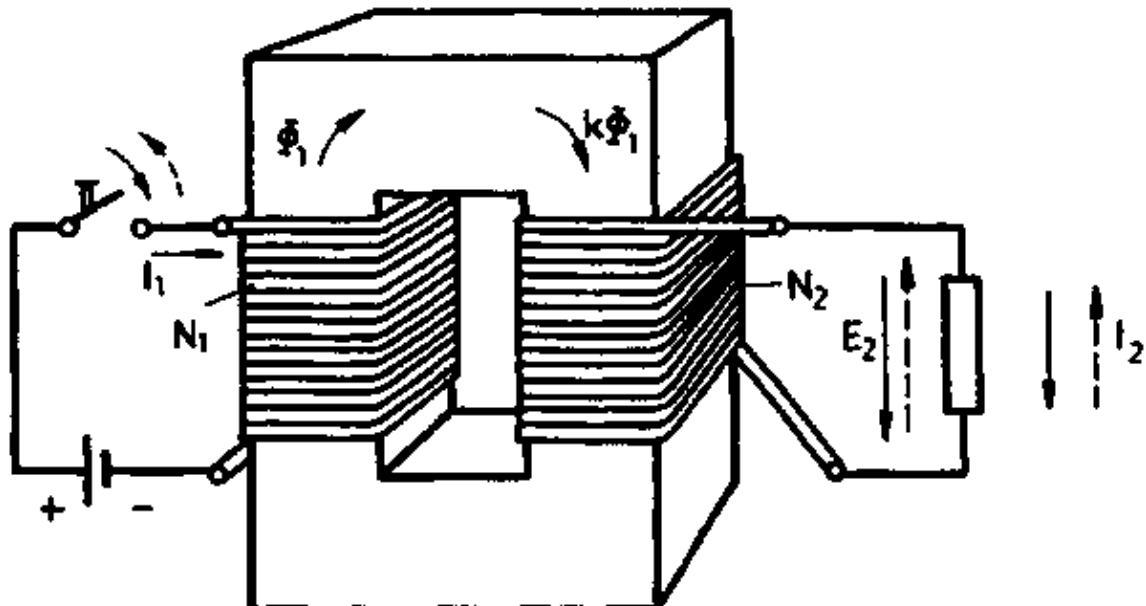


Fig. 5.18. Induction of rest in magnetically coupled coils

When the switch is closed, a current flows through the primary coil and produces a magnetomotive force. It drives a magnetic flux  $\Phi_1$  whose greater part, namely  $k\Phi_1$ , also penetrates the secondary coil. The quantity  $k$  is called coefficient of coupling and indicates how many per cent of the produced magnetic flux penetrates the matching coil. In closed ferromagnetic circuits (as in the present example)  $k \approx 1$  (or 100 %).

$$E = -N_2 \frac{\Delta k\Phi_1}{\Delta t}$$

Consequently, an electromotive force of  $E = -N_2 \frac{\Delta k\Phi_1}{\Delta t}$  is induced in the secondary coil according to equation (5.12.).

When the switch is opened, the magnetic flux diminishes and the voltage induced in the secondary coil inverts its direction.

$$\Phi_1 = \frac{N_1 I_1}{R_m}$$

Since  $\Phi_1 = \frac{N_1 I_1}{R_m}$  [see equations (5.3.) and (5.1.)], hence,  $\Delta\Phi_1 = \frac{N_1}{R_m} \Delta I_1$ , we have

$$E_2 = -N_2 \frac{k N_1 \cdot \Delta I_1}{R_m \Delta t}$$

This process is called mutual induction. The latter equation indicates that, in the event of a current variation in the primary coil, an electromotive force is induced in the secondary coil whose magnitude is proportional to

$\frac{\Delta I_1}{\Delta t}$  the rate of current change  $\frac{\Delta I_1}{\Delta t}$ . The conditions are analogous when secondary and primary coils are exchanged. The proportionality factor is called mutual inductance M.

$$M = k \frac{N_1 N_2}{R_m} \quad (5.13.)$$

where:

M mutual inductance

k coefficient of coupling

$N_1$  number of turns in coil 1

$N_2$  number of turns in coil 2

$R_m$  magnetic resistance

$$[M] = \frac{1}{V \cdot s} = \frac{V \cdot s}{A} = H$$

$$E_2 = -M \frac{\Delta I_1}{\Delta t} \quad (5.14.)$$

where:

$E_2$  electromotive force induced in coil 2  
 $M$  mutual inductance

$\frac{\Delta I_1}{\Delta t}$   
 $\Delta t$  rate of current change in coil 1

$$[E_2] = \frac{V \cdot s \cdot A}{A \cdot s} = V$$

Example 5.5.

The mutual inductance of two coils is 100 mH. Calculate the electromotive force induced in the secondary coil when the current in the primary coil is uniformly changed by 500 mA within 20 ms!

Given:  $M = 100 \text{ mH}$  To be found:  $E_2$

$$\frac{\Delta I_1}{\Delta t} = \frac{500 \text{ mA}}{20 \text{ ms}}$$

Solution:  $E_2 = -M \frac{\Delta I_1}{\Delta t}$

$$E_2 = -100 \text{ mH} \cdot \frac{500 \text{ mA}}{20 \text{ ms}}$$

$$E_2 = -100 \cdot 10^{-3} \frac{V \cdot s}{A} \cdot 25 \frac{A}{s} = -2500 \cdot 10^{-3} V$$

$$E_2 = -2.5 V$$

=====

$$E_2 = -2.5 V$$

The general law of induction does not state anything about the origin of the magnetic flux but only the fact that, as a consequence of changes in the magnetic flux, voltages are induced in the turns of a coil encircling it. When we omit the secondary coil in Fig. 5.18., the following conditions are brought about:

When we close the switch, a current passes through the coil producing a magnetomotive force. The latter drives a magnetic flux which penetrates all turns of the coil generating an electromotive force in these turns. When the switch is opened, the magnetic field breaks down, the magnetic flux is reduced to zero. This reduction of flux will also cause an induced voltage in the coil itself. This process of production of electromotive force in the turns of the coil generating the field is called self-induction.

The magnitude of the self-induced voltage can easily be determined on the basis of the general law of induction.

$$E = -N \frac{\Delta \emptyset}{\Delta t}$$

Since  $\emptyset = \frac{NI}{R_m}$  [see equations (5.3.) and (5.1.)], hence

$$\emptyset = \frac{N}{R_m} \Delta I \quad \text{we have}$$

$$E = -N \frac{N}{R_m} \frac{\Delta I}{\Delta t}$$

The latter equation indicates that, in a current-carrying coil, an electromotive force is induced by the magnetic flux associated with the current in the coil; the magnitude of the electromotive force is proportional to the rate

of current change  $\frac{\Delta I}{\Delta t}$ .

The proportionality factor is called inductance L (more precisely self-inductance).

$$L = \frac{N^2}{R_m} \quad (5.15.)$$

where:

L self-inductance

N number of turns of the coil

R<sub>m</sub> magnetic resistance

$$[L] = \frac{1}{V \cdot s} = \frac{V \cdot s}{A} = H$$

$$E = -L \frac{\Delta I}{\Delta t} \quad (5.16.)$$

where:

E = self-induced voltage

L self-inductance

$\frac{\Delta I}{\Delta t}$  rate of current change in the coil

$$[E] = \frac{V \cdot s}{A} \cdot \frac{A}{s} = V$$

### Example 5.6.

A coil has a self-inductance of 1.35 H. Calculate the self-induced voltage when the current is uniformly reduced by 3.7 mA within 20  $\mu$ s.

Given: L = 1.35 H      To be found: E

$$\frac{\Delta I}{\Delta t} = \frac{3.7 \text{ mA}}{20 \mu\text{s}}$$

Solution:  $E = -L \frac{\Delta I}{\Delta t}$

$$E = -1,35 H \cdot \left( \frac{3,7 \text{ mA}}{20 \mu\text{s}} \right) = 1,35 \frac{\text{V} \cdot \text{s}}{\text{A}} \cdot 185 \frac{\text{A}}{\text{s}}$$

$$E = 250 \text{ V}$$

=====

The correlation between mutual inductance and self-inductance results from the equations (5.13) and (5.15.)

$$M = k \frac{N_1 N_2}{R_m} \rightarrow M^2 = k^2 \frac{N_1^2}{R_m} \frac{N_2^2}{R_m}$$

$$L = \frac{N^2}{R_m}$$

with  $L = \frac{N^2}{R_m}$  we have  $M = k L L$

$$M = k \sqrt{L_1 L_2} \quad (5.17.)$$

The mutual inductance of two magnetically coupled coils is equal to the product of coupling factor times geometric mean of the self-inductances.

In case of the induction of motion, a conductor (a conductor loop, a coil) is moved through a magnetic field constant as to time. This may be effected, for example, by moving a conductor loop through a homogeneous magnetic field having the magnetic flux density  $B$ , length  $l$  and width  $s$  in accordance with Fig. 5.19.

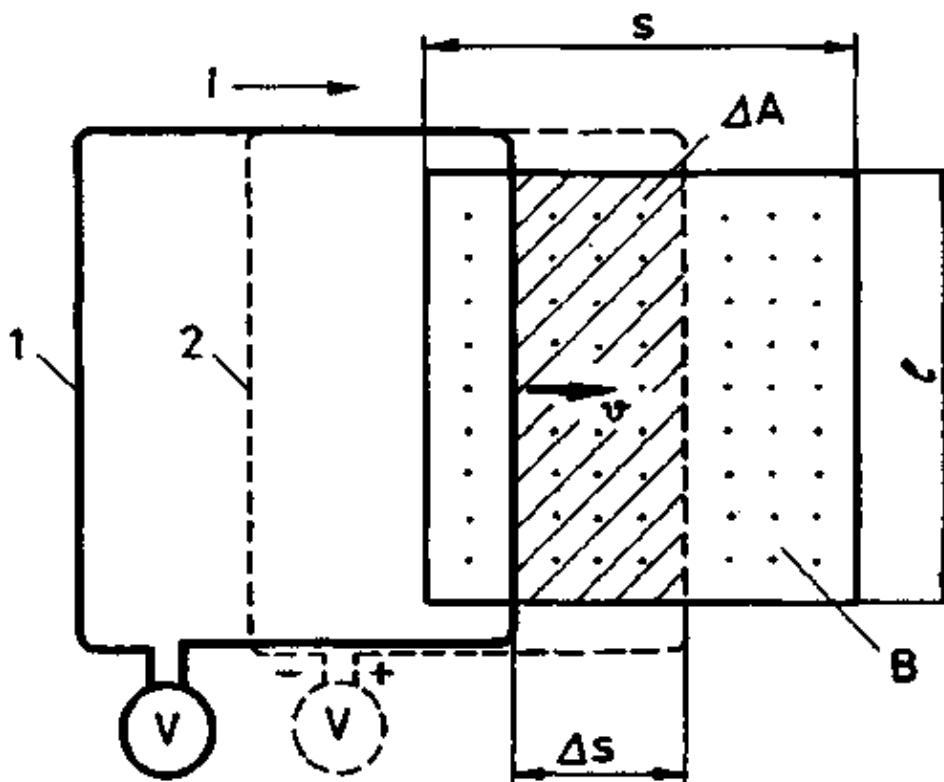


Fig. 5.19. Induction of motion

1 Position of the conductor at the beginning of motion

2 Position of the conductor after a certain period

Assume, the conductor loop is moved at a constant speed  $v$ . At the commencement of motion, the conductor loop has the position 1 and after a certain period of time  $\Delta t$ , position 2. During the time  $\Delta t$ , the distance  $\Delta s$  has

$$v = \frac{\Delta s}{\Delta t}$$

been covered, a fact, which corresponds to a rate of motion of  $\frac{\Delta s}{\Delta t}$ . The magnetic flux encircled by the conductor loop becomes greater by the share which, in this time, enters through the area  $\Delta A$ . Thus, the

induced voltage for N turns is

$$E = -N \frac{\Delta \Phi}{\Delta t}$$

$$\Delta \Phi = B \cdot A$$

$$\Delta A = l \cdot s$$

$$E = -N \frac{B \cdot l \cdot \Delta s}{\Delta t}$$

$$v = \frac{\Delta s}{\Delta t} \text{ we have}$$

$$E = -N B l v \quad (5.18.)$$

where:

E induced voltage

N number of turns of the coil

B magnetic flux density of the constant magnetic field

l length of the magnetic field v rate of motion of the coil or conductor

$$[E] = \frac{V \cdot s}{m^2 \cdot m \cdot \frac{m}{s}} = v$$

Example 5.7.

A wire is moved at a constant speed of 12.5 cm/s across a homogeneous magnetic field of 500 mT and 8 cm in length. Calculate the value of the voltage induced in this wire!

Given:  $N = 1$  (coil with 1 turn!) To be found:  $|E|$

$$B = 500 \text{ mT}$$

$$l = 8 \text{ cm}$$

$$v = 12.5 \text{ cm/s}$$

Solution:  $E = -N B l v$

$$|E| = N B l v$$

$$|E| = 1 \cdot 0.5 \text{ T} \cdot 8 \text{ cm} \cdot 12.5 \frac{\text{cm}}{\text{s}}$$

$$|E| = 1 \cdot 0.5 \frac{V \cdot s}{m^2} \cdot 8 \cdot 10^{-2} \cdot 12.5 \cdot 10^{-2} \frac{m}{s} = 50 \cdot 10^{-4} V$$

$$|E| = 5 \text{ mV}$$

=====

According to the Lenz law, the direction of the induced electromotive force is such that the magnetic field caused by the induced current acts against the cause of origin, in this case an increase in flux. From this follows the indicated current direction. For the direction of the induced voltage or of the current driven by this voltage in the induction of motion, the right-hand rule (generator rule) holds in general.

When extending the opened right hand into the magnetic field in such a way that the field lines enter the inner palm and the spread out thumb points in the direction of motion of the conductor, then the extended fingers point in the direction of the induced electromotive force (Fig. 5.20).

### 5.3.2. Utilisation of the Phenomena of Induction

Phenomena of induction are utilised very frequently in engineering. The examples given below are a very limited selection of typical applications.

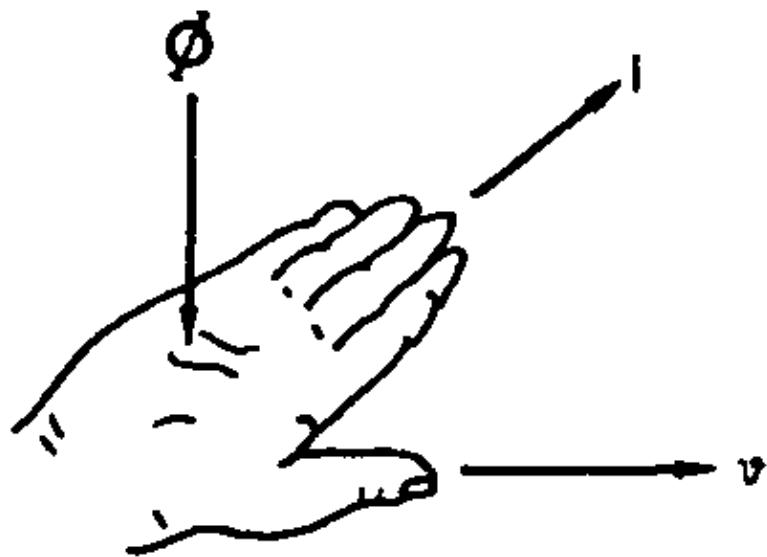


Fig. 5.20. Right-hand rule (generator rule)

- Magnetic head of magnetophone (induction of rest)

The magnetophone process is a high-grade sound storage method. The sound store is a thin (18 to 50  $\mu\text{m}$ ) polyester or acetate tape with a ferromagnetic film applied to it. The sound information is entered in the tape in the form of a more or less intense magnetisation in the running direction of the tape and thus stored. For the reproduction of the stored information, a magnetic replay head is required. This is a magnetic circuit with a very small air gap (1 to 100  $\mu\text{m}$ ) which carries an induction coil. The magnetised tape is transported past the air gap. The flux caused by the individual "permanent magnets" of the tape in the ferromagnetic circuit induces a voltage proportional to the sound information in the induction coil surrounding the circuit (Fig. 5.21.). This voltage is amplified and fed to a loudspeaker.

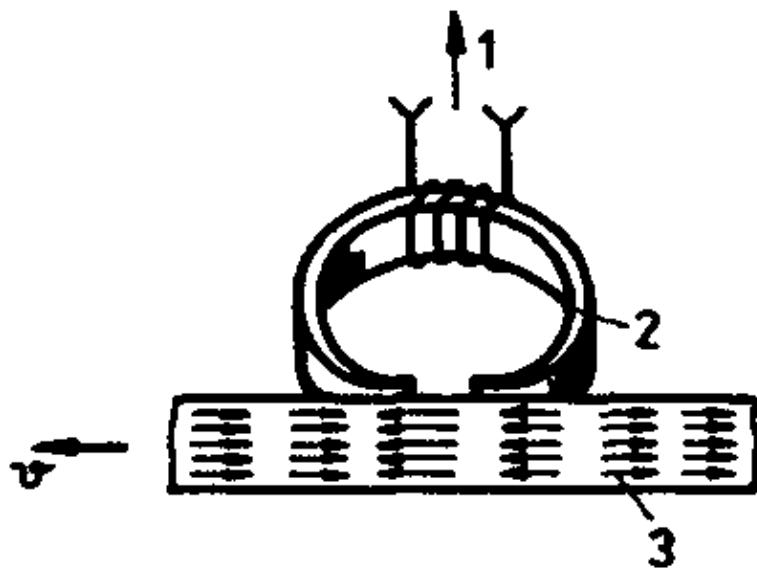


Fig. 5.21. Reproducing head of a magnetic recorder

1 to amplifier; 2 Iron core; 3 Magnetic tape

- Transformer (induction of rest)

The transformer is a stationary electrical machine and one of the most important components of electrical engineering. In accordance with Fig. 5.18., the transformer is provided with two coils which are galvanically separated but magnetically coupled. When a voltage is applied to the primary coil which periodically changes

as to magnitude and direction (such a voltage is called alternating voltage, see Section 7), a magnetic field is produced in both coils which also changes periodically its magnitude and direction. In an ideal case, a 100 per cent coupling is effected, i.e. the coefficient of coupling  $k = 1$ . Then the magnetic flux  $\Phi_1$  completely penetrates the secondary coil as  $\Phi_2$ , hence,  $\Phi_1 = \Phi_2 = \Phi$  (Fig. 5.22.).

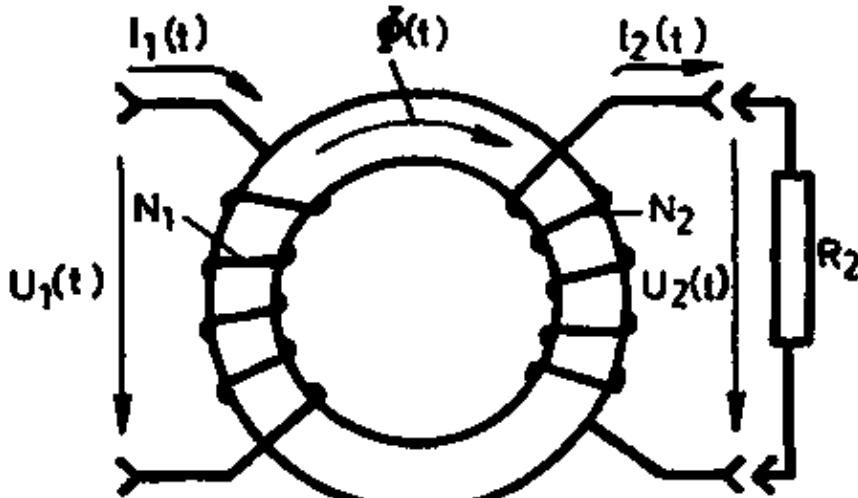


Fig. 5.22. Transformer

According to the law of induction, the induced voltage is directly proportional to the number of turns when the rate of flux variation is given.

The ratio of the primary voltage to the secondary voltage is called ratio of transformation  $tr_r$  and is written as

$$tr_r = \frac{U_1}{U_2} = \frac{N_1}{N_2} \quad (5.19.)$$

In a loss-free transformer, the ratio of the voltages is equal to the ratio of turns in the coils.

According to the law of conservation of energy, the primary power must be equal to the secondary power, hence,  $P_1 = P_2$ .

According to Section 4.1., equation (4.3.), power is written as  $P = UI$ , that is to say,

$$U_1 I_1 = U_2 I_2$$

$$\frac{U_1}{U_2} = \frac{I_1}{I_2}$$

Taking equation (5.19.) into account, we have

$$tr_r = \frac{U_1}{U_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} \quad (5.19a)$$

In a loss-free transformer, the currents are in inverse ratio to the numbers of turns of the coils.

This shows that a given alternating voltage can be transformed into any desired higher or lower alternating voltage by means of a transformer. Therefore, the transformer is an important connecting link between energy generator and the distribution network or between the distribution network and the consumers. In information electrical engineering, the transformer is frequently used for impedance matching. Since  $P_1 = P_2$ , we have in accordance with Section 4.1.,

$$I_1^2 R_1 = I_2^2 R_2$$

$$R_1 = \left( \frac{I_2}{I_1} \right)^2 R_2 \quad \text{with } tr_r = \frac{I_2}{I_1} \text{ we read}$$

$$R_1 = tr_r^2 R_2 \quad (5.19b)$$

The load resistance  $R_2$  acts on the primary with the square of the transmission ratio.

- Generator (induction of motion)

In a homogeneous magnetic field, a conductor loop or a coil is arranged. If it is turned about its central axis which is perpendicular to the field direction, then the magnetic flux penetrating the coil area varies (see Fig. 5.23.).

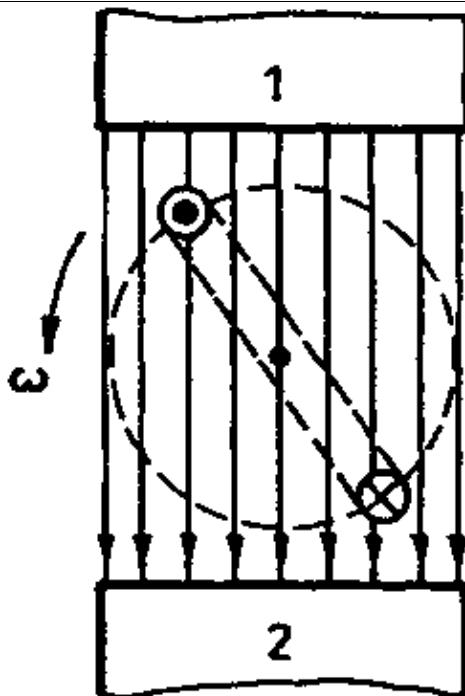


Fig. 5.23. Generator (principle)

1 North pole; 2 South pole

Since, according to the law of induction, any change of the magnetic flux causes an electromotive force, a voltage is induced in the rotating coil. Its direction can be determined with the help of the right-hand rule. A generator is the inversion of the motor principle described in Section 5.2. Generalising, we can say that a pivoted coil in a magnetic field is the basic design of all rotating electrical machines (motors, generators).

- Eddy-current brake

In planar conductors voltages are induced by magnetic flux variations in the same manner as in wires and coils. The induction currents associated with these induced voltages are high because the current paths in a planar conductor are closed in themselves and act as short-circuits (see Fig. 5.25.). An experiment sketched in Fig. 5.24. shows the action of such induced currents. A metal plate of copper (or of another electrically conductive material such as aluminium) is suspended in such a way that it is allowed to swing through a magnetic field like a pendulum. In this manner, currents are induced which, according to the Lenz rule, built up magnetic fields of opposite direction and thus damp the motion. The pendulum will come to rest very quickly.

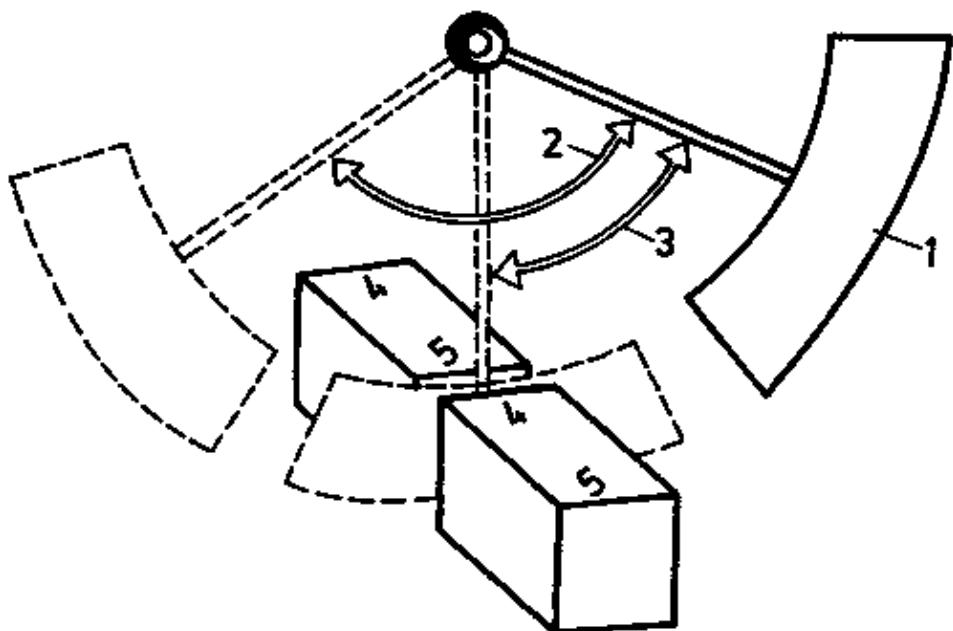


Fig. 5.24. Electromagnetic induction in planar conductors

1 Pendulum of non-ferromagnetic conductor material; 2 Total deflection of oscillation; 3 Oscillation in the magnetic field; 4 North pole; 5 South pole

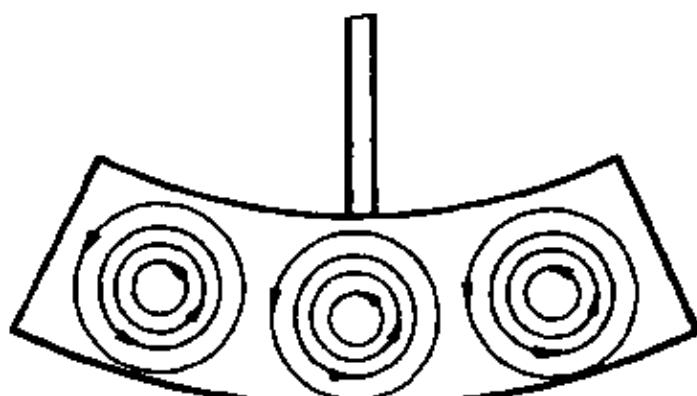


Fig. 5.25. Eddy currents in planar conductors

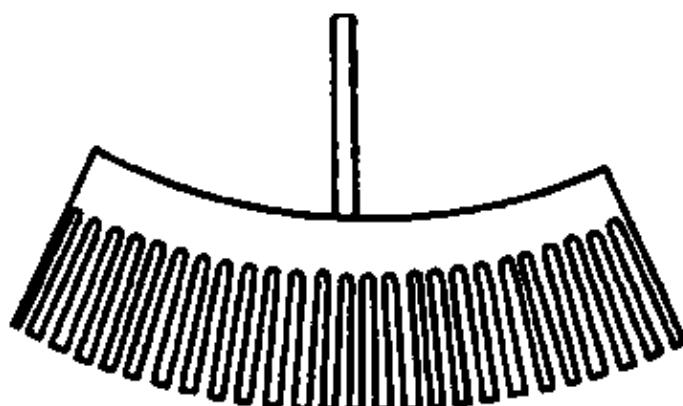


Fig. 5.26. Reduction of the eddy currents by means of slots in planar conductors

Fig. 5.25. shows the closed current paths in the metal surface. Because of the apparently irregular course taken by the current, these currents are called eddy currents. Eddy currents can be avoided to a great extent when fine slots are made into the metal surface as shown in Fig. 5.26. The pendulum of such a slotted metal plate will hardly be damped; the braking action and thus the eddy-current formation are cancelled to a great extent.

Eddy-current brakes operate on the above described principle. They are used now and then for the braking of rail vehicles, for damping the deflection of electrical indicator operating mechanisms, and for braking electrical machines.

In most cases, eddy currents are not desired. They occur both by induction of rest in stationary electrical machines and by induction of motion in rotating electrical machines. Because of their short – circuit character, they heat the metal mass involved, thus, uselessly doing work. These eddy-current losses must be avoided as far as possible. This is achieved by avoiding compact metal masses. This is possible by composing metallic bodies of individual sheets insulated against each other and arranged in parallel to the direction of flux. Further eddy-current losses can be avoided when using ferromagnetic materials having a small electric conductivity, for example, sheet iron alloyed with silicon or certain iron–oxide compounds.

### 5.3.3. Inductance

- Inductance and coil

A wire usually wound on ferromagnetic core is called coil. This component stores energy at a certain current. The storage capacity for magnetic energy is called inductance of a coil.

$$L = N \frac{\emptyset}{I} \quad (5.20.)$$

where:

L inductance (more precisely self-inductance)

$\emptyset$  magnetic flux

I current

N number of turns

$$L = \frac{Vs}{A} = H$$

The following subunits are most frequently used:

$$1 \text{ mH} = 1 \text{ millihenry} = 10^{-3} \text{ H}$$

$$1 \text{ ?H} = 1 \text{ microhenry} = 10^{-6} \text{ H}$$

The storage capacity of the coil is dependent on the number of turns, the dimensions and the permeability of the core. From the equations (5.5.) and 5.15.) we have

$$L = N^2 \mu \frac{A}{l} \quad (5.21.)$$

where:

N number of turns

$\mu$  permeability (material constant)

A coil (core) cross-section

l length of coil

Like resistors, coils can be connected in series or in parallel. In series connection according to Fig. 5.27., the same current passes through the coils with the individual inductances of  $L_1$  and  $L_2$ . In case of a current variation, voltage proportional to the individual inductances of the coils is induced in the latter. The equivalent inductance of this arrangement is

$$L_{\text{equ}} = L_1 + L_2 \quad (5.22.)$$

This equation has the same structure as the equation for the determination of  $R_{\text{equ}}$  of a series connection of resistors.

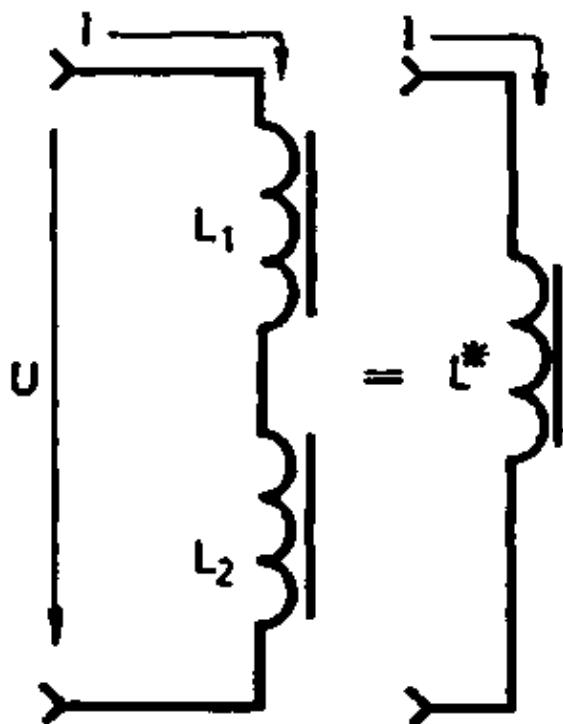


Fig. 5.27. Series connection of two coils

$$L_{\text{ers}} = L_{\text{equ}}$$

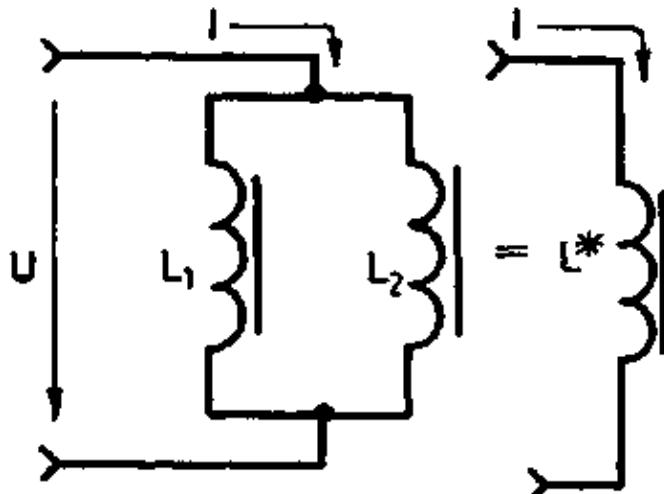


Fig. 5.28. Parallel connection of two coils

$$L_{\text{ers}} = L_{\text{equ}}$$

The parallel connection of two coils is shown in Fig. 5.28. The same voltage is applied to the two coils and the equivalent inductance is analogous to the equivalent resistance of resistors connected in parallel.

$$\frac{1}{L_{\text{equ}}} = \frac{1}{L_1} + \frac{1}{L_2} \quad (5.23.)$$

From the equations (5.22.) and (5.23.), the following general statement can be derived: In a series connection of coils, the equivalent inductance is always greater than the greatest individual inductance and in a parallel connection of coils, the equivalent inductance is always smaller than the smallest individual inductance.

#### Example 5.8.

Two coils having the inductances of 1.5 H and 3 H have to be connected in series and then in parallel. Determine the equivalent inductances for these two types of connections!

Given:  $L_1 = 1.5 \text{ H}$  To be founds  $L_{\text{equ}}$  in series connection and in parallel connection  
 $L_2 = 3 \text{ H}$

Solution:

series connection of  $L_1$  and  $L_2$

$$\begin{aligned}L_{\text{equ}} &= L_1 + L_2 \\L_{\text{equ}} &= 1.5 \text{ H} + 3 \text{ H} \\L_{\text{equ}} &= 4.5 \text{ H}\end{aligned}$$

=====

parallel connection of  $L_1$  and  $L_2$

$$\frac{1}{L_{\text{equ}}} = \frac{1}{L_1} + \frac{1}{L_2} = \frac{L_2 + L_1}{L_1 L_2}$$

$$L_{\text{equ}} = \frac{L_1 L_2}{L_1 + L_2}$$

$$L_{\text{equ}} = \frac{1.5 \text{ H} \cdot 3 \text{ H}}{1.5 \text{ H} + 3 \text{ H}} = \frac{4.5 \text{ H}}{4.5}$$

$$L_{\text{equ}} = 1 \text{ H}$$

=====

- Behaviour of a coil in a direct-current circuit

A coil is connected to a direct voltage source according to Fig. 5.29. (switch position 1). At the instant of switching on (time  $t_1$ ), current starts flowing. The maximum current limited by  $R$  cannot flow immediately because self-induction counteracts any current change. After a short time, the current has reached a certain value and the magnetic flux the value proportional to the current. The current causes a voltage drop at the resistor  $R$ ; consequently, the voltage across the coil is reduced. In the following time, the current is not allowed to rise as quickly as immediately after the instant of switching on. All this shows that, after switching on, the current first increases rapidly and then more and more slowly while the coil voltage first drops rapidly and then more and more slowly.

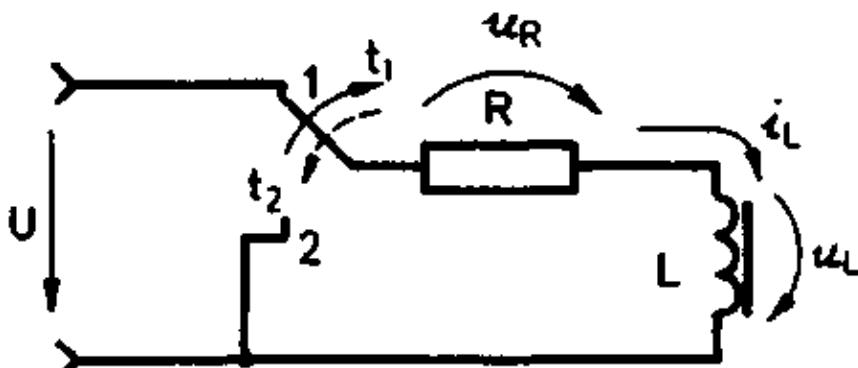


Fig. 5.29. Circuit for switching on and off of a direct voltage in a coil

Now, the voltage source is to be switched off from the coil (switch position 2). At the instant of switching off (time  $t_2$ ), the current passing through the coil is not immediately interrupted because self-induction opposes any current change. The starting change in current causes a self-induced voltage which, according to the Lenz rule, is so directed that it counteracts the cause of origin. An induced current is driven in the same direction as before when the voltage source was connected. Now, the magnetic field gradually dies out and the stored magnetic energy is converted into heat energy in resistor  $R$ .

The course taken by current and voltage during switching on and off is shown in Fig. 5.30. It is evident that at the instant of switching on and at the instant of switching off the coil voltage reaches its highest value rapidly and, after some time, drops to zero. The current, however, changes its value only slowly in switching. In coils,

there are no sudden current changes.

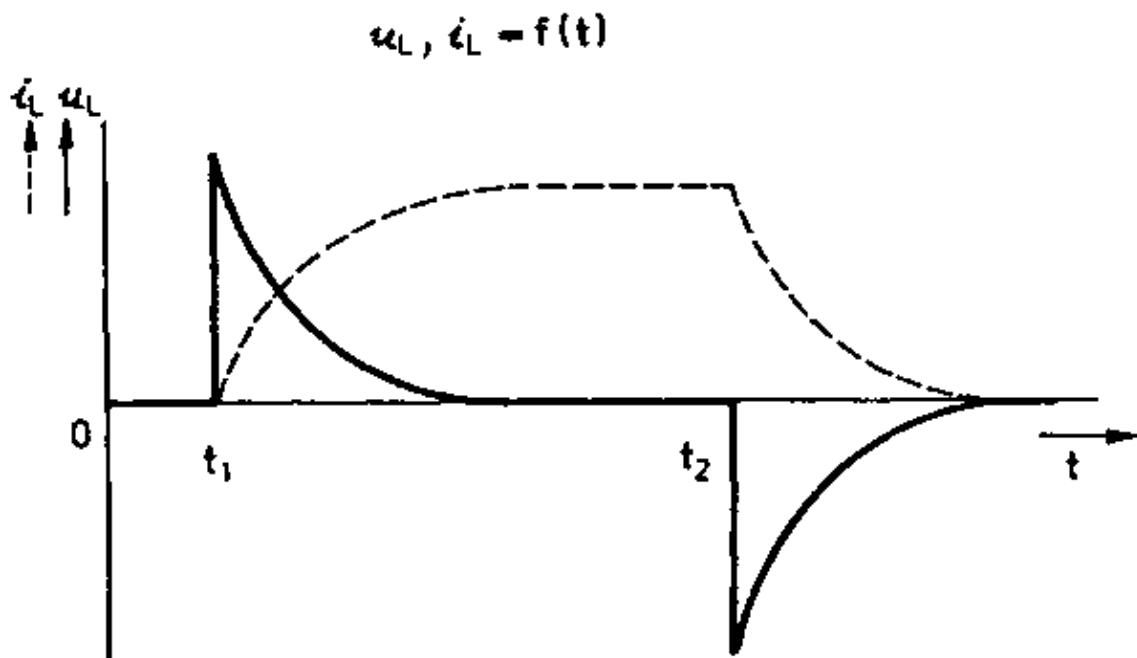


Fig. 5.30. Behaviour of current and voltage in a coil when a direct voltage is being switched on and off

When switching off a coil, the following should be observed: The energy stored in the magnetic field is only maintained by a current flow. In case of an interruption (instant of switching off), the field must disappear and the energy be converted into another form of energy. An instantaneous interruption ( $\Delta t = 0$ ) according to the law of induction leads to a high induced voltage which can attain values of such a magnitude that connected components and the insulation of the coil winding may be destroyed.

When circuits include coils, caution is imperative at any time. In switching off, dangerous overvoltages can occur. They are prevented by closing the current path for the induced current. For this purpose, a resistor, a capacitor or a semiconductor diode is connected in parallel to the coil.

The coil is a storage element. The energy stored by a coil in the form of magnetic energy is

$$W = \frac{L}{2} I^2 \quad (5.24.)$$

where:

W energy  
L inductance  
I current

$$W = \frac{V \cdot s}{A} \cdot A^2 = V \cdot A \cdot s = W \cdot s$$

In a magnetic field considerably higher energies can be stored than in a dielectric field (see Section 6.2.2.). Therefore, large force actions can be achieved with magnetic fields.

Example 5.9.

A coil having an inductance of  $L = 10 \text{ H}$  carries a current of  $5 \text{ A}$ . Calculate the energy stored!

Given:  $I = 5 \text{ A}$  To be found:  $W$

$$L = 10 \text{ H}$$

Solution:  $W = \frac{L}{2} I^2$

$$W = \frac{10 \text{ Vs}}{2 \text{ A}} \cdot 5^2 \text{ A}^2 = 5 \cdot 25 \text{ Ws}$$

$$W = 125 \text{ Ws}$$


---

Any magnetic flux variation causes an electromotive force (electromagnetic induction). It is directed in such a way that the magnetic field caused by the induced current counteracts the cause of its origin. A distinction is made between induction of rest and induction of motion. The electromagnetic induction forms the basis of a large number of technical applications including generators, motors, transformers and measuring instruments.

When a coil carries a current, the latter is associated with a magnetic flux. When the current varies, the magnetic flux also varies inducing an electromotive force. When this takes place in another, galvanically separated coil, this is called mutual induction; when it takes place in the same coil, it is called self-induction. In any case, the magnitude of the induced electromotive force is proportional to the rate of current variation. The proportionality factor in mutual induction is called mutual inductance M, that in self-induction is called inductance L.

The characteristic circuit parameter of a coil is the inductance; its unit is henry. The equivalent inductance in series and parallel connections of coils is expressed by the equations (5.22.) and (5.23.).

Loss-free coils (in practice, low-loss coils are only possible) allow a direct current to pass without any restriction. In switching on and off, however, a certain sluggishness is imparted to the current by the self-induced voltage, that is to say, there are no sudden current changes in a coil. When switching off a coil, very high over-voltages may occur which have to be limited in the circuit.

A current-carrying coil stores energy in the form of magnetic energy by means of which great force actions can be attained.

Questions and problems:

1. Describe in which way induced voltages are brought about!
2. In which way are self-induction and mutual induction physically related?
3. Compare generator principle and motor principle and explain the relations!
4. Explain the mode of action of an eddy-current brake with the help of the law of induction!
5. Which property of a coil is described by inductance?
6. Explain the course taken by current and voltage in a coil when it is switched on or off a direct voltage source!
7. Why can a very high overvoltage occur in a coil when it is switched off a voltage source? By which measures can this overvoltage be limited or avoided?
8. The inductance of a coil is 4 H. The current flowing through the coil changes uniformly by - 150 mA within 5 ms. Calculate the self-induced voltage!
9. Calculate the inductance of a coil when a self-induced voltage of 100 V is brought about with a rate of current change of 50 A/s.
10. Calculate the energy that is required for the building up of the magnetic field of a coil having an inductance of 500 mH when the coil carries a current of 2 A!

## 6. Electrical Field

### 6.1. Electrical Phenomena in Non-conductors

Electrical phenomena also occur in non-conductors. This becomes clearly evident in lightning during thunderstorms. A lightning may occur between two clouds or between cloud and earth. The cause of a lightning is a sudden charge equalisation between differently charged clouds or between different states of charge of cloud and earth. The form of discharge usually is a forked lightning. The voltage involved in lightning is about  $100 \cdot 10^6$  V, the current intensity about 50 kA. With a time of discharge of 1 ms, an energy of 1000 kWh is released. Unfortunately, advantage cannot be taken of this enormous quantity of energy. But we can protect ourselves from the dangerous effects of a lightning stroke with the help of modern technical means.

Charging with considerably lower energy takes place due to friction between different materials. When different materials contact each other, electrons from one material can migrate to the other one. When the two materials are separated, they show different charges. Due to frequent repetition of these contacts and separations (as involved in friction), high differences in charge may occur so that discharge via a spark takes place. This phenomenon will occur only when the materials involved are extremely well insulating (in a high atmospheric humidity, many materials lose their high insulating capacity and the charges can flow off). Due to the low energy involved in the way of charging up described here, a primary danger is not given for man, but other dangers may occur due to the effects of fire or shock. Spark discharges may become dangerous when they occur in close vicinity of easily combustible liquids or explosive substances. For example, protective measures are necessary when petrol is pumped from a bulk lorry into a storage tank. To avoid spark discharge, bulk lorry and storage tank must be properly connected electrically conductive before petrol should be pumped. Force actions occur between charges. Dissimilar charges attract each other and correspondent charges repel each other (see also force actions between magnetic poles). In the printing industry this force action is disturbing. During the rapid passage of paper through the machine, the paper may be charged so that proper transport of the paper will be prevented. Similar phenomena occur in the textile industry.

Direction and intensity of the force action is described by field lines like in the magnetic field. In contrast to magnetic field lines, however, electrical field lines arise from and end in charges. The extent of the electrical field is three-dimensional. Fig. 6.1. shows a few typical courses of field lines.

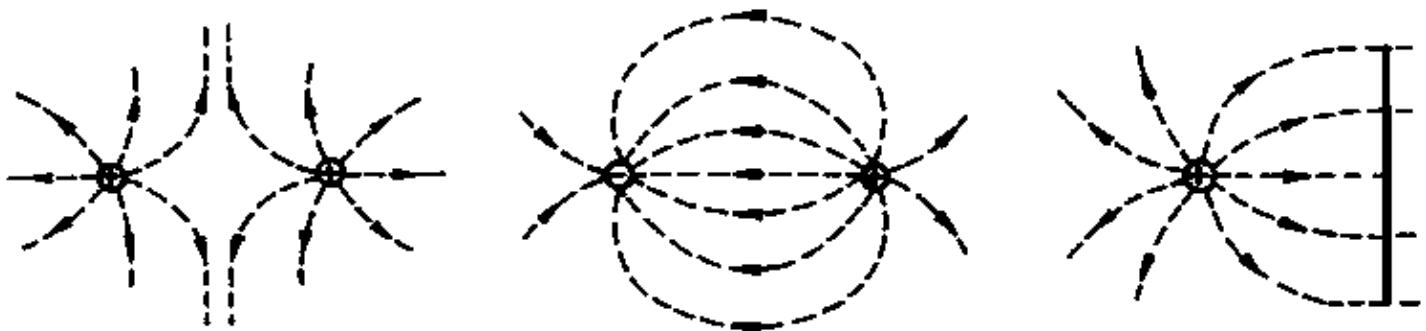


Fig. 6.1. Characteristic patterns of electric field lines

When an electrical conductor is placed in an electrical field (Fig. 6.2.), the freely movable electrons are displaced. The side facing the negative charge is positively charged and the side facing the positive charge is negatively charged. This phenomenon is called electrostatic induction or influence.

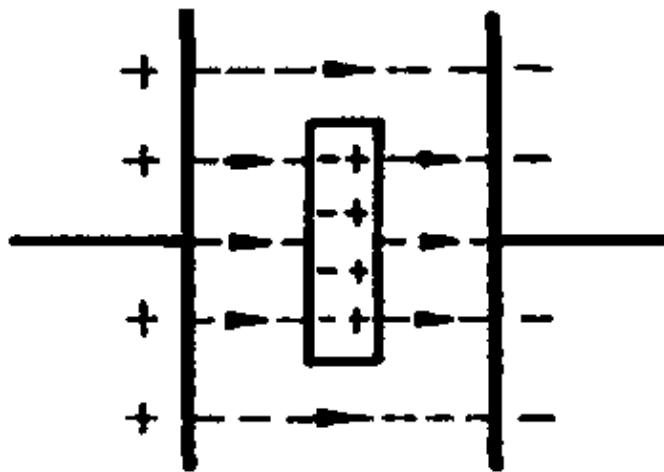


Fig. 6.2. Influence in an electric field

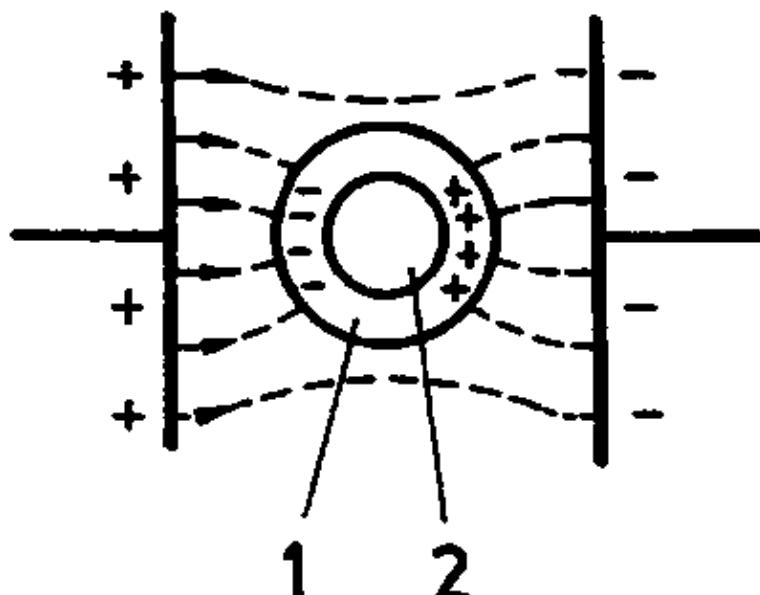


Fig. 6.3. Electrical shielding

1 Metal ring; 2 Field-free space

When a conductive ring is placed in an electrical field, in the interior of the ring, a field-free space is brought about (Fig. 6.3.). This phenomenon is called electrical shielding. It is used in practice to shield from interference fields. Complicated electronic measurements are taken in Faraday's cage (working room surrounded by a double wall of copper foil) or aerial lines and other signal lines are screened.

When the electrical field acts on a non-conductor (also known as dielectric), the not freely movable electrons can be displaced only insignificantly in the direction of the positive charge. This phenomenon is called dielectric polarisation (Fig. 6.4.).

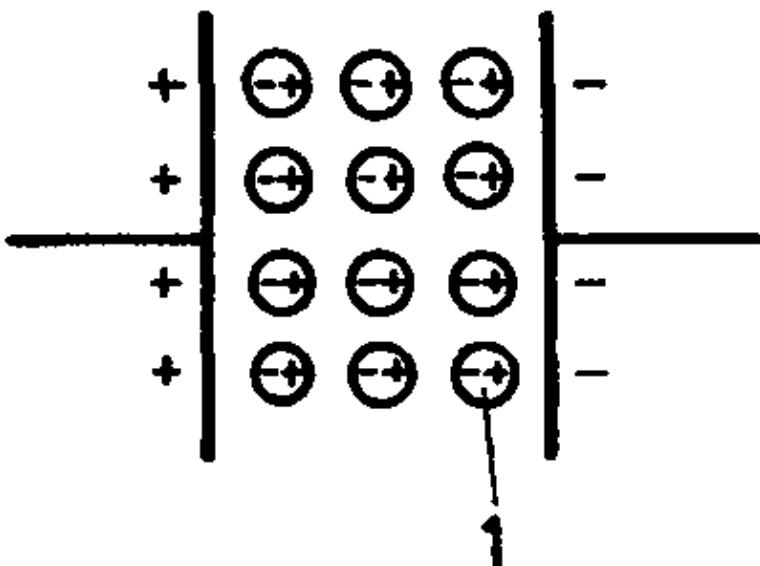


Fig. 6.4. Dielectric polarisation

1 Elementary particle of the dielectric

If dust particles are in an electrical field, they will be charged negatively or positively, depending on their composition, attracted by the electrode having the opposite charge and deposited there. Advantage is taken of this effect in flue-gas cleaning. Dust can be removed almost completely from flue gases by means of electrical filters. The consumption of electrical energy for 1000 m<sup>3</sup> of flue gas to be cleaned is about 1 kWh. A voltage of about 50 kV is applied to the electrodes.

When relating the voltage between two charged plates to the distance between the latter, we obtain the field strength E.

$$E = \frac{U}{l}$$

$$[E] = \frac{V}{m} \quad (6.1)$$

where:

E electrical field strength

U voltage

l distance between the plates

This simple method of calculating the field strength is only applicable to parallel field lines (in a homogeneous field). At points and edges, the field strength is considerably higher than in the vicinity of large-area electrodes.

When the field strength reaches a critical value, the dielectric is subjected to a flashover or breakdown and, hence, to a spark discharge. The field strength required for a breakdown is called breakdown field strength. It is a quantity which depends on material (see Table 6.1.). The breakdown field strength of air is considerably lower than that of strong insulating materials. Therefore, the distance between conductors carrying high-voltage in air must be larger than between these conductors sheathed with strong insulating materials. For example, a voltage of 330 kV can break through an air gap of about 100 mm and through a rubber insulation of maximum 13.3 mm, however.

Table 6.1. Dielectric Strength of a Few Insulating Materials

Insulating material	Dielectric strength in kV/mm
air	3.3

paper	10
rubber	25
porcelain	15
paraffin	40
aluminium oxide	1000

Since at points and edges (small surface areas) a particularly high field strength is prevalent, flashovers preferably start from them. This phenomenon is utilised for lightning protection pointed metal rods are fastened to the highest point of buildings, the rods are connected with the ground in a properly conducting manner and so are capable of arresting lightnings and carry them off to ground without any damage to the building. This point effect also entails remarkable disadvantages. Thus, charges are sparked off from lines carrying high-voltage; this leads to considerable energy losses. Therefore, it is necessary to enlarge the electrically effective surface of high-voltage lines and provide a smooth surface. An enlargement of the surface is obtained by bunch lines. The line is divided into 4 conductors which are combined into one stranded conductor by means of spacers (Fig. 6.5.). Potential rings are attached to Insulators. Great store should be set by carefully rounded edges and smooth undamaged surfaces.

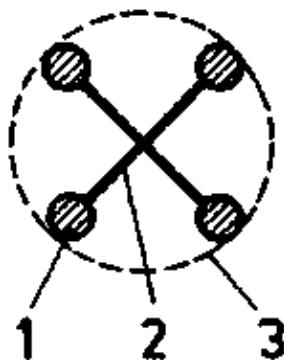


Fig. 6.5. Sectional view of a stranded conductor for highest voltage

1 Individual conductor; 2 Spacer; 3 Electrically effective surface

Brush discharges nevertheless occurring in extra-high voltage lines may cause luminous phenomena under certain atmospheric conditions which are called corona because of their ring shape.

Electrical phenomena also occur in non-conductors. They show different effects (lightning, spark discharge, force action). In electric filters, the force action is used for dust separation. Charges can be produced by a continuous contacting and separating (e.g. friction). Spark discharges can lead to uncontrolled actions of man due to fear, to fire and explosions. Spark discharges occur when the breakdown field strength is exceeded. Discharges primarily take place at points and edges; that is why in high-voltage engineering all conductive parts should be provided with large and smooth surfaces.

Questions and problems:

1. Quote examples of electrical phenomena in non-conductors!
2. How are charges brought about?
3. What are the facts described by the breakdown field strength?
4. Why have points and edges to be avoided in high-voltage engineering?

## 6.2. Capacity

### 6.2.1. Capacity and Capacitor

Figs. 6.2. to 6.4. show certain phenomena between two charged plates. Two plates provided with connections and separated by a dielectric are called capacitor (Fig. 6.6.). This component is capable of storing a certain charge when a certain voltage is present. This storage capability is called capacity of a capacitor.

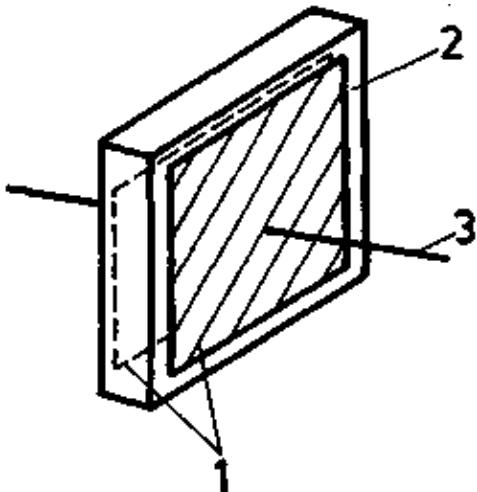


Fig. 6.6. Design of a capacitor

1 Conducting metal plates (electrodes); 2 dielectric; 3 Connections

$$C = \frac{Q}{U}$$

$$[C] = \frac{AS}{V} = F \quad (6.2.)$$

where:

C capacity  
Q charge  
U voltage

Since the unit 1 F (farad) is very great, the capacity of the capacitors manufactured only reaches fractions of 1 F. These fractions are designated by the prefixes specified by legal regulation:

$$1 \text{ pF} = 1 \text{ picofarad} = 10^{-12} \text{ F}$$

$$1 \text{ nF} = 1 \text{ nanofarad} = 10^{-9} \text{ F}$$

$$1 \text{ ?F} = 1 \text{ microfarad} = 10^{-6} \text{ F}$$

The storage capacity of a capacitor is dependent on the area of the electrodes, the distance between them and the type of dielectric.

$$C = \frac{\epsilon A}{d} \quad (6.3.)$$

where:

$\epsilon^1)$  dielectric constant  
A area of the electrodes  
d distance between the electrodes

<sup>1)</sup> ? Greek letter epsilon

The material constant is usually stated for the dielectric in question in the form of the product of the absolute dielectric constant times the relative dielectric constant.

$$\epsilon = \epsilon_0 \epsilon_r \quad (6.4.)$$

where:

$\epsilon_0$  absolute dielectric constant  
 $\epsilon_r$  relative dielectric constant

The absolute dielectric constant applies to vacuum and is

$$\epsilon_0 = 8.86 \cdot 10^{-12} \frac{\text{As}}{\text{Vm}}$$

Table 6.2.  $\epsilon_r$  of Some Insulating Materials

Insulating material	$\epsilon_r$
air	1
paper	2
transformer oil	2.5
rubber	2.7
porcelain	5
Epsilan (special ceramic compound for the production of capacitors)	up to 10,000

Like resistors, capacitors can be connected in series or in parallel. The total capacity obtained in this way is to be determined. Fig. 6.7. shows the series connection of two capacitors. The two capacitors have the same charge  $Q$ . The following holds:

$$Q_{AB} = Q_1 = Q_2$$

$$U_{AB} = U_1 + U_2$$

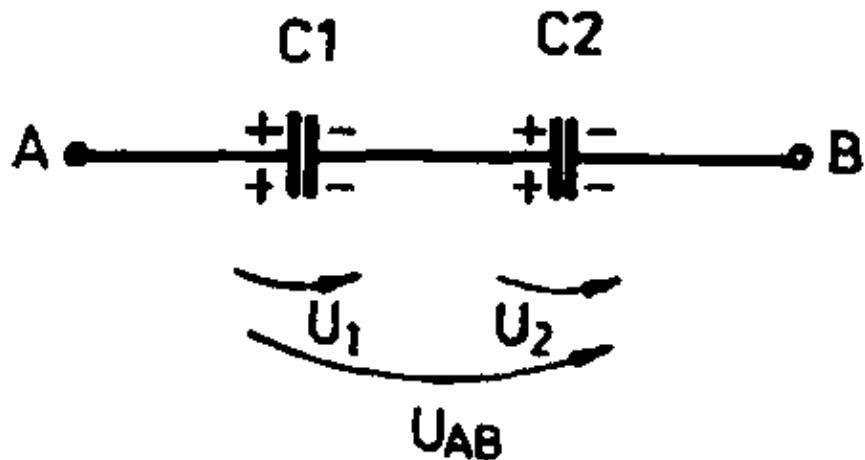


Fig. 6.7. Capacitors connected in series

When dividing the voltage equation by the charge, we have

$$\frac{U_{AB}}{Q} = \frac{U_1}{Q} + \frac{U_2}{Q}$$

After inversion, we obtain from equation 6.2.

$$\frac{1}{C} = \frac{U}{Q}$$

and, for the total capacity of a series connection of capacitors we have

$$\frac{1}{C_{\text{equ}}} = \frac{1}{C_1} + \frac{1}{C_2} \quad (6.5.)$$

This equation has the same structure as the equation for the determination of  $R_{\text{equ}}$  of a parallel connection of resistors.

The parallel connection of two capacitors is shown in Fig. 6.8. The same voltage is applied to the two capacitors, and each capacitor has stored a charge in accordance with its capacity.

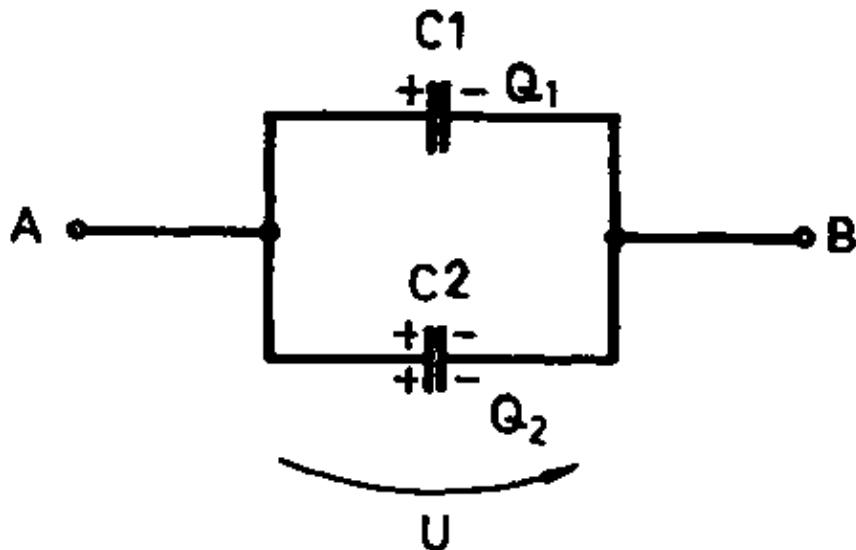


Fig. 6.8. Capacitors connected in parallel

Thus, we have

$$\begin{aligned} U &= U_{C1} = U_{C2} \\ Q_{AB} &= Q_1 + Q_2 \end{aligned}$$

After division, we obtain and with equation 6.2. we have

$$\frac{Q_{AB}}{U} = \frac{Q_1}{U} + \frac{Q_2}{U}$$

$$C_{\text{equ}} = C_1 + C_2 \quad (6.6.)$$

This equation has the same structure as the equation for the determination of  $R_{\text{equ}}$  of a series connection of resistors.

From the equations 6.5. and 6.6., the following general statement can be derived: In a series connection of capacitors, the total capacity is always smaller than the smallest individual capacity, and in a parallel connection of capacitors, the total capacity is always greater than the greatest individual capacity.

#### Example 6.1.

Two capacitors with a capacity of 470 nF and of 680 nF have to be connected in series and then in parallel. Determine the total capacity of each of the two types of connections!

Given:  $C_1 = 470 \text{ nF}$  To be found:  $C$  in series connection and in parallel connection

Solution:

Series connection of  $C_1$  and  $C_2$

$$\frac{1}{C_{\text{equ}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_2 + C_1}{C_1 \cdot C_2}$$

$$C_{\text{equ}} = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

$$C_{\text{equ}} = \frac{470\text{nF} \cdot 680\text{nF}}{470\text{nF} + 680\text{nF}}$$

$$C_{\text{equ}} = 277.9 \text{nF}$$

=====

Parallel connection of  $C_1$  and  $C_2$

$$C_{\text{equ}} = C_1 + C_2$$

$$C_{\text{equ}} = 470 \text{nF} + 680 \text{nF}$$

$$C_{\text{equ}} = 1150 \text{nF}$$

$$C_{\text{equ}} = 1.15 \text{ ?F}$$

=====

In series connection, a total capacity of 277.9 nF is obtained while in parallel connection the total capacity is 1.15 ?F.

### 6.2.2. Behaviour of a Capacitor in a Direct Current Circuit

An uncharged capacitor is connected to a direct voltage source according to Fig. 6.9. (switch position 1).

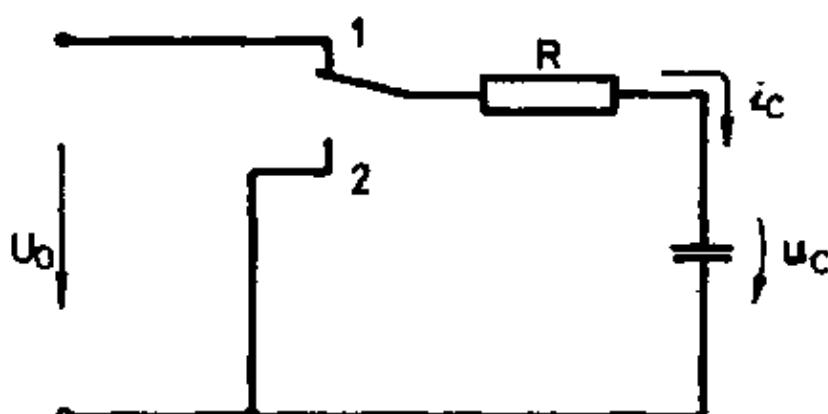


Fig. 6.9. Circuit for charging and discharging a capacitor

The terminal voltage of the uncharged capacitor is 0. In order that, between the plates of the capacitor, the voltage can be applied to which it is connected, the capacitor must be charged. This means that a charging current must flow at the instant of switching on. The intensity of the current at the instant of switching on is determined by the large difference between charging voltage and terminal voltage of the capacitor and the resistance. With increasing charge of the capacitor, the voltage of it increases and will reach the value of the charging voltage when the process of charging is finished. With increasing charge, the voltage difference between charging voltage and terminal voltage of the capacitor also drops and, consequently, the charging current also drops. At the end of charging, the voltage difference and the charging current are 0. A direct current is no longer allowed to flow now. This is also due to the design of the capacitor because a dielectric (insulating material) is between its two connections.

When the capacitor is now discharged via a resistor according to Fig. 6.9. (switch position 2), at the first instant of discharge, a discharge current must flow which is only limited by the resistance. Since, due to the discharge, the terminal voltage of the capacitor drops, the discharge current must also drop in the course of time. After complete discharge, the terminal voltage and the discharge current have dropped to 0. The course taken by current and voltage during charging and discharging is shown in Fig. 6.10. Charging commences at time  $t_1$  and discharging at time  $t_2$ . It is evident that charging and discharging currents suddenly reach their maximum value at the beginning of the charging or discharging process and then they reach the value of 0 after some time. Both in charging and in discharging, the voltage changes its value only slowly. There are no

sudden voltage changes in capacitors.

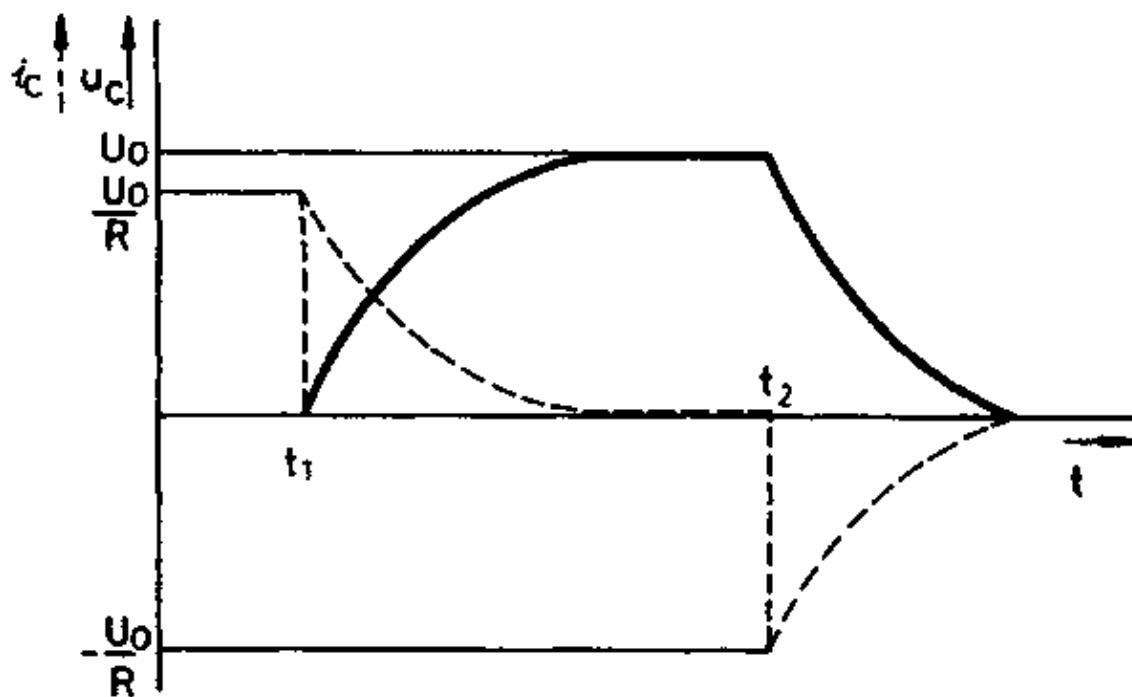


Fig. 6.10. Behaviour of current and voltage during the charging and discharging of a capacitor  $t_{\text{ein}} = t_{\text{on}} = t_{\text{aus}} = t_{\text{off}}$

Since the peak value of the discharge current is limited only by the discharge resistance, a capacitor should not be discharged via a short circuit. If this would occur, however, at the first instant of discharge, an extremely high current would flow for a short time which might cause the destruction of the capacitor or a fusing of the shorting bridge. As a capacitor retains its charge for some time after charging without external discharge, particular caution is necessary when working at installations containing capacitors. After disconnection from the mains, the fact that the capacitors are completely discharged must be checked or discharge via a resistor must be effected. In many installations, a discharge resistance is incorporated in order to avoid dangers to man.

During discharging, a capacitor acts as an electrical energy source (during a certain time, its terminal voltage drives a current). The energy stored in a capacitor is written as

$$W = \frac{C}{2} U^2 \quad (6.7.)$$

where:

W energy  
C capacity  
U voltage

The energy that can be stored in a capacitor is relatively small. It is of advantage however, that it is available as a short-time energy release. Advantage of this effect is taken in a photoflash device and in some spot-welding equipment.

Example 6.2.

Which energy is stored in a capacitor of 47 ?F charged up to 100 V?

Given:  $U = 100 \text{ V}$  To be found:  $W$

$$C = 47 \text{ ?F}$$

Solution  $W = \frac{C}{2} U^2$

$$W = 23.5 \cdot 10^{-6} \frac{\text{As}}{\text{V}} \cdot 10^4 \text{V}^2$$

$$W = 0.235 \text{ Ws}$$

$$W = 235 \text{ mWs}$$

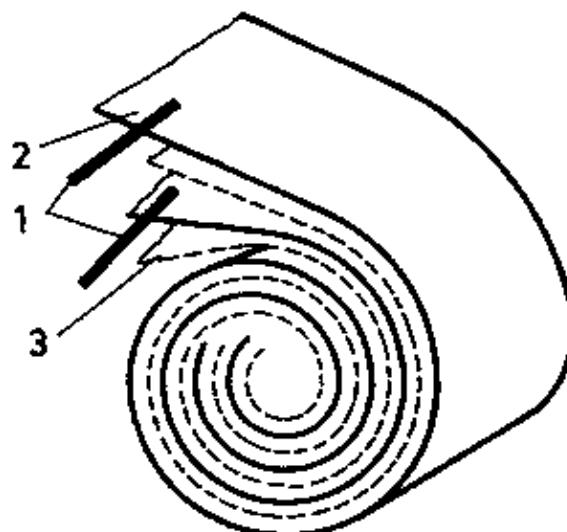
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The energy stored in the capacitor is 235 mWs.

### 6.2.3. Types of Capacitors

For the different fields of application, a great variety of designs of capacitors is available. In heavy current engineering, primarily paper capacitors in a metal cup are used (Fig. 6.11.). Two metal foils and two paper strips are placed one upon the other in the way shown in the illustration and then properly rolled up. The two metal foils are attached to connections and the roll is mounted in a metal cup. Such a paper capacitor is also known as roll-type capacitor. The MP-capacitor (metal–paper capacitor) is designed in a similar manner; in this case, the foil is replaced by a coat of metal which is produced by vapour deposition. These capacitors are smaller than paper capacitors of the same capacity.

**Fig. 6.11. Encased capacitor**



**Fig. 6.11.a Design of the roll**

1 Connections of the foils; 2 Metal foil; 3 Paper strips

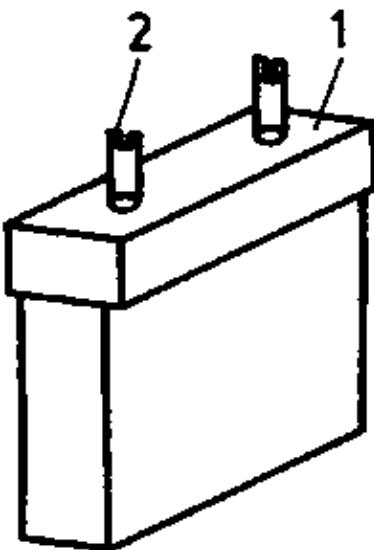


Fig. 6.11.b External view of the capacitor

1 Metal enclosure; 2 Connections

Another advantage of MP-capacitors is the fact that, after a breakdown or puncture of the dielectric, the extremely thin metal coat in the close vicinity of the puncture evaporates and, thus, removing the short-circuit – that is why MP-capacitors are called “self-healing” capacitors.

For the practical use, the value of the capacity printed on the device and the rated voltage up to which the capacitor may be used have to be observed.

An arrangement consisting of two plates with a dielectric between them is called capacitor. The capacity of a capacitor is a measure of the charge which the capacitor is capable of storing at a certain voltage, and it is also dependent on the design. The total capacity in series connection and in parallel connection of capacitors is expressed by the equations 6.5. and 6.6.

When a capacitor is connected to a direct voltage, a current will only flow during charging and discharging. There are not sudden voltage changes in a capacitor. A charged capacitor can retain its charge for a longer period of time (danger!) and it should never be discharged via a short circuit. The capacitor may be used as an energy store.

In heavy current engineering, the paper capacitor arranged in a metal cup is used. For use, pay particular attention to the value of the capacity and the rated voltage.

Questions and problems:

1. Describe the basic design of a capacitor and, in particular, the design of a paper capacitor!
2. Which property of the capacitor is described by the capacity?
3. Explain the course taken by current and voltage during charging and discharging of the capacitor!
4. Why should capacitors not be discharged via a short circuit?
5. What should be strictly observed when working at installations incorporating capacitors?

## 7. Alternating Current

## 7.1. Importance and Advantages of Alternating Current

In the preceding Sections, we have explained the electro-technical conformities with natural laws under the restricting condition that current intensity and magnitude of voltage remain constant with respect to time. In practice, however, especially in power electrical engineering, mainly alternating current is used.

Alternating current is a current whose magnitude and direction varies periodically; this also applies to alternating voltage.

The electrical laws naturally also apply to alternating current engineering; a few peculiarities have to be observed, however.

Of the various possible forms, the sinusoidal alternating current has the greatest importance. Its substantial advantages are as follows:

- simple and economical generation
- transformation into other values (principle of mutual induction)
- low-loss energy transmission even through large distances
- the sinusoidal form is not changed by the basic components R, L and C

Because of these and other advantages, alternating current engineering is of paramount importance. If direct current is required (e.g. for the operation of the majority of electronic devices), it can easily be produced by rectifying the alternating current. In practice, especially in power electrical engineering, alternating current is used because of many advantages. This is a current whose magnitude and direction varies periodically. The sinusoidal alternating current has the greatest importance.

## 7.2. Characteristics of Alternating Current

Alternating current and alternating voltage are produced in generators. In Section 5.3.2. the generator principle has been pointed out. Below, explanations in great detail are given.

When turning a coil (represented by a conductor loop in Fig. 7.1.) in a magnetic field, a voltage which can drive a current is induced in this coil. The direction can be determined with the help of the right-hand rule. At a constant rotational speed, magnitude and direction of the induced voltage is dependent on the position of the conductor loop.

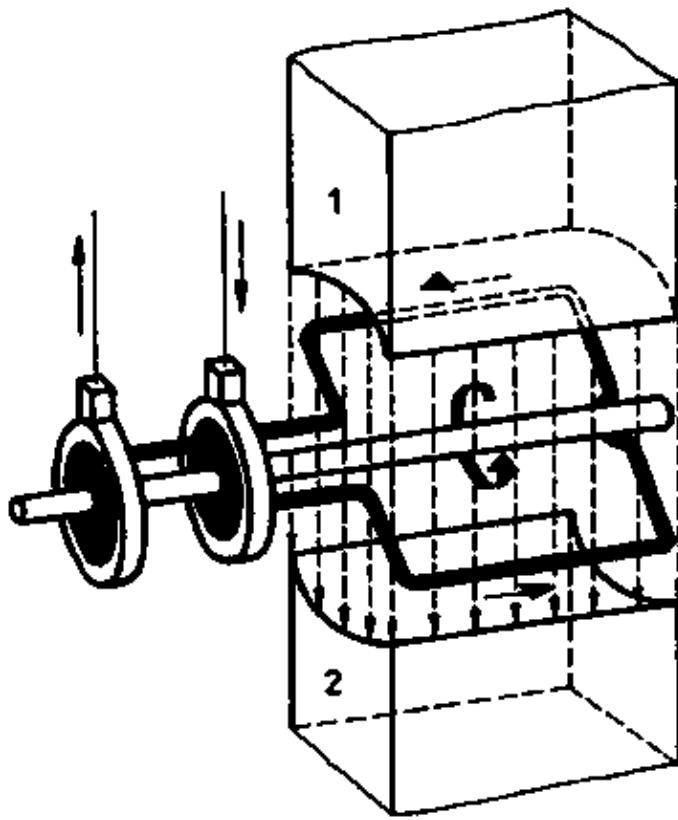


Fig. 7.1. Model of an alternating current generator – a) Position of the loop of conductor at a certain instant

1 North pole; 2 South pole

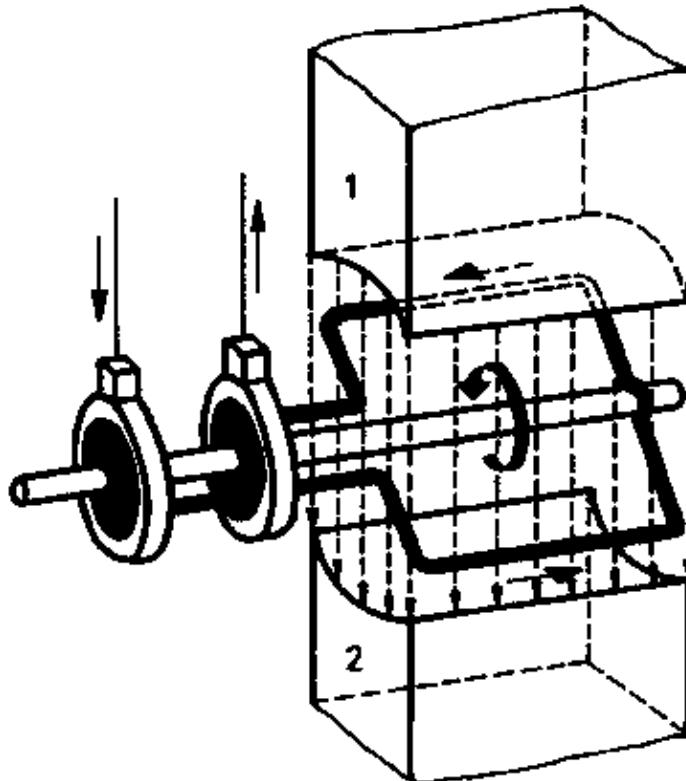


Fig. 7.1. Model of an alternating current generator – b) Position of the loop of conductor after half a revolution

1 North pole; 2 South pole

In horizontal position, the entire magnetic flux penetrates the loop,  $\frac{\partial\Phi}{\partial t}$  has the smallest value and the induced voltage is equal to zero. In vertical position, (parallel to the magnetic fields), the rate of flux variation  $\frac{\partial\Phi}{\partial t}$  is maximum and the induced voltage has the highest value. Upon further rotation, the voltage again

drops and after half a revolution reaches the value of zero. In the further course, the voltage changes its direction and reaches its negative maximum value in vertical position. After one full revolution, the initial condition is again reached, the voltage has dropped to zero and another cycle with exactly the same course can begin. In Fig. 7.2., the described conditions are shown for eight selected positions of the conductor. The behaviour of the curve is a sine function.

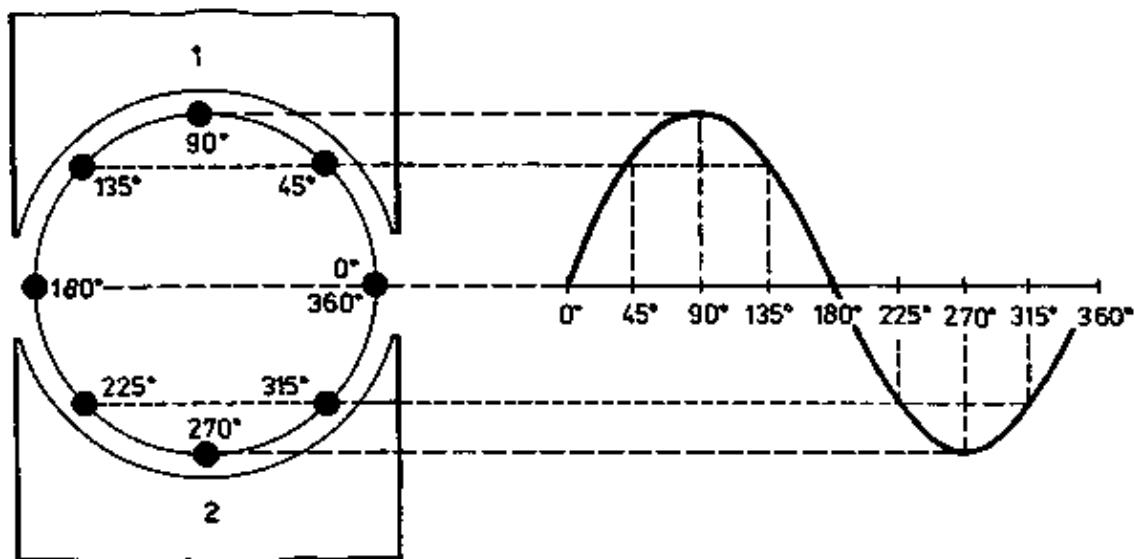


Fig. 7.2. Development of a sinusoidal voltage or current curve

1 North pole; 2 South pole

Frequently, the angle is expressed in circular measure; the correlation is easily given with the circumference of a circle having the radius  $r = 1$ . We have:

Circumference of a circle  $C = 2 r \pi = 2 \pi$ ; the perigon of a circle is  $\pi = 360^\circ$ ; this means that  $360^\circ \hat{=} 2\pi$  and, hence,  $180^\circ \hat{=} \pi$ ,  $90^\circ \hat{=} \frac{\pi}{2}$ ,  $45^\circ \hat{=} \frac{\pi}{4}$  etc.

The voltage and current course represented in Fig. 7.2. is called oscillation, cycle or wave. Each wave is made up of a positive (1) and a negative (2) half wave. The time for a full revolution of the conductor loop is called time of oscillation or duration of a cycle (formula sign T). The angular velocity at which the conductor loop rotates is the angle through which the loop has passed in a certain unit of time. With a perigon of  $360^\circ = 2\pi$ , the duration of a cycle T is required. The angular velocity  $\omega$  is usually called angular frequency  $\omega^1$  in electrical engineering and it is written as

<sup>1)</sup>  $\omega$  Greek letter omega

$$\omega = \frac{2\pi}{T} \quad (7.1.)$$

where:

$\omega$  angular frequency

T duration of a cycle

$2\pi$  circular measure of the circle (= circumference of the unit circle)

$$[\omega] = \frac{1}{s}$$

The product of  $\omega t$  is the angle  $\alpha$  at time t and at the angular frequency  $\omega$ ;  $\alpha = \omega t$ . This angle is called phase angle or, in short, phase. The phase is the condition of oscillation given at a certain time which is repeated at

the same time intervals. When, for example,  $t = \frac{T}{4}$ , then  $\alpha = \omega \frac{T}{4} = \frac{2\pi}{T}$ . Using  $\omega = \frac{2\pi}{T}$  from equation (7.1), we

$\alpha = \frac{2\pi}{T} \cdot \frac{T}{4} = \frac{\pi}{2} \approx 90^\circ$ . This is the phase angle at a quarter of a cycle. The number of cycles or waves produced in a certain time (e.g.  $t = 1$  s) is called frequency  $f$  of the alternating voltage or alternating current. The greater the duration of a cycle  $T$ , the smaller the frequency  $f$ . Fig. 7.3. shows two different curves for a duration of 1 s. The curve drawn as a solid line is a cycle, its duration  $T = 1$  s. The curve represented by a dashed line covers five cycles (the frequency is higher), and the duration of a cycle

$$T = \frac{1}{5} \text{ s} = 0.2 \text{ s.}$$

$$f = \frac{1}{T} \quad (7.2.)$$

where:

$f$  frequency  
 $T$  duration of a cycle

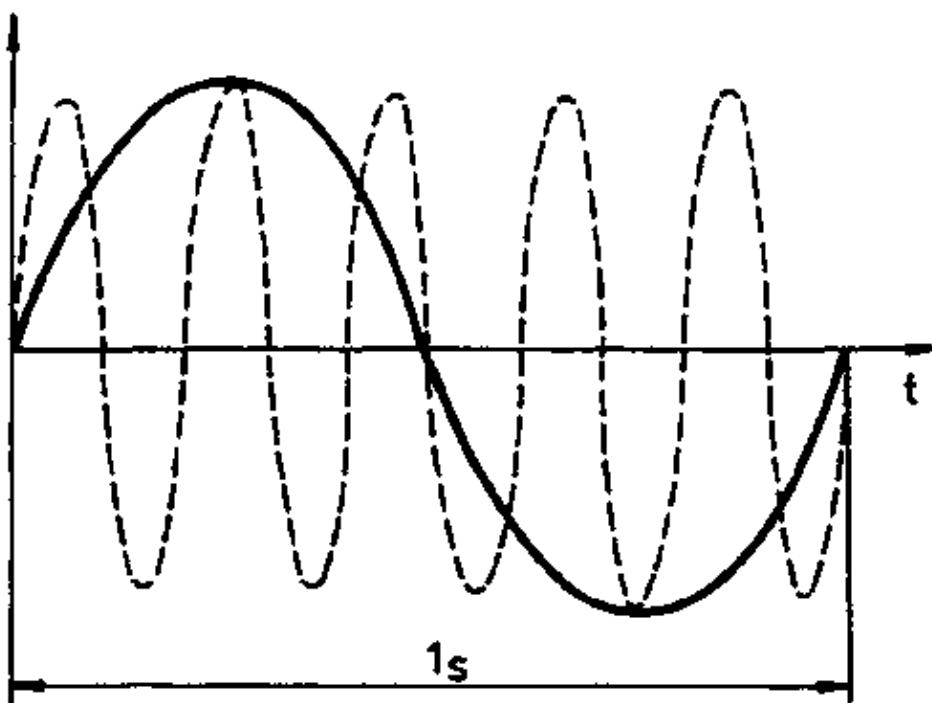


Fig. 7.3 Alternating currents with frequencies of 1 Hz and 5 Hz

In honour of the German physicist Heinrich Hertz (1857 – 1894), the unit of frequency is called Hertz = Hz.

$$[f] = \text{Hz} = \frac{1}{\text{s}}$$

The following subunits are frequently required

$$1 \text{ kHz} = 1 \text{ kilohertz} = 10^3 \text{ Hz}$$

$$1 \text{ MHz} = 1 \text{ megahertz} = 10^6 \text{ Hz}$$

$$1 \text{ GHz} = 1 \text{ gigahertz} = 10^9 \text{ Hz}$$

The correlation between the angular frequency and frequency is given by the equations (7.1.) and (7.2.).

$$\omega = 2\pi f \quad (7.3.)$$

Another rarely used characteristic for the electrical wave is the wavelength  $\lambda$ <sup>1)</sup>.

<sup>1)</sup>  $\lambda$  Greek letter lambda

Wavelength is the length of a wave measured in a unit of length (e.g. m, km, cm, mm). As the electrical energy propagates with light velocity

$$c = 3000,000 \text{ km/s}$$

the distance over which a wave extends can be calculated on the basis of a given frequency. We have:

$$c = ? f;$$

after inversion we obtain for the wavelength

$$\lambda = \frac{c}{f} \quad (7.4.)$$

where:

? wavelength

c velocity of propagation ( $c = 300,000 \text{ km/s} = 3 \cdot 10^8 \text{ m/s}$ )

f frequency

$$[\lambda] = \frac{\text{m/s}}{\text{1/s}} = \text{m}$$

The magnitude of the alternating voltage or the alternating current can be determined on the basis of the sine curve developed in Fig. 7.2. The maximum value occurring at  $90^\circ$  and  $270^\circ$  is called peak value, maximum value or amplitude and is designated by  $\hat{U}$  (or  $\hat{i}$ ). All other values which very continuously and, thus, are different at any time are called instantaneous values and are designated by  $u$  (or  $i$ ). When the maximum value is known the instantaneous values can be determined at any time. The general equation of a sinusoidal alternating current is

$$u = \hat{U} \sin \omega t \quad (7.5.)$$

where:

$u$  instantaneous value

$\hat{U}$  maximum value, peak value or amplitude  
 $\sin \omega t$  factor of the sine function at angle  $\omega t$

Besides an exclusively mathematical treatment, alternating current processes are frequently represented in diagrams which offer a better survey. Particularly suitable for this purpose are vector and line diagrams.

Vector diagram

The alternating voltage or the alternating current is represented by a pointer (vector) capable of rotating whose length corresponds to the peak value. This pointer rotates anti-clockwise at the angular velocity  $\omega$ . The pointer position at any time indicates the position of the conductor loop. In order to determine the instantaneous of voltage or current for any desired position, a straight line is drawn from the pointer tip to the horizontal axis which passes through the centre of the circle. The length of the straight line corresponds to the instantaneous value in question (Fig. 7.4.a). The particular advantage of the vector diagram is lucidity; a disadvantage is the fact that the conditions can be represented only for one point of time or for a few selected instants.

Line diagram

The alternating voltage or alternating current is represented by a sine curve from which the values for all instants can be read off (Fig. 7.4.b). The line diagram can be developed from the vector diagram in the following manner. Close by the vector diagram, a horizontal line is drawn. This line is divided into periods, the smallest one being equal to the duration of one cycle  $T$ , or into angular degrees up to  $360^\circ \approx 2\pi$ . Perpendicular lines are drawn from the points of division which resemble the lines on the vector diagram as to

size and direction. When connecting the end points of the perpendicular lines, the sine curve is obtained. Fig. 7.4. shows the construction described. For reasons of clearness, the vector diagram (Fig. 7.4.a) only shows the vectors for  $30^\circ$ ,  $60^\circ$ ,  $90^\circ$ ,  $180^\circ$  and  $225^\circ$ . The advantage of the line diagram is the possibility of representing all of the instantaneous values; a disadvantage is the restricted lucidity especially when several curves have to be represented.

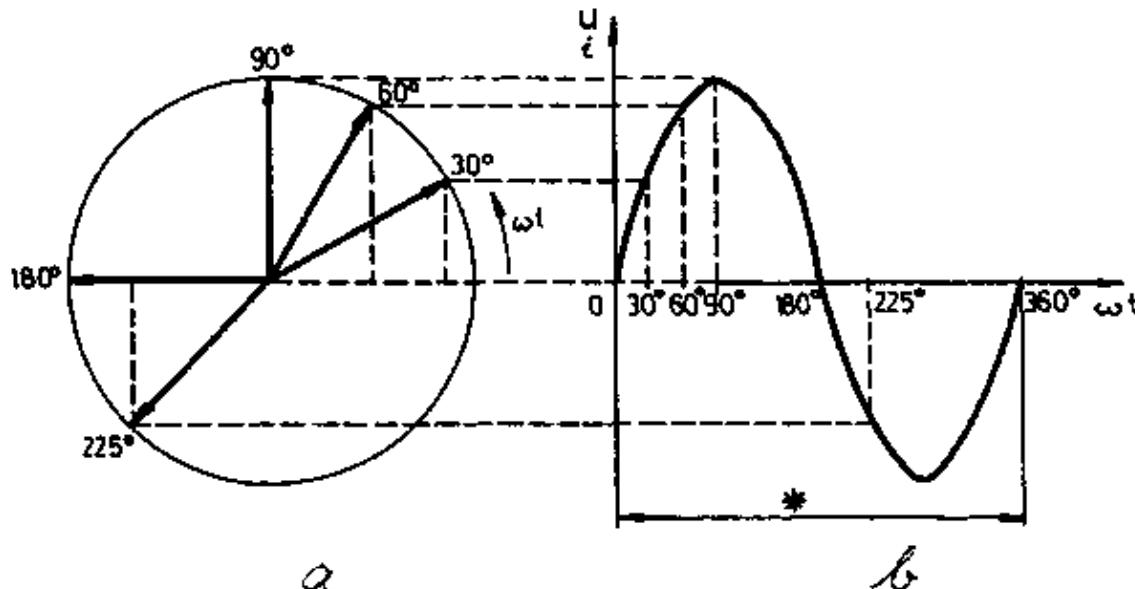


Fig. 7.4. Graphical representation of the alternating current and alternating voltage

- a) Vector diagram;
- b) Line diagram;
- 1 Periode = 1 cycle

Example 7.1.

A sinusoidal alternating voltage having a frequency of  $f = 50$  Hz has a peak value  $\hat{U} = 311$  V. Draw the vector diagram for the angles of rotation  $\omega t = 30^\circ, 45^\circ, 60^\circ$  and develop the line diagram on this basis! Further, determine the angular frequency, the duration of a cycle T, and the wavelength  $\lambda$ ?

Given:  $\hat{U} = 311$  V      To be found: vector and line diagrams

$f = 50$  Hz      ?

$\omega t = 30^\circ, 45^\circ, 60^\circ$       T

?

Solution: (Fig. 7.5.)

$$\omega = 2\pi f$$

$$\omega = 2 \cdot 3.14 \cdot 50 \frac{1}{s}$$

$$\omega = 3.14 \frac{1}{s}$$

=====

$$T = \frac{1}{f}$$

$$T = \frac{1}{50} \text{ s} = 0.02 \text{ s}$$

$$T = 20 \text{ ms}$$

=====

$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{3 \cdot 10^8 \text{ m/s}}{50 \text{ Hz}} = 6 \cdot 10^6 \text{ m} = 6,000 \cdot 10^3 \text{ m}$$

$$? = 6,000 \text{ km}$$

=====

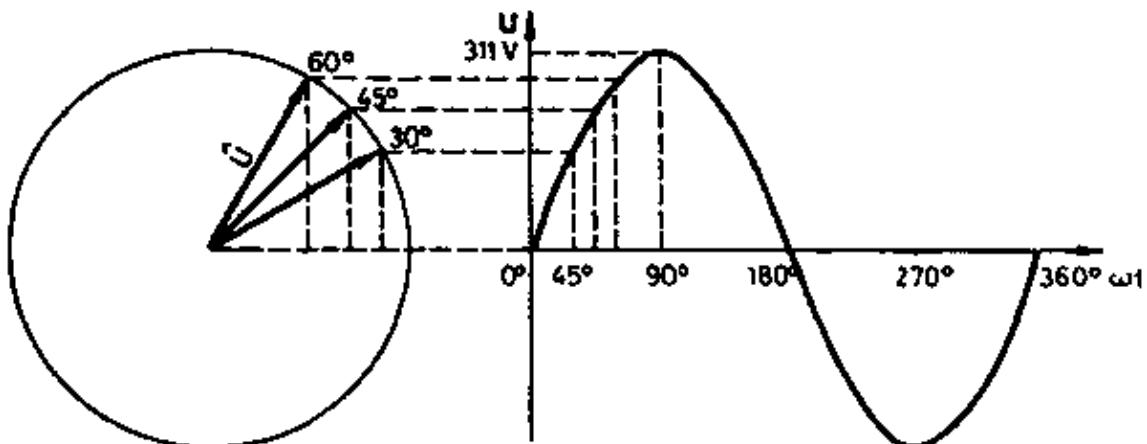


Fig. 7.5. Vector diagram and line diagram for example 7.1.

Like direct current, alternating current lends itself to the operation of heating and thermal appliances as well as incandescent lamps. Alternating current motors are used for the conversion into mechanical energy. Since alternating current, in contrast to direct current, continuously changes its magnitude and direction, a mean value must be found which has the same effect as a corresponding direct current. This mean value is called effective value (or root mean square value = r.m.s. value).

In Section 4.1., the energy conversion and the power of the current have been represented in general and the

$$P = I^2 R t = \frac{U^2}{R} t$$

relation has been derived. Evidently, it is the square of the current intensity and of the voltage that matters. In case of alternating current, we have to square all instantaneous values. Of all squared values of a cycle, the arithmetic mean must be formed. In this way, the square of the effective value is obtained. This is illustrated by Fig. 7.6. The sine curve has been squared.

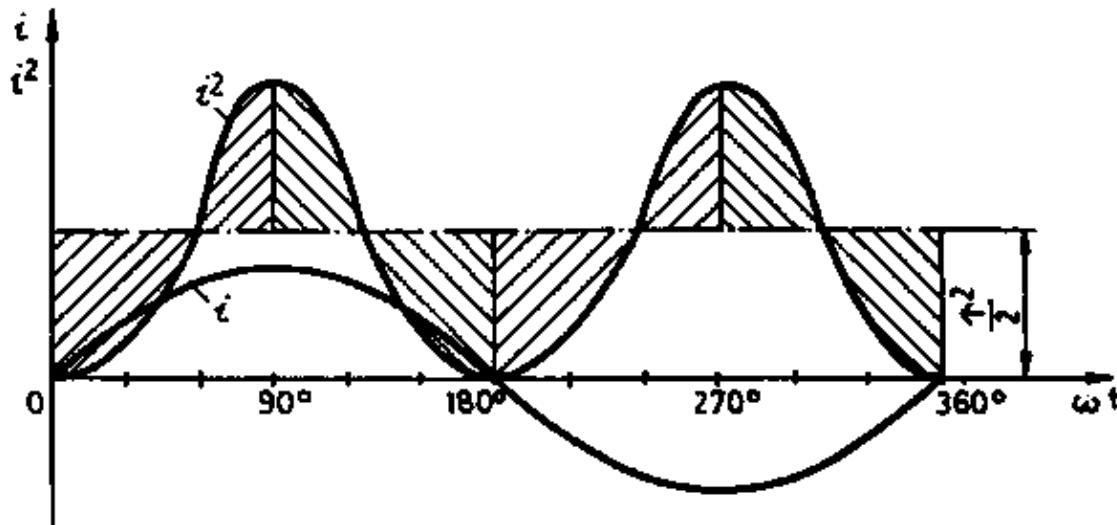


Fig. 7.6. Determination of the effective value of the alternating current

In this way, all values, even those of the negative half wave, become positive. The squared sine curve is also sinusoidal but has double the frequency of the original curve. The arithmetic mean is  $\frac{1}{2}$ . This value is the square of the effective value  $I$ , hence,

$$I^2 = \frac{\hat{i}^2}{2} \quad \text{and, with respect to voltage,} \quad U^2 = \frac{\hat{U}^2}{2}$$

By extraction of roots in the equation, we obtain

$$I = \frac{\hat{i}}{\sqrt{2}} \quad \text{or} \quad U = \frac{\hat{U}}{\sqrt{2}}$$

Since  $\sqrt{2} = 1.414$  and  $\frac{1}{\sqrt{2}} = 0.707$ , we obtain for the current

$$I = 0.707\hat{i} \quad \text{or} \quad \hat{i} = 1.414I \quad (7.6.a)$$

and for the voltage

$$U = 0.707\hat{U} \quad \text{or} \quad \hat{U} = 1.414U \quad (7.6.b)$$

A sinusoidal alternating current causes the same thermal effect as a direct current of intensity  $I$  if its peak value  $\hat{i}$  is 1.414 times the current intensity of the direct current. Analogous conditions hold for the sinusoidal alternating voltage.

Example 7.2.

Two usual mains voltages have the following values:

a)  $U = 220 \text{ V}$ ; b)  $\hat{U} = 535 \text{ V}$ . Determine the peak voltage for a) and the r.m.s. voltage for b)

Given: a)  $U = 220 \text{ V}$  To be found: a)  
 $\hat{U}$

b)  $\hat{U} = 535 \text{ V}$  b)  
 $U$

Solution:

Given: a)  $\hat{U} = 1.414U$       b)  $U = 0.707\hat{U}$

$$\hat{U} = 1.414 \cdot 220 \text{ V} \quad U = 0.707 \cdot 535 \text{ V}$$
$$\hat{U} = 311 \text{ V} \quad U = 380 \text{ V}$$

In the general use of alternating voltages and alternating currents, always effective (r.m.s.) values are involved.

When turning a coil in a magnetic field, a voltage is induced in the coil which changes periodically with respect to magnitude and direction. The voltage and current path produced during one revolution is called oscillation, cycle or wave. The most important characteristics of a wave are duration of a cycle, frequency, angular frequency, wavelength, phase, instantaneous value, peak value or maximum value or amplitude. Any sinusoidal quantity can be described mathematically, namely, analytically by an equation, graphically by a vector diagram or a line diagram.

The value of an alternating quantity (voltage or current) is called effective value (or r.m.s. value) if the same thermal effect is produced as caused by a corresponding direct quantity (voltage or current). Current and voltage data without special designation are always effective values in alternating current engineering.

Questions and problems:

1. Determine the duration of a cycle, angular frequency, and wavelength of the oscillations with the following frequencies

- |  |                                 |
|--|---------------------------------|
| a) technical alternating current                     | $f = 50 \text{ Hz}$             |
| b) test tone for electrical paths                    | $f = 16 \frac{2}{3} \text{ Hz}$ |
| c) test tone for telecommunication installations     | $f = 1 \text{ kHz}$             |
| d) transmitter frequency of a long-wave transmitter  | $f = 182 \text{ kHz}$           |
| e) transmitter frequency of a short-wave transmitter | $f = 6115 \text{ kHz}$          |
| f) transmitter frequency of a VHF transmitter        | $f = 97.15 \text{ MHz}$         |

2. What are the advantages and disadvantages of the vector diagram as compared with the line diagram?

3. Draw the vector and line diagrams of an alternating voltage whose frequency is 50 Hz, peak value 156 V and zero-phase angle  $30^\circ$ ! Select a suitable scale!

4. Determine the peak values of the following sinusoidal quantities:

- a) 6 V
- b) 380 V
- c) 15 kV
- d) 200 mA
- e) 10 A
- f) 25 A

5. The peak values of the following sinusoidal quantities are given:

- a) 311 V
- b) 70.7 mV
- c) 8.5 A
- d) 4.25 mA

Find the effective values!

### 7.3. Resistances in an Alternating Current Circuit

Effective resistance R

Loads which completely convert the electrical energy into heat energy are called effective resistances. They include thermal appliances, incandescent lamps and wire and film resistors used for current limitation. The behaviour of effective resistances in alternating current is the same as in direct current. Ohm's law dealt with in Section 3.2. is applicable to them without any restriction. Its resistance value R is independent of the frequency of the alternating current (Fig. 7.7.). The voltage has the same phase as the current. In Fig. 7.8., the vector diagram and the line diagram for current and voltage with an effective resistance is represented. Ideal effective resistances, also known as active resistances, have no inductance and no capacity.

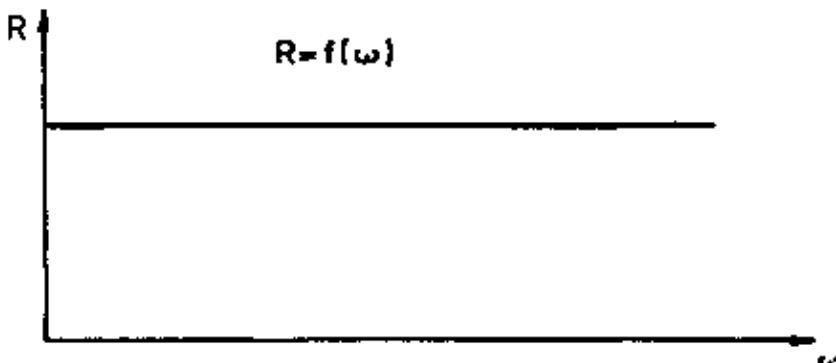


Fig. 7.7. Effective (or active) resistance as a function of frequency

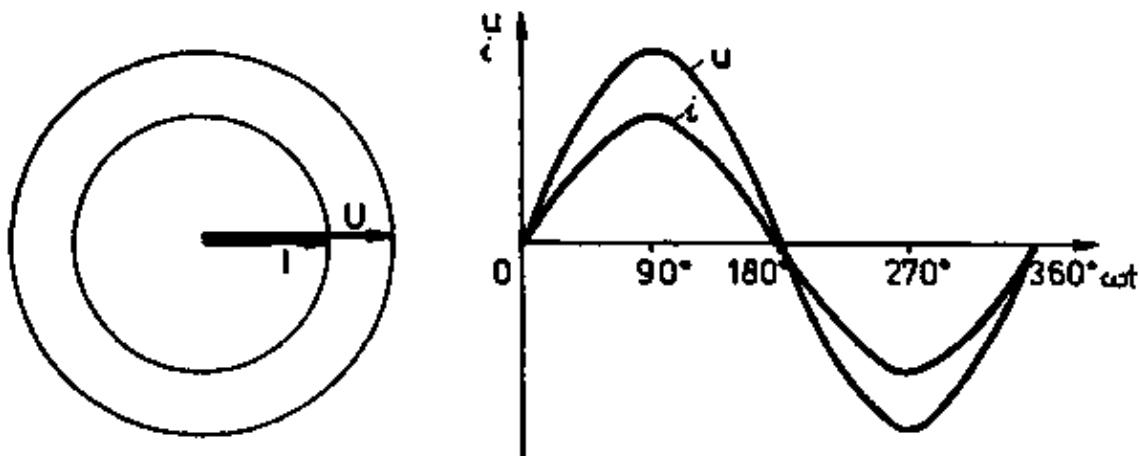


Fig. 7.8. Current and voltage curves for an effective resistance

Reactance X

Coils and capacitors do not convert the electrical energy into heat energy but store it in a magnetic or electrical field. Such components have a reactance. A distinction is made between inductive reactances and capacitive reactances.

- Inductive reactance  $X_L$

When an alternating current flows through a coil, a voltage is induced in the latter which offers a resistance to the passage of current. This capability of offering resistance is the greater, the greater the inductance and the rate of current variation (hence, the frequency) are. Consequently, the coil has a resistance which increases with increasing frequency.

$$X_L = \omega L = 2\pi f L \quad (7.7.)$$

where:

$X_L$  inductive reactance

? angular frequency

f frequency

L inductance

$$[X_L] = \frac{1 \text{ V}\cdot\text{s}}{\text{s} \cdot \text{A}} = \frac{\text{V}}{\text{A}} = \Omega$$

Fig. 7.9. shows the function  $X_L = f(\omega)$  (?)

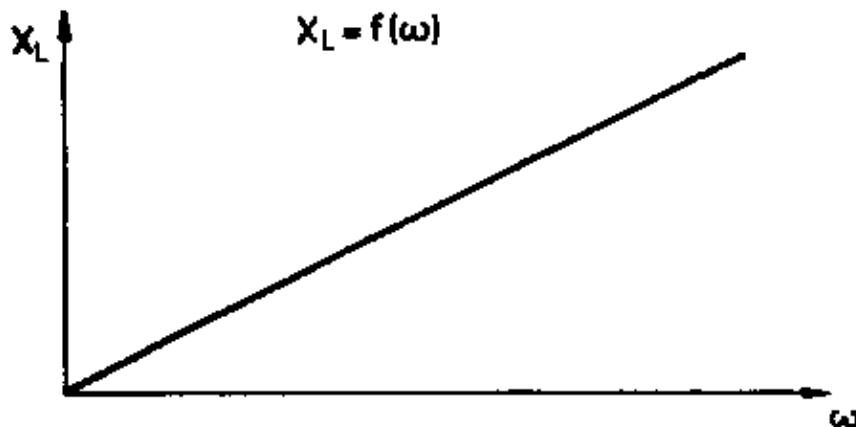


Fig. 7.9. Inductive reactance as a function of frequency

In Section 5.3.3. proof has been given of the fact that a coil imparts sluggishness to a current. When the

$$\frac{I}{4} \hat{=} 90^\circ$$

current passes through its maximum value (for alternating current this is  $\frac{1}{4}$  ), its rate of variation has the smallest value and the counter-voltage induced in the coil or the voltage drop is equal to zero.

$$90^\circ \hat{=} \frac{\pi}{2}$$

Consequently, there is a phase shift produced between current and voltage of  $\frac{\pi}{2}$ , that is to say, the current lags behind the voltage. Fig. 7.10. shows vector and line diagrams to illustrate these correlations. Ideal coils do not have an effective resistance and no capacity.

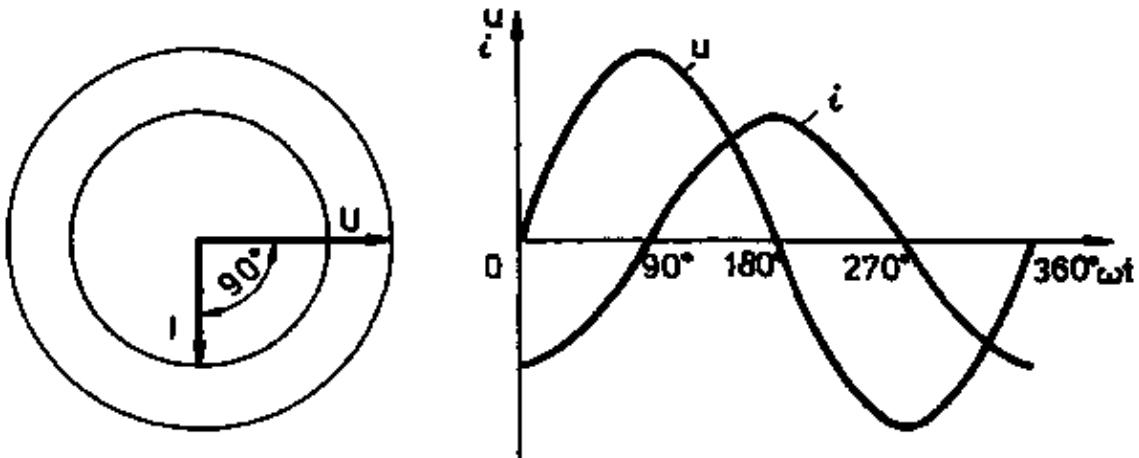


Fig. 7.10. Current and voltage curves for an inductive reactance

### Capacitive reactance $X_C$

When an alternating voltage is applied to a capacitor, then a continuously varying charging and discharging current is produced which apparently penetrates the capacitor. This current is the greater, the greater the capacity and the rate of voltage variation (i.e. the frequency) are. Consequently, the capacitor has a resistance which becomes smaller with increasing frequency.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad (7.8.)$$

where:

$X_C$  capacitive reactance  
 $\omega$  angular frequency  
 $f$  frequency  
 $C$  capacity

$$[X_L] = \frac{1}{\text{As}} = \frac{\text{V}}{\text{A}} = \Omega$$

Fig. 7.11. shows the function  $X_C = f(\omega)$ .

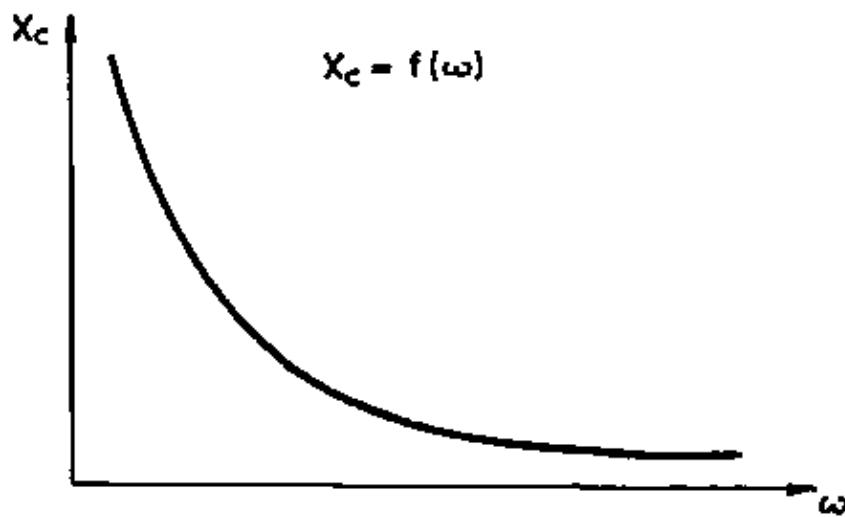


Fig. 7.11. Capacitive reactance as a function of frequency

In Section 6.2.2. it has been explained that no sudden voltage changes are possible in a capacitor. First a

current must flow before a voltage can be brought about. Like in a coil, in this case a phase shift of  $90^\circ \triangleq \frac{\pi}{2}$  between voltage and current takes place so that the current is in advance of the voltage. Fig. 7.12. shows vector diagram and line diagram illustrating these correlations.

Ideal capacitors have no effective resistance and no inductance.

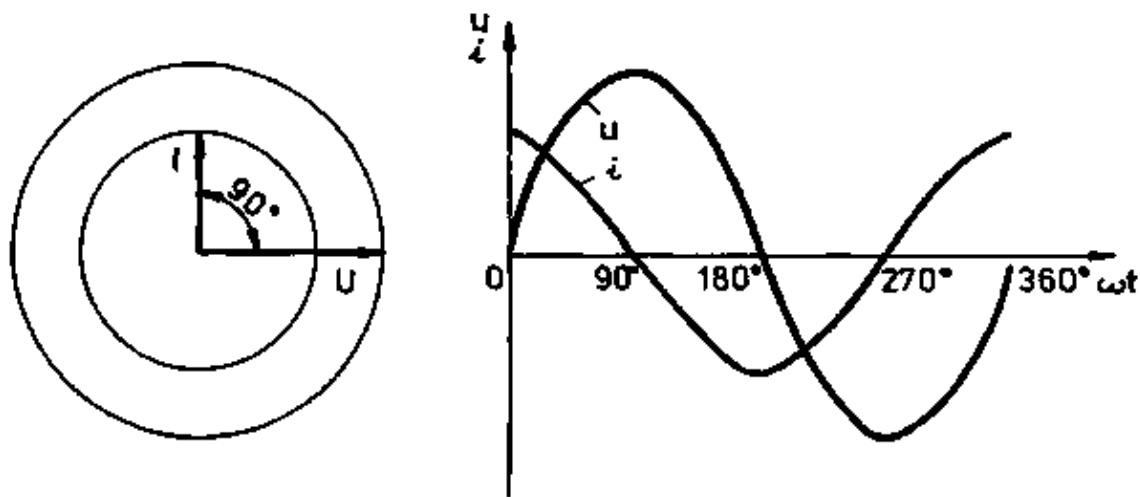


Fig. 7.12. Current and voltage curves for a capacitive reactance

## Impedance Z

When effective resistances and reactances are connected together, either in series or parallel or series-parallel, then the equivalent resistance of the overall circuit is called impedance Z. It is the quotient of effective voltage value and effective current value.

$$Z = \frac{U}{I} \quad (7.9.)$$

where:

Z impedance  
U effective voltage value  
I effective current value

$$[Z] = \frac{V}{A} = \Omega$$

The reciprocal value of the impedance is called admittance Y:

$$Y = \frac{1}{Z}$$

Since the phase shift between current and voltage is  $0^\circ$  in effective resistances,  $90^\circ$  in reactances, the following important facts are found:

- An impedance causes a phase shift between current and voltage which is greater than  $0^\circ$  but smaller than  $90^\circ$

The effective resistances and reactances resulting in the impedance are made up at right angles. This means that, in series connection, Z is always greater than the larger partial resistance but smaller than the algebraic sum of the two. Analogous conditions apply to the admittances in parallel connection.

Written in formular form, we have;

series connection

$$Z = \sqrt{R^2 + X^2} \quad (7.10.)$$

parallel connection

$$Z = \frac{1}{\sqrt{\frac{1}{R^2} + \frac{1}{X^2}}} \quad (7.11.)$$

$$\tan \varphi_{(u)} = \frac{X}{R} \quad (7.12.)$$

$$\tan \varphi_{(l)} = \frac{R}{X} \quad (7.13.)$$

If, besides effective resistances, inductive and capacitive reactances are contained in a circuit, attention must be paid to the fact that the phase-shifting effect of the coil (I lags behind U) is opposed to the action of a capacitor (I is in advance of U). Consequently, these two effects are partly neutralised or, in a special case, fully neutralised. The latter case is called resonance.

In a series connection of L and C, we have for X

$$X = |X_L - X_C| \quad \text{If } X_L > X_C, \text{ then } X \text{ is inductive}$$

$$X_L < X_C, \text{ then } X \text{ is capacitive}$$

$$X_L = X_C, \text{ then } X = 0 \text{ (resonance)}$$

In case of resonance, the highest current is flowing; it is only limited by the effective resistance (current increase).

In a parallel connection of L and C, we have for X

$$\frac{1}{X} = \left| \frac{1}{X_L} - \frac{1}{X_C} \right|$$

$$X = \left| \frac{1}{\frac{1}{X_L} - \frac{1}{X_C}} \right|$$

If  $X_L > X_C$ , then  $X$  is capacitive  
 If  $X_L < X_C$ , then  $X$  is inductive  
 If  $X_L = X_C$ , then  $X$  ?? (resonance)

In case of resonance, the smallest current is flowing, namely, only the current through an effective resistance connection in parallel. When current is supplied at a constant rate, a maximum voltage drop is brought about (voltage increase).

### Example 7.3.

A coil with an inductance  $L = 200 \text{ mH}$  is connected in series with an effective resistance  $R = 100 \Omega$ . A current of 500 mA with a frequency of 50 Hz is flowing through the circuit (Fig. 7.13.).

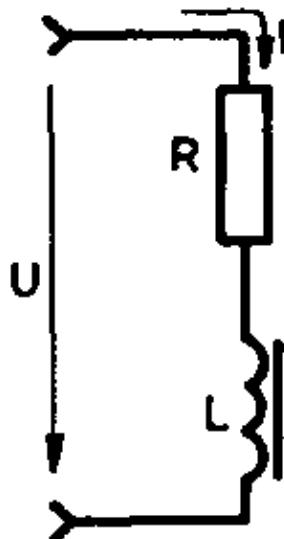


Fig. 7.13 Circuit for example 7.3

Draw the vector diagram for current and voltage true to scale. Calculate the partial voltages, the total voltage and the phase angle between current and voltage!

Given:  $L = 0.2 \text{ H}$  To be founds vector diagram  
 $H$   $U_R; U_L; U$   
 $R = ?(u)$   
 $100 \Omega$   
 $I = 0.5 \text{ A}$   
 $f = 50 \text{ Hz}$

Solution:  $U_R = R I = 100 \Omega \cdot 0.5 \text{ A}$

$$U_R = 50 \text{ V}$$

=====

$$U_L = X_L I$$

$$X_L = ? \quad L = 2 \quad fL = 2 \cdot 3.14 \cdot 50 \\ 1/s \cdot 0.2 \text{ H}$$

$$X_L = 62.8 ?$$

$$U_1 = 62.8 ? \cdot 0.5 \text{ A}$$

$$U_L = 31.4 \text{ V}$$

=====

Now, the vector diagram can be drawn with the data obtained. The total voltage  $U$  60 V and the phase angle  $\varphi = 32^\circ$  are indicated by the length of the vectors.

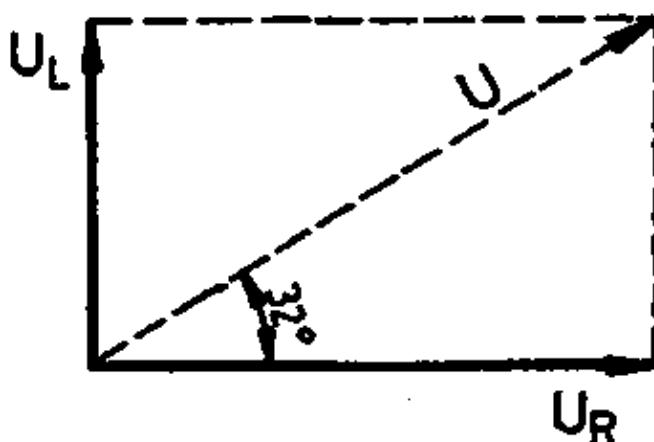


Fig. 7.14. Vector diagram for example 7.3.

Calculation:

$$U = Z I$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{10,000 + 3,944} \Omega = \sqrt{13,944} \Omega$$

$$Z = 118 ?$$

$$U = 118 ? \cdot 0.5 \text{ A}$$

$$U = 59 \text{ V}$$

=====

$$\text{Proof: } U = \sqrt{U_R^2 + U_L^2} = 59 \text{ V}$$

$$\tan \varphi = \frac{X_L}{R}$$

$$\tan \varphi = \frac{62.8}{100} = 0.628$$

$$\tan \varphi = 32^\circ$$

=====

Example 7.4.

A capacitor with a capacity of  $C = 5 \text{ nF}$  is connected in parallel to an effective resistance  $R = 100 \text{ k}\Omega$ . A voltage of 10 V having a frequency of 300 Hz is applied to the circuit (Fig. 7.15.).

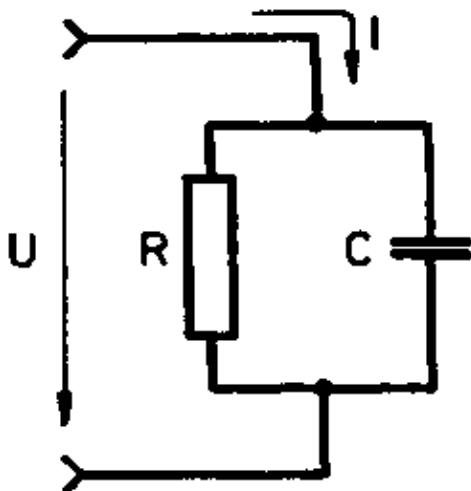


Fig. 7.15. Circuit for example 7.4.

Draw the vector diagram for current and voltage true to scale. Calculate the partial currents, the total current and phase angle between current and voltage!

Givens     $C =$  To be found: vector diagram

$$5 \text{ nF} \quad I_R; I_C; I$$

$$R = ?_{(i)}$$

$$100$$

$$\text{k?}$$

$$U =$$

$$10 \text{ V}$$

$$f =$$

$$300$$

$$\text{Hz}$$

Solution:  $I_R = \frac{U}{R} = \frac{10\text{V}}{100\text{k}\Omega}$

$$I_R = 100\mu\text{A}$$

=====

$$I_C = \frac{U}{X_C}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$= \frac{1}{2 \cdot 3.14 \cdot 300 \frac{1}{\text{s}} \cdot 5 \cdot 10^{-9} \text{F}}$$

$$X_C = 106\text{k}\Omega$$

$$I_C = \frac{10\text{V}}{106\text{k}\Omega}$$

$$I_C = 94\mu\text{A}$$

=====

Now, the vector diagram can be drawn on the basis of the values obtained above.

The total current  $I = 140 \text{ mA}$  and the phase angle of  $45^\circ$  are indicated by the lengths of the vector.

Calculation:

$$I = \sqrt{I_R^2 + I_C^2} = \sqrt{10,000 + 8,836} \mu\text{A}$$

$$I = 137 \mu\text{A}$$

=====

Proof:

$$I = \frac{U}{Z}$$

$$Z = \frac{1}{\sqrt{\frac{1}{R^2} + \frac{1}{X^2}}} = 73 \text{ k}\Omega$$

$$I = \frac{10V}{73 \text{ k}\Omega} = 137 \mu\text{A}$$

$$\tan \varphi = \frac{R}{X_C}$$

$$\tan \varphi = \frac{100}{106} = 0.943$$

$$\varphi = 43^\circ$$

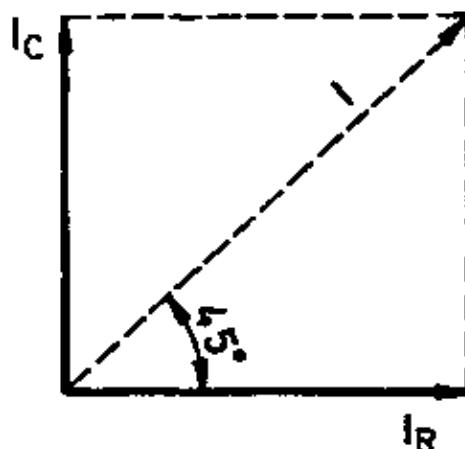


Fig. 7.16. Vector diagram for example 7.4.

Example 7.5.

An alternating voltage of 500 mV is applied to a series connection of  $R = 250 \Omega$ ,  $L = 200 \text{ mH}$  and  $C = 125 \text{ pF}$ . Calculate the maximum possible current and the frequency  $f_r$  at which this current will flow!

Given:  $R = 250 \Omega$  To be found:  $I_{\max}$   
 $L = 200 \text{ mH}$   $f$  (resonance frequency)  
 $C = 125 \text{ pF}$   
 $U = 500 \text{ mV}$

Solution:  $I_{\max} = \frac{U}{R}$  (case of resonance)

$$I_{\max} = \frac{0.5V}{250\Omega} = 2 \text{ mA}$$

$$I_{\max} = 2 \text{ mA}$$

resonance at  $X_L = X_C$

$$X_L = 2\pi f L \quad X_C = \frac{1}{2\pi f C}$$

$$2r_L = \frac{1}{2\pi f_r C}$$

Inversion for results in

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$f_r^2 = \frac{1}{2^2 \pi^2 LC}$$

$$f_r = \frac{1}{2 \cdot 3.14 \sqrt{200 \cdot 10^{-6} \frac{\text{Vs}}{\text{A}} \cdot 125 \cdot 10^{-12} \frac{\text{As}}{\text{V}}}}$$

$$f_r = \frac{10^9}{2 \cdot 314 \sqrt{25,000}}$$

$$H_z = \frac{10^9}{2 \cdot 314 \cdot 158}$$

$$H_z = \frac{1000 \cdot 10^6}{992}$$

Hz = 1.01 MHz

$f_r ? 1 \text{ MHz}$

=====

There are effective resistance, reactances and impedances in the form of loads.

Effective resistance convert the electrical energy completely into heat energy; They are independent of frequency and do not cause phase shifts between current and voltage.

Storage elements such as coils and capacitors have a reactance. It is frequency dependent and causes a 90° phase shift between current and voltage. There are inductive and capacitive reactances.

In inductive reactances, the current lags behind the voltage, and in capacitive reactances, the current is in advance of the voltage.

Impedances are interconnections of effective resistances and reactances. They are dependent on frequency because of the reactance included in the system. The magnitude of the impedance can be found by diagrams or by calculation by geometric addition. Depending on the preponderance of the inductive component or the capacitive component, either the voltage is ahead of the current or vice versa. The phase angle is always between 0° and 90°. If, in one circuit, inductive and capacitive components are present, they neutralise each other partly or completely. The special case where the inductive reactance is equal to the capacitive reactance is called resonance. The frequency in the presence of which resonance occurs is called resonant frequency or frequency of resonance. When resonance is present, a circuit has the behaviour of an effective resistance.

Questions and problems:

1. What is the essence of effective (or active) resistances, reactances and impedances?
2. A coil has a reactance of 100 Ω at a frequency of 50 Hz. What is the size of the inductance?
3. Represent graphically the curve of the reactance in dependence of the frequency from 0 to 10 kHz for a coil having an inductance of 5 H.

4. At a frequency of 50 Hz, a capacitor has an impedance of about 65  $\Omega$ . What is the size of its capacity?

5. Represent graphically the curve of the impedance in dependence of the frequency from 0 to 10 kHz for a capacitor with a capacity of 100  $\mu\text{F}$ !

6. A coil with a loss resistance (effective resistance) of 12  $\Omega$  connected in series has an impedance of  $Z = 20 \Omega$  at a frequency of 50 Hz. The phase angle is  $53^\circ$ . Determine the magnitude of the inductive reactance and the inductance by graph and by calculation!

7. What is the frequency  $f$  which resonance occurs in a circuit?

#### 7.4. Power of Alternating Current

When loads carry current, a voltage drop is caused. The product of the instantaneous values of current and voltage is called instantaneous power. Normally, the instantaneous power changing from time to time is of less interest than its mean value.

In an effective resistance, current and voltage are in phase. The electrical power becomes completely, i.e. effectively, utilisable. It is called effective power  $P_e$  (active power).

It can be determined on the basis of the effective values according to the relations derived in Section 4.1.

$$[P_e] = W \text{ (watt)}$$

In a reactance (ideal coil, ideal capacitor), current and voltage are subjected to a phase shift of  $90^\circ$ . The electrical power is required for the duration of a quarter of a cycle for the building up of the magnetic field (in a coil) or of the electrical field (in a capacitor) and delivered in the subsequent quarter of a cycle. There is no power conversion in a temporal mean. This power is called reactive power  $P_r$ .

$$[P_r] = \text{Var} \text{ (voltampere – reactive)}$$

In an impedance, the phase shift between current and voltage is between  $0^\circ$  and  $90^\circ$ . The electrical power is converted partly as effective power and partly as reactive power. This power is called apparent power  $P_a$ .

$$[P_a] = \text{VA} \text{ (voltampere)}$$

Since effective resistances and reactances are made up at right angles, this analogously applies to the powers. Consequently, effective and reactive powers are the legs and the apparent power is the hypotenuse of a right-angled triangle having the phase angle (Fig. 7.17.).

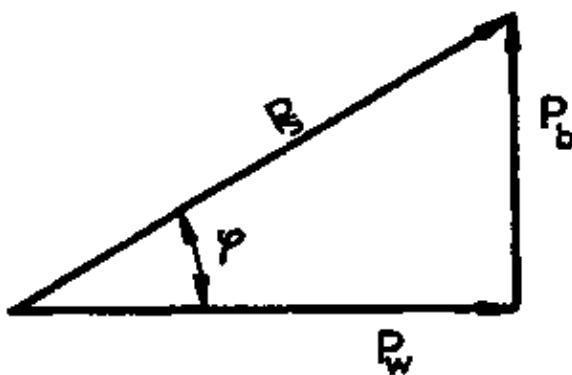


Fig. 7.17. Correlation between effective power, reactive power and apparent power

$$P_w = P_e, P_b = P_r, P_s = P_a$$

From this follows

$$P_a = U I \quad (7.14.)$$

where:

$P_a$  apparent power  
 $U$  effective value of voltage  
 $I$  effective value of current

From Fig. 7.17. we can derive

$$P_a = \sqrt{P_e^2 + P_r^2} \quad (7.15.)$$

$$P_e = P_a \cos \varphi \quad (7.16.a)$$

$$P_r = P_a \sin \varphi \quad (7.16.b)$$

The expression  $\cos \varphi$  in equation (7.16.a) is called power factor.

From equation (7.16.a) we have

$$\cos \varphi = \frac{P_e}{P_a} \quad (7.16.c)$$

The power factor  $\cos \varphi$  can be between 0 and 1;

$$0 \leq \cos \varphi \leq 1$$

With a low power factor, the reactive power is high. Since reactive power unnecessarily loads the generators of the power stations and the distribution network, the reactive power must be kept as small as possible for economical reasons; in other words,  $\cos \varphi$  must be as high as possible. A power factor of  $\cos \varphi = 1$  is an ideal case.

In networks of power electrical engineering, an inductive phase shift occurs always because of the necessary transformers and connected motors; this phase shift always worsens the power factor.

The power factor can be improved up to the value of  $\cos \varphi = 1$  by means of an additional capacitive component. In practice, capacitors are connected in parallel having a total capacity of

$$C = \frac{P_r}{\omega U^2} \quad (7.17.)$$

where:

$C$  capacity required to attain a  $\cos \varphi = 1$   
 $P_r$  reactive power  
 $\omega$  angular frequency  
 $U$  alternating voltage (effective value)

Of the energy or work has to be determined, then the product of power times time must be formed according to Section 4.1. In accordance with the various types of power, there are effective work, reactive work and apparent work.

Example 7.6.

An enterprise is connected to a 380 V network (50 Hz). A current of 66 A passes through the loads with a power factor of  $\cos \varphi = 0.5$ . Determine the apparent power, effective power and reactive power and the capacity of the capacitor necessary to improve the power factor to  $\cos \varphi = 1$ !

Given:

$U = 380 \text{ V}$   
 $I = 66 \text{ A C}$   
 $f = 50 \text{ Hz}$   
 $\cos\phi = 0.5$

To be founds  $P_a$ ,  $P_e$ ,  $P_r$   
C

Solution:  $P_a = U I$

$$P_a = 380 \text{ V} \cdot 66 \text{ A} = 25,000 \text{ VA}$$

$$P_a = 25 \text{ kVA}$$

=====

$$P_e = P_a \cos\phi$$

$$P_e = 25 \text{ kVA} \cdot 0.5$$

$$P_a = 12.5 \text{ kW}$$

=====

$$P_r = P_a \sin\phi$$

$$\sin\phi = \sqrt{1 - \cos^2\phi} = \sqrt{1 - 0.25} = 0.865$$

$$P_r = 25 \text{ kVA} \cdot 0.865$$

$$P_r = 21.6 \text{ kVar}$$

=====

Proof with equation (7.15.):

$$P_a = \sqrt{P_e^2 + P_r^2}$$

$$P_a = \sqrt{156 + 469} \text{ kVA} = \sqrt{625} \text{ kVA} = 25 \text{ kVA}$$

$$C = \frac{P_r}{\omega U^2}$$

$$C = \frac{21600 \text{ V} \cdot \text{A}}{2 \cdot 3.14 \cdot 50 \frac{1}{s} \cdot 380^2 \text{ V}^2} \approx 476 \mu\text{F}$$

$$C \approx 500 \text{ } \mu\text{F}$$

=====

With a capacitor connected in parallel with the loads having a capacity of  $C = 500 \text{ } \mu\text{F}$ , the bad power factor of  $\cos\phi = 0.5$  can be increased to the ideal value of  $\cos\phi = 1$ .

With alternating current, there are three types of powers, namely, apparent power, effective power and reactive power. Effective power and reactive power are made up at right angles. The apparent power is always greater than effective power or reactive power but it is always smaller than the algebraic sum of the latter two. Efforts are always made to achieve a high effective power and a reactive power which is as small as possible. The ratio of effective power to apparent power is called power factor. It can reach the value of 1 in the most favourable case. The power factor can be improved when a capacitor is connected in parallel with the loads.

## Questions and Problems:

1. What are the differences between effective power, reactive power and apparent power?
2. What are the values of the effective power and reactive power when the apparent power is 23 kVA with a phase angle of  $\phi = 30^\circ$ ?
3. Why is a high power factor desired in energy supply systems? Explain measures by means of which the power can be improved!
4. Two motors are connected in parallel and to a supply system of 220 V. One motor has a power factor of 0.65 and carries a current of 2 A; the other motor having a power factor of 0.85 carries a current of 3.5 A. Calculate for the total circuit the effective power, reactive power and apparent power and the total power factor!

## 8. Three-phase Current

### 8.1. Generation of Three-phase Current

In the preceding Chapter we have shown that a sinusoidal alternating voltage is induced in a conductor loop which is rotated in a homogeneous magnetic field. Several conductor loops which are mechanically connected together can also be turned in a magnetic field at the same time. Then, in each conductor loop, a sinusoidal alternating voltage is induced. An arrangement where three conductor loops displaced by  $120^\circ$  to each other are used has gained great importance in practice (Fig. 8.1.).

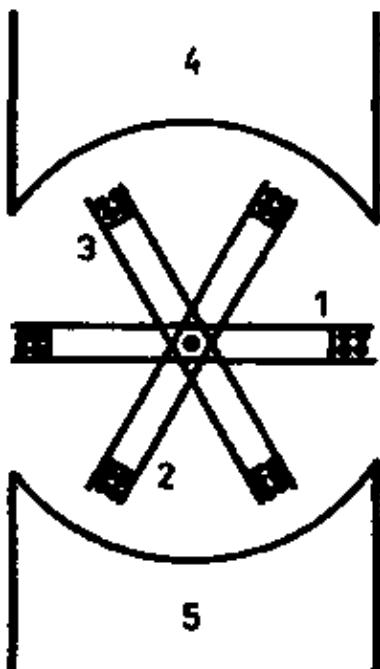


Fig. 8.1. Principle of the three-phase overhung-type alternator

1,2,3 Rotatable coils; 4 North pole; 5 South pole

In order to conduct the electrical energy generated in this overhung-type alternator to the consumer, sliding contacts must be provided for all three conductor loops. This disadvantage is not associated with the inner-pole alternator. In this type, the three conductor loops are fixed in the stator of the alternator (an alternating-current generator is also called alternator) while the magnetic field in the interior is rotated (Fig. 8.2.). The relatively low electrical energy for the production of the magnetic field must be fed to the rotor of the generator via sliding contacts.

In each of the three coils, a sinusoidal voltage is produced when the magnetic field is rotating which – in accordance with the arrangement of the coils – exhibits a phase shift of  $120^\circ$ . (Consequently, a voltage maximum is always reached, when the magnetic pole is turned past the coil. For the three coils, this always occurs after a rotation of the magnetic pole through  $120^\circ$ ). The line diagram of these three voltages is shown in Fig. 8.3.

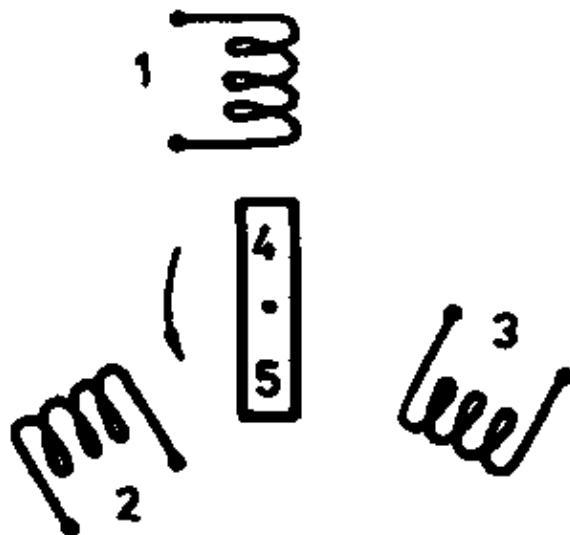


Fig. 8.2. Principle of the three-phase inner-pole alternator

1, 2, 3 Fixed coils; 4 North pole; 5 South pole

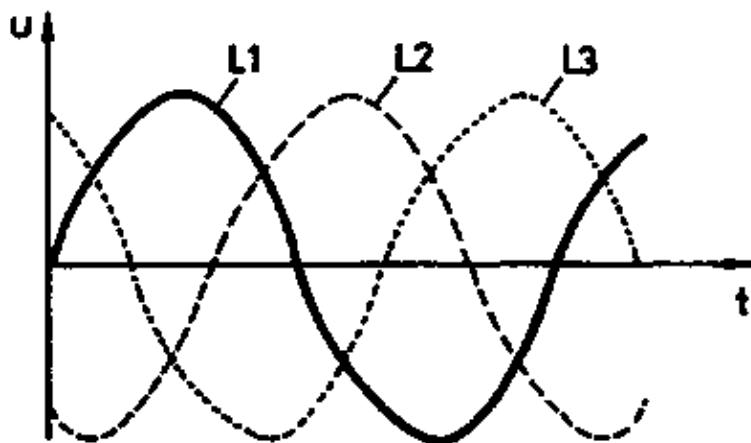


Fig. 8.3. Line diagram of voltages of three-phase current

The three voltages subjected to a phase-shift of  $120^\circ$  are called three-phase current. In order to distinguish safely between the three voltages and their three coils, the three phases are marked by L1, L2 and L3 and coloured (L1: yellow; L2: green; L3: violet) according to an IEC-recommendation.<sup>1)</sup> The connections of the starts of the coils can be designated by U, V and W and the ends of the coils by X, Y and Z. For the phases L1 to L3, sometimes R, S and T are used as designation.

<sup>1)</sup> IEC = International Electrotechnical Commission

When a magnetic field is rotated inside of three coils which are displaced by  $120^\circ$  to each other, then three sinusoidal voltages are produced which are called three-phase current. A phase shift of  $120^\circ$  exists between every two of the three voltages.

Questions:

1. What is the difference between overhung-type alternator and inner-pole alternator?
2. What are the advantages of the inner-pole alternator over the overhung-type alternator?
3. How are the three phases distinguished from each other by markings?

4. What is the phase angle between the voltages of the three phases?

## 8.2. The Rotating Field

The most important property of the three-phase current is discussed here. For this purpose, we again start from three coils displaced by 120° from each other which are connected to a three-phase current according to Fig. 8.3. (Fig. 8.4.).

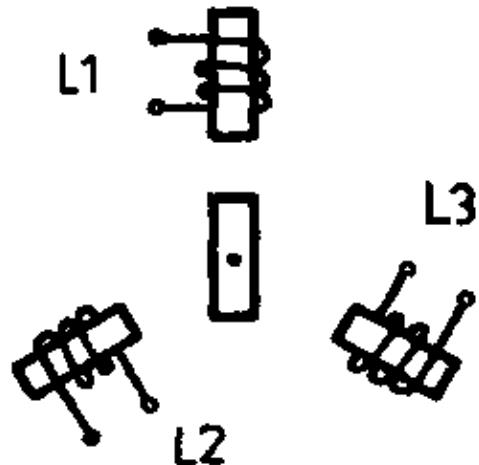
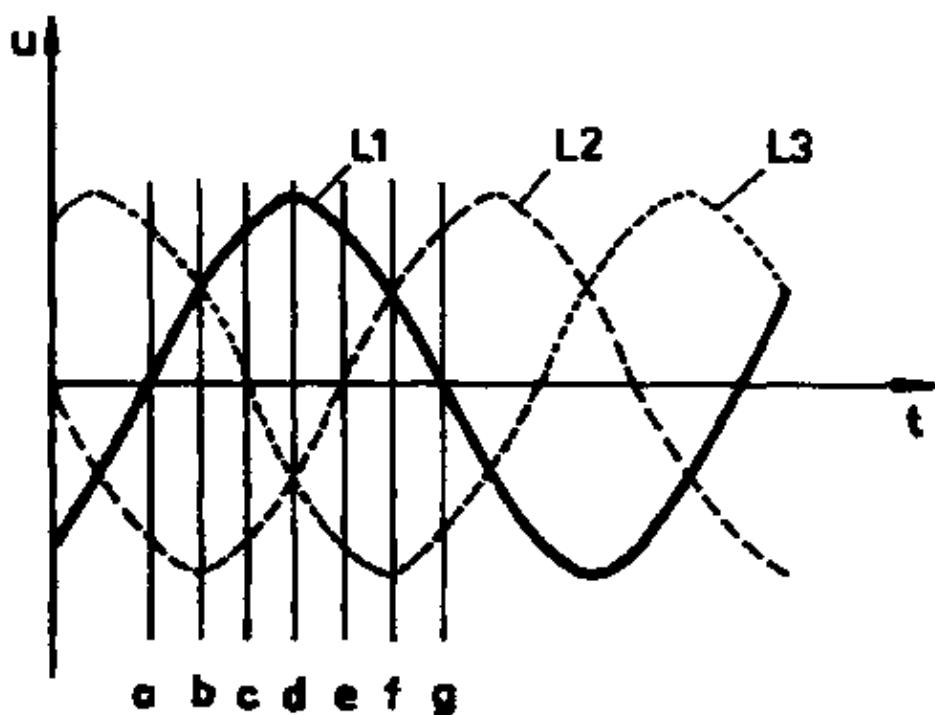
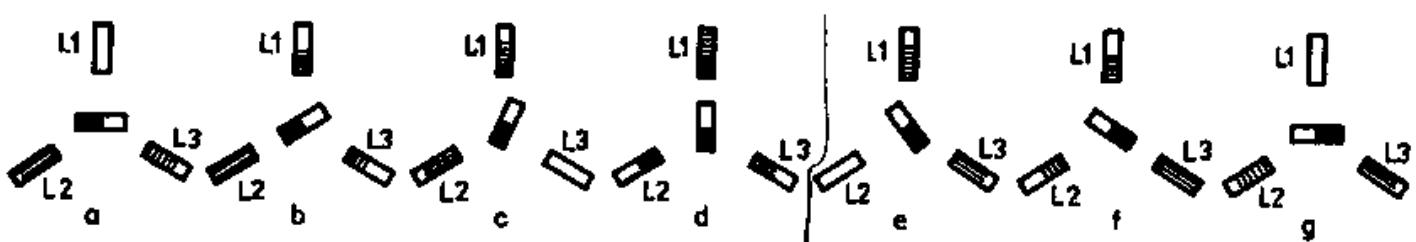


Fig. 8.4. Three-phase winding of a motor

Fig. 8.5. Development of the rotating field



a) Three-phase line diagram with points of time plotted on the diagram



b) Resulting magnetic field for the points of time plotted on

In each of the three coils, an alternating field will be brought about in accordance with the alternating voltage applied. The total magnetic field resulting from the magnetic fields of the 3 coils is subject to a closer examination below. For this purpose, we have to give some explanations regarding the representations. When the positive half wave of the alternating voltage is applied to the coil, a magnetic north pole will be formed at the end of the coil which is inside the arrangement in accordance with the sense of winding of the coil. This pole is represented by cross hatching. The density of the lines of hatching corresponds to the amplitude of the voltage present at the instant of observing. When the negative half wave is applied, the south pole is formed which is represented by longitudinal hatching. For the points of time entered into the three-phase current line diagram in Fig. 8.5.a, the magnetic flux produced in the three coils at the points of time a to g is shown in Fig. 8.5.b. The position of the total magnetic field brought about by the three individual fields is represented by the position of a rod magnet whose north pole is marked black. (A representation of the coils as given in Fig. 8.4. is omitted for the sake of clearness.) The position of the total magnetic field changes with the interval of time under consideration of the three-phase current line diagram. It is evident that the total magnetic field has turned through half a revolution from point of time a to point of time g. When further partial pictures would be represented, then a full revolution of the total magnetic field would be performed during the duration of a cycle of the three-phase current. This shows that, in a three-phase winding connected to three-phase current, a rotating magnetic field is formed which is called rotating field. The majority of electric motors and their mode of operation are based on the presence of the rotating field which enables extremely simple and sturdy designs of electric motors. Within a full cycle of the three-phase current one revolution of the rotating field is performed in an arrangement according to Fig. 8.5. When two times three coils are arranged at the circumference, then the rotating field will perform only half a revolution within one cycle. Such an arrangement is termed as design with two pairs of poles. There are also arrangements with a higher number of pole pairs. For the calculation of the rotational speed of the rotating field we have

$$n_D = \frac{f}{p} \quad (8.1.)$$

where:

$n_D$  rotational speed of the rotating field  
 $f$  frequency  
 $p$  number of pole pairs

Example 8.1.

A three-phase current winding (also known as polyphase winding) with three pairs of poles is connected to a three-phase current having a frequency of 50 Hz. Determine the rotational speed of the rotating field per minute!

Given:  $f = 50 \text{ Hz}$  To be found:  $n_D$  in rpm  
 $p = 3$

Solution:  $n_D = \frac{f}{p}$

$$n_D = \frac{50}{3} \frac{1}{\text{s}}$$

Since the speed per minute is required, the above calculation will, however, result in the revolutions per

second, multiplying by  $60 \frac{\text{s}}{\text{min}}$  is necessary.

$$n_D = \frac{50 \frac{1}{\text{s}} \cdot 60 \frac{\text{s}}{\text{min}}}{3}$$

$$n_D = 1000 \text{ rpm}$$


---

The rotating field has a speed of 1000 revolutions per minute.

When exchanging two phases shown in Fig. 8.5. in the manner represented in Fig. 8.6., the sense of rotation of the rotating field is inverted. This property of the three-phase current is also of advantage to the construction of electric motors.

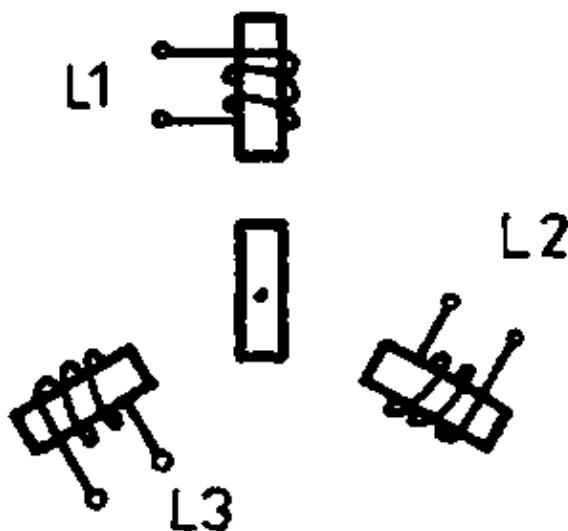


Fig. 8.6. Three-phase winding with the phases exchanged as compared to Fig. 8.5.

Inside a polyphase winding connected to three-phase current a rotating field is formed (rotating magnetic field) whose sense of rotation can be inverted by exchanging two connections. The rotational speed of the rotating field is dependent on the frequency and the number of pole pairs. Simple designs of motors are possible due to the rotating field involved in three-phase current.

Questions and problems:

1. Under which conditions is a rotating field formed?
2. Demonstrate the reversal of the sense of rotation due to the exchange of two phases as shown in Fig. 8.6. with the help of a representation as given in Fig. 8.5.b!
3. What is the technical importance of the rotating field?
4. Calculate the possible rotational speeds of the rotating field for arrangements with 1 to 10 pairs of poles when the frequency is 50 Hz!

### 8.3. Interlinking of the Three-phase Current

The explanations given in Section 8.1. show that three-phase current involves three phases with two connections each. Consequently, six lines would be necessary as connection between generator and consumer or load. Such an open three-phase system is not used in practice. By a combination of certain conductors, which is termed as interlinking, connecting lines can be saved. In practice two of such interlinking connections are used, namely, the star connection (also known as Y-connection) and the delta connection.

The interlinking of the three phases of a three-phase current into a star connection is shown in Fig. 8.7. For this purpose, the ends of the three generator coils (X, Y and Z) are connected and, in most cases, the connecting point is brought out as neutral conductor N. Then, four lines are required between generator and consumer.

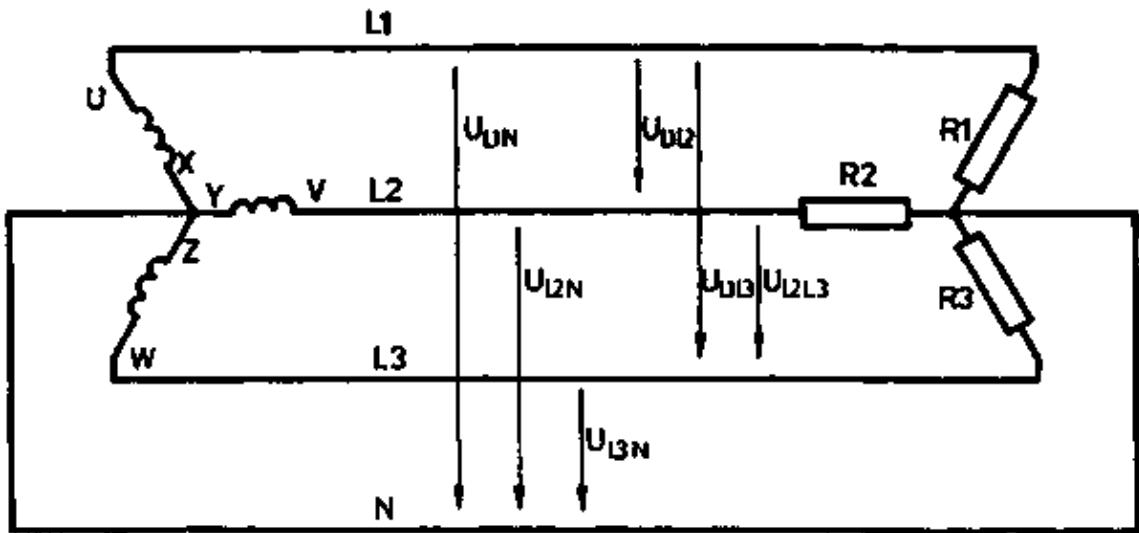


Fig. 8.7. Star connection

When the load resistors also arranged in star connection have the same resistance values and cause an equal phase angle between current and voltage (in Fig. 8.7., pure active load with a phase angle of 0 is represented), then a line diagram will be brought about for the three currents which resembles that shown in Fig. 8.3. When we take a closer look at the sum of the three instantaneous values at any point of time in this line diagram, we will find that this sum is always equal to 0. Let us study this fact with particular respect to certain points of time on the basis of the line diagram shown in Fig. 8.5.a. For example, at point of time b, the phases L1 and L3 have half of the positive peak value and phase L2 the negative peak value. At point of time c, phase L3 has the value of 0, and the values of phases L1 and L2 are equal but of opposed directions. From this follows that no current is flowing in the neutral conductor N with the same phase load. In practice, equal phase load is seldom given so that a current flows in the neutral conductor which, however, is smaller than the current in the phase conductors L1, L2 and L3.

Fig. 8.7. also shows that a voltage can be tapped between phase conductor and neutral conductor ( $U_{L1N}$ ,  $U_{L2N}$ ,  $U_{L3N}$ ) which is designated by  $U_{Str}$ , and between two phase conductors ( $U_{L1L2}$ ,  $U_{L1L3}$ ,  $U_{L2L3}$ ) which is designated by  $U_L$ . The voltage between the phase conductors is produced by series connection of two generator coils. In order to be in a position to calculate the magnitude relations between  $U_{Str}$  and  $U_L$ , a similar circuit in a direct current circuit will be considered as a repetition (Fig. 8.8.).

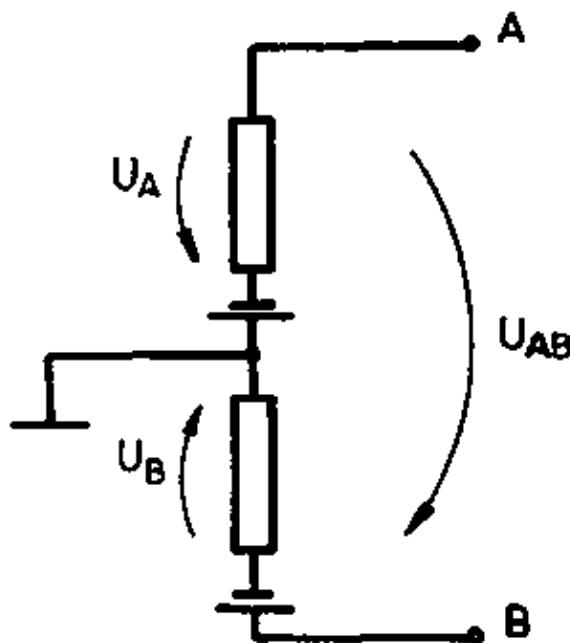


Fig. 8.8. Determination of the voltage between the points A and B

Two voltage sources are connected in series and the point of connections is considered as the reference point (e.g. frame or earth). In this example, point A has a positive voltage ( $U_A$ ) with respect to the reference point and point B has a negative voltage ( $U_B$ ) with respect to the reference point. Between the points A and B is the

difference of the voltages  $U_A$  and  $U_B$  related to the reference point.

$$U_{AB} = U_A - U_B$$

When we assume that  $U_A = 2 \text{ V}$  and  $U_B = -2 \text{ V}$ , then we have for  $U_{AB}$

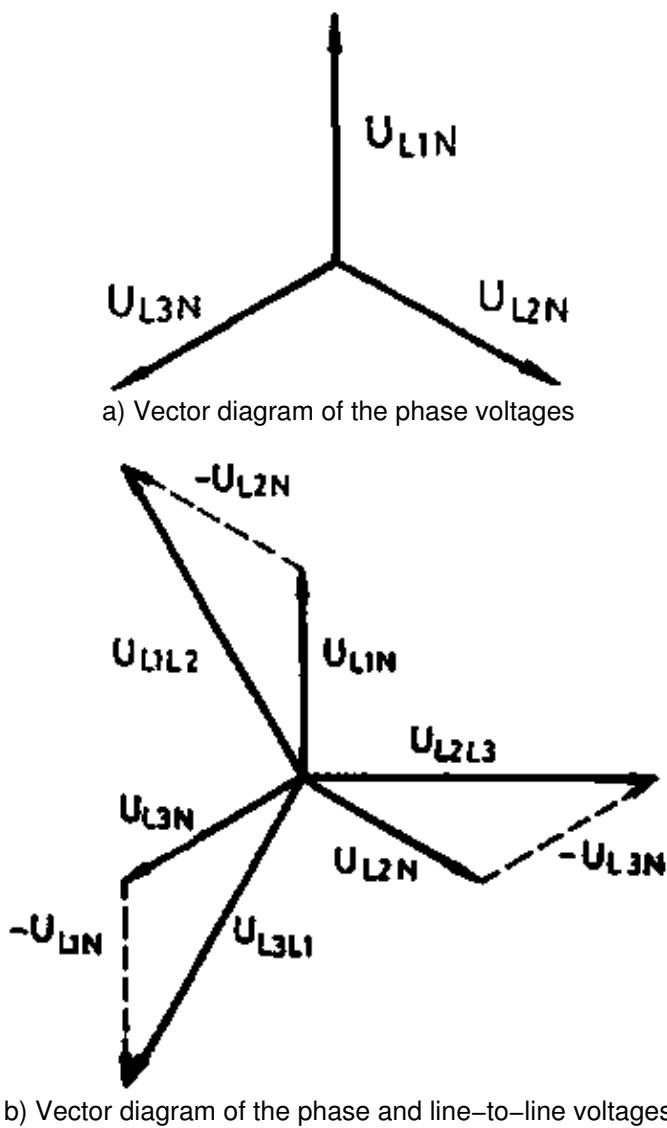
$$U_{AB} = +2 \text{ V} - (-2 \text{ V})$$

$$U_{AB} = 4 \text{ V}$$

=====

Similar conditions are given in a star connection; however, the phase shift between the voltage sources connected in series must be taken into consideration. In order to ascertain the voltage between the phase conductors, we first have to represent the three phase voltages (also known as star voltage) with their phase shift of  $120^\circ$  to each other in the form of voltage vectors.

**Fig. 8.9. Graphical determination of the line-to-line voltage**



In order to be in a position to determine the voltage between the phase conductors, two voltages related to the neutral conductor as reference point in each case must be (...) from each other by adding a further vector to the tip of a given vector but having opposite direction. As resultant vector we obtain the line-to-line voltage. By comparison of magnitudes in a vector graph true to scale, we obtain as magnitude relation between phase voltage and line-to-line voltage

$$U_L = 1.73 \cdot U_{\text{Str}}$$

The numerical factor 1.73 is the root extracted from 3 and, consequently, the equation can be written as

$$U_L = \sqrt{3} \cdot U_{Str} \quad (8.2.)$$

where:

$U_L$  line-to-line voltage (or phase-to-phase voltage)  
 $U_{Str}$  phase voltage or star voltage or voltage to neutral

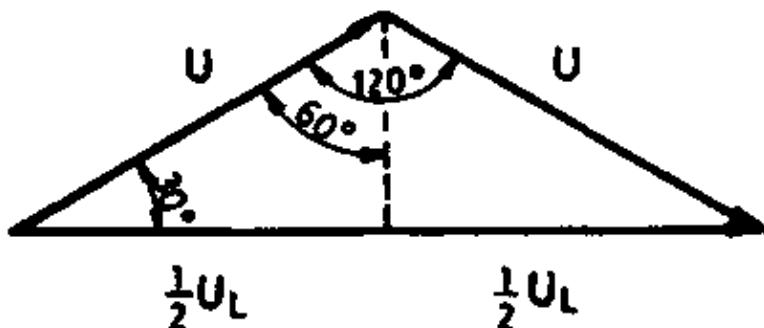


Fig. 8.10. Triangle for calculating the magnitude relation between phase voltage and line-to-line voltage

$U_{Str}$  = phase voltage

$U_L$  = line-to-line voltage

The relation of magnitude between  $U_L$  and  $U_{Str}$  can also be determined mathematically. For this purpose, Fig. 8.10. shows a triangle as a part of the representation given in Fig. 8.9. Using the trigonometric functions, we have

$$\frac{U_L}{2} = U_{Str} \cdot 30^\circ$$

$$U_L = 2 \cdot U_{Str} \cdot \cos 30^\circ$$

$$U_L = 2 \cdot U_{Str} \frac{1}{2} \sqrt{3}$$

$$U_L = \sqrt{3} \cdot U_{Str}$$

=====

### Example 8.2.

The phase voltage of a three-phase current network is 220 V. Which line-to-line voltage is available in this network?

Given:  $U_{Str} = 220$  To be found:  $U_L$

Solution:  $U_L = \sqrt{3} \cdot U_{Str}$

$$U_L = 1.73 \cdot 220 \text{ V}$$

$$U_L = 380 \text{ V}$$

=====

A line-to-line voltage of 380 V is available.

Example 8.2. demonstrates a typical case. With such a network, the voltage of 220 V desired by households can be supplied as the phase voltage while a line-to-line voltage of 380 V is available from the same network for industrial enterprises.

In practice, sometimes the phase relation between phase voltage and line-to-line voltage is utilised. Thus, Fig. 8.9.b shows that, for example  $U_{L1L2}$  exhibits a phase shift of  $90^\circ$  with respect to  $U_{L3N}$ . In general, it holds that the line-to-line voltage has a phase shift of  $90^\circ$  with respect to the phase voltage of the phase conductor not under consideration.

In the delta connection shown in Fig. 8.11., one starts from the consideration that the sum of the individual voltages is always 0 at any time. Therefore, no balance current can flow within the generator in delta connection. In this circuit, only the line-to-line voltage occurs. But in the junctions, a current division is obtained. For the upper junction we have

$$I_{L1} = I_{WZ} - I_{UX}$$

When the three line-to-line currents have the same intensity, then the vector representation given in Fig. 8.12. is obtained. For the relation of magnitude between phase current and line-to-line current a relation is given similar to the relation of magnitude between phase voltage and line-to-line voltage. With the same phase load we have

$$I_L = \sqrt{3} \cdot I_{Str} \quad (8.3.)$$

=====

where:

$I_L$  line-to-line current (or phase-to-phase current)

$I_{Str}$  phase current

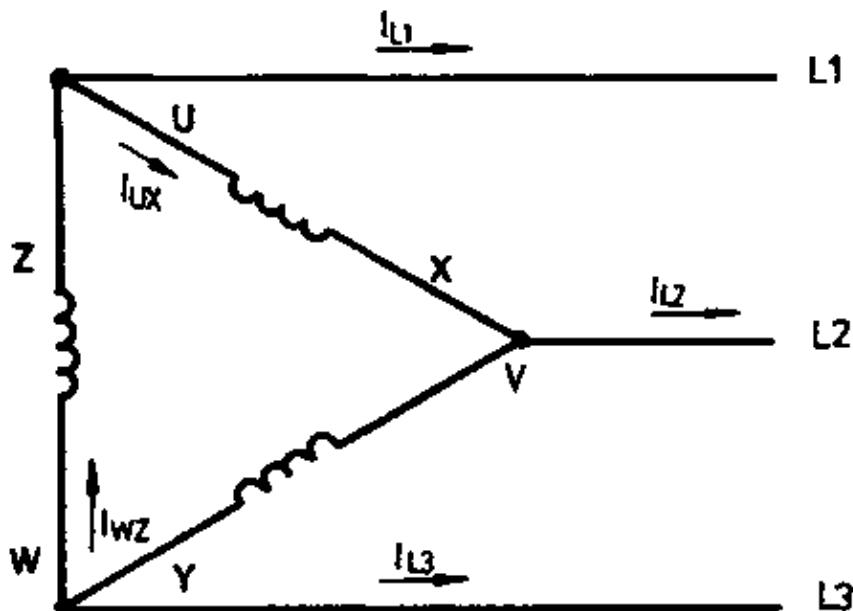


Fig. 8.11. Vector representation of the currents

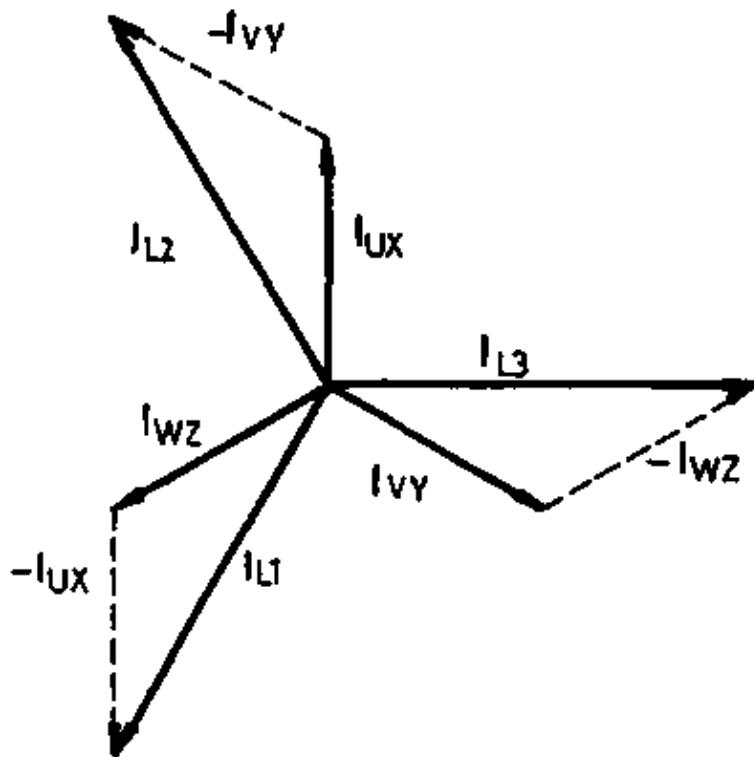


Fig. 8.12. Delta connection

In three-phase systems, two methods of interlinking are possible – the star connection and the delta connection. In the star connection, voltages can be tapped both between the phase conductors and between the neutral conductor and the phase conductors. In the delta connection, only the line-to-line (phase-to-phase) voltage is available, however, the difference between line-to-line current and phase current must be taken into consideration.

Questions and problems:

1. Sketch the star connection and the delta connection of the three coils of a generator!
2. Determine the relations of magnitude between phase voltage (voltage to neutral) and line-to-line (phase-to-phase) voltage by means of sketch true to scale! (Note: Select a length of 50 MM for the phase voltage!)
3. Calculate the voltage to neutral with a phase-to phase voltage of 220 V!
4. What is the value of the current flowing in the coils of a generator in delta connection when the current in the phase conductors is 34.6 A?

#### 8.4. Power of Three-phase Current

For calculating the power of three-phase systems, the same relations are applicable as for the calculation of the power of alternating current systems. In accordance with the phase angle involved, a distinction is also made between effective power, reactive power and apparent power.

The star connection of three equal resistors is shown in Fig. 8.13.

For the total power, we have

$$P = 3 \cdot U_{\text{Str}} \cdot I_L \cdot \cos \varphi$$

When the power is to be determined on the basis of the line-to-line voltage  $U_L$ , the following holds when using equation u.2.

$$P = 3 \cdot \frac{U_L}{\sqrt{3}} \cdot I_L \cdot \cos \varphi \quad (8.4.)$$

$$P = \sqrt{3} \cdot U_L \cdot I_L \cos \varphi$$

=====

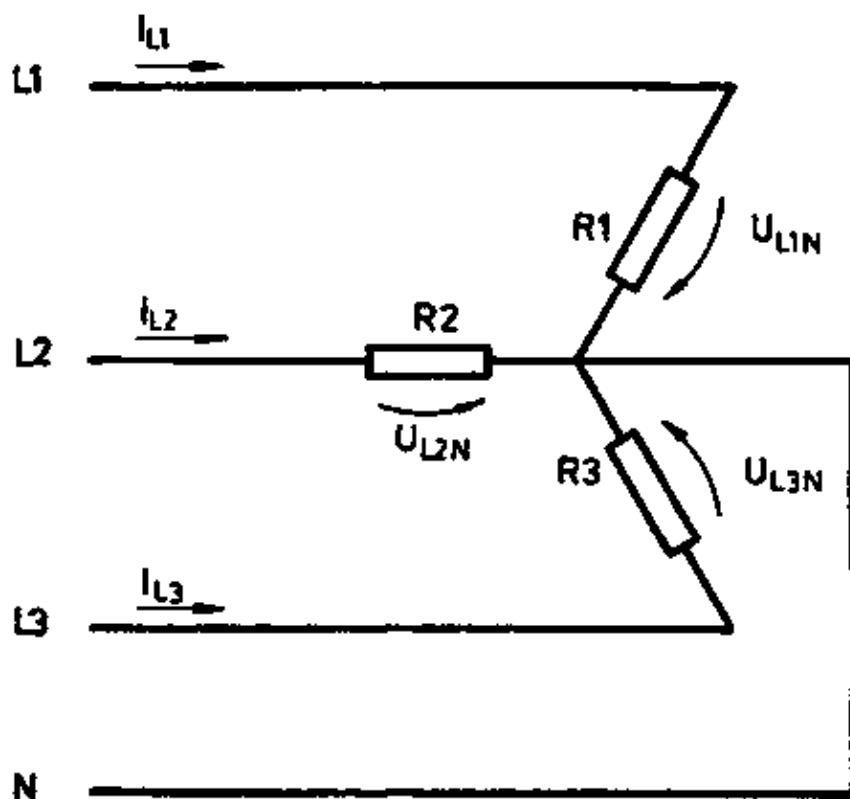


Fig. 8.13. Star connection of three resistors

When three equal resistors are connected in delta (Fig. 8.14.), the total power is written as

$$P = 3 \cdot U_L \cdot I_{Str} \cdot \cos \varphi$$

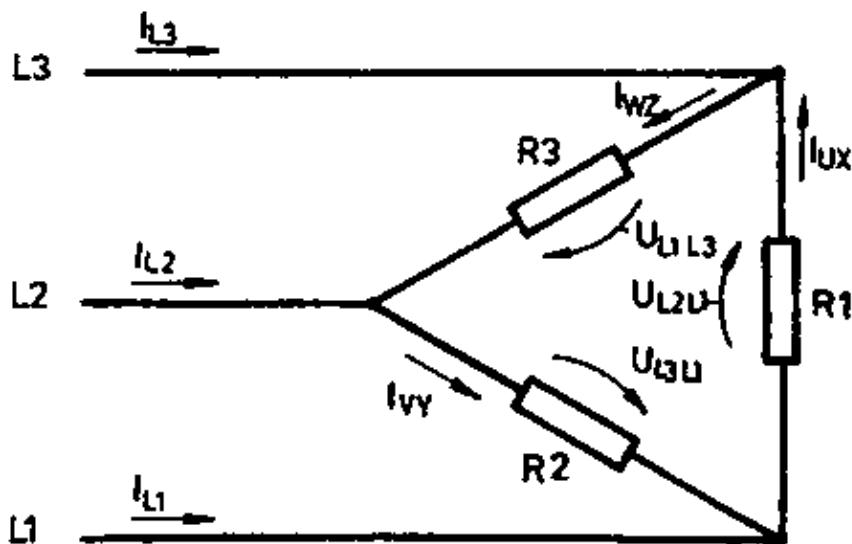


Fig. 8.14. Delta connection of three resistors

When the line-to-line current is used, the following holds when using equation 8.3.

$$P = 3 \cdot U_L \cdot \frac{I_L}{\sqrt{3}} \cdot \cos\varphi \quad (8.5.)$$

$$P = \sqrt{3} \cdot U_L \cdot I_L \cos\varphi$$

=====

where:

$U_L$  line-to-line voltage  
 $I_L$  line-to-line current  
 $\cos\varphi$  power factor

A comparison of the equations 8.4. and 8.5. shows that, independent of the given type of connection, the same equations for calculating the power are given.

When the phase load is unequal, the total power is obtained in the form of the sum of the powers in the three phases to be determined individually.

Example 8.3.

Three resistors of  $800 \Omega$  each have to be interposed in a three-phase network of 380 V one time in star connection and another time in delta connection. Calculate the effective power involved in each case.

Given:  $U_L = 380 \text{ V}$  To be found: effective power  $P$  for star connection and for delta connection  
 $R = 800 \Omega$   
 $\cos\varphi = 1$

Solution:

star connection of the three resistors

$$P = \sqrt{3} \cdot U_L \cdot I_L \cos\varphi$$

In star connection, only the phase voltage drops at the three resistors. Hence, for the current  $I_L$  we have

$$I_L = \frac{U_L}{\sqrt{3} \cdot R}$$

This expression is entered in the initial equation

$$P = \sqrt{3} \cdot U_L \cdot \frac{U_L}{\sqrt{3} \cdot R} \cdot \cos\varphi$$

$$P = \frac{(380 \text{ V})^2}{800 \Omega}$$

$$P = 180.5 \text{ W}$$

=====

delta connection of the three resistors

$$P = \sqrt{3} \cdot U_L \cdot I_L \cos\varphi$$

$$P = 3 \cdot U_L \cdot I_{Str} \cdot \cos\varphi$$

Since the full line-to-line voltage is applied to each resistor, we have for the phase current

$$I_{Str} = \frac{U_L}{R}$$

This expression is entered in the initial equation

$$P = 3 \cdot \frac{U_L^2}{R} \cdot \cos\varphi$$

$$P = 3 \cdot \frac{(380 \text{ V})^2}{800\Omega}$$

$$P = 541.5 \text{ W}$$

At the three resistors, a total power of 180.5 W is obtained, in star connection and of 541.5 W in delta connection.

In practice frequently advantage is taken of the possibility of obtaining different powers by changing the type of connection of the various loads. For example, for three-phase motors, there are special switching devices which enable the chaning over from star connection to delta connection and vice versa.

The power of a three-phase system can be determined from the sum of the individual powers in the three phases in case of unequal phase loads or from the relation given in equation 8.4. in case of equal loads irrespective of the type of connection.

Questions and problems:

1. Determine the effective power of three resistors in star connection of 200 ? if the latter are connected to a three-phase network with a line-to-line voltage of 220 V!
2. Determine the effective power when the three resistors of problem 1. are in delta connection!
3. Three unequal effective resistances (80 ?, 200 ?, 500 ?) have to be connected in star and in delta arrangement to a three-phase network with a line-to-line voltage of 380 V.  
Determine the total effective power!

## 9. Protective Measures in Electrical Installations

### 9.1. Danger to Man by Electric Shock

The electrical current exerts effects on man (also see Chapter 1). A useful effect is produced in a few electromedical therapies. On the other hand, hazards to man can be caused by an electric shock (inadvertent passage of current through the human body). The consequences of an electric shock are dependent on the intensity of the current passing through the human body and on the duration of action. The current path through the human body is also of particular importance. A great danger is given when the current path passes through the heart; this is given when the voltage acts from hand to hand or from hand to foot. Depending on intensity and duration of the passage of current through the body, muscle contraction, which may reder impossible the release of the touched electrode, will occur. Consequences are burns, unconsciousness and ventricular fibrillation which is an extreme danger of life.

To prevent accidents due to electrical current, comprehensive safety regulations have been en acted in all countries which must be observed in any case. The increasing exchange of goods necessitates in future a standardisation of these regulations on an international level. Efforts are made by the "International Electrotechnical Commission" (IEC), and the "Standing Commission for Standardisation" of the "Council for Mutual Economic Aid" (CMEA) to arrive at generally accepted rules.

The definition of a few physical quantities in connection with protective measures is illustrated in Fig. 9.1. Here, it is assumed that the phase conductor L1 has body contact (low-resistance connection of the phase conductor with the casing of the motor) and that the protective conductor with the function of a neutral conductor (PEN) is interrupted. The contact voltage  $U_B$  is the voltage directly acting on man in the event of a fault while the failure voltage  $U_F$  in the present case is given by the voltage between phase conductor and

neutral conductor. The current flowing in case of a fault is called fault current  $I_F$ ! The operational earthing resistance  $R_B$  is the resistance given when the neutral conductor at the supply side is connected with reference earth and the position transition resistance  $R_{St}$  is the resistance between the position (of the installation and the like) and reference earth. Depending on the condition of the position,  $R_{St}$  can become very small (e.g. wet basement floor) while  $U_B$  practically becomes equal to  $U_F$ . The danger to man at a well conducting position is, therefore, particularly high.

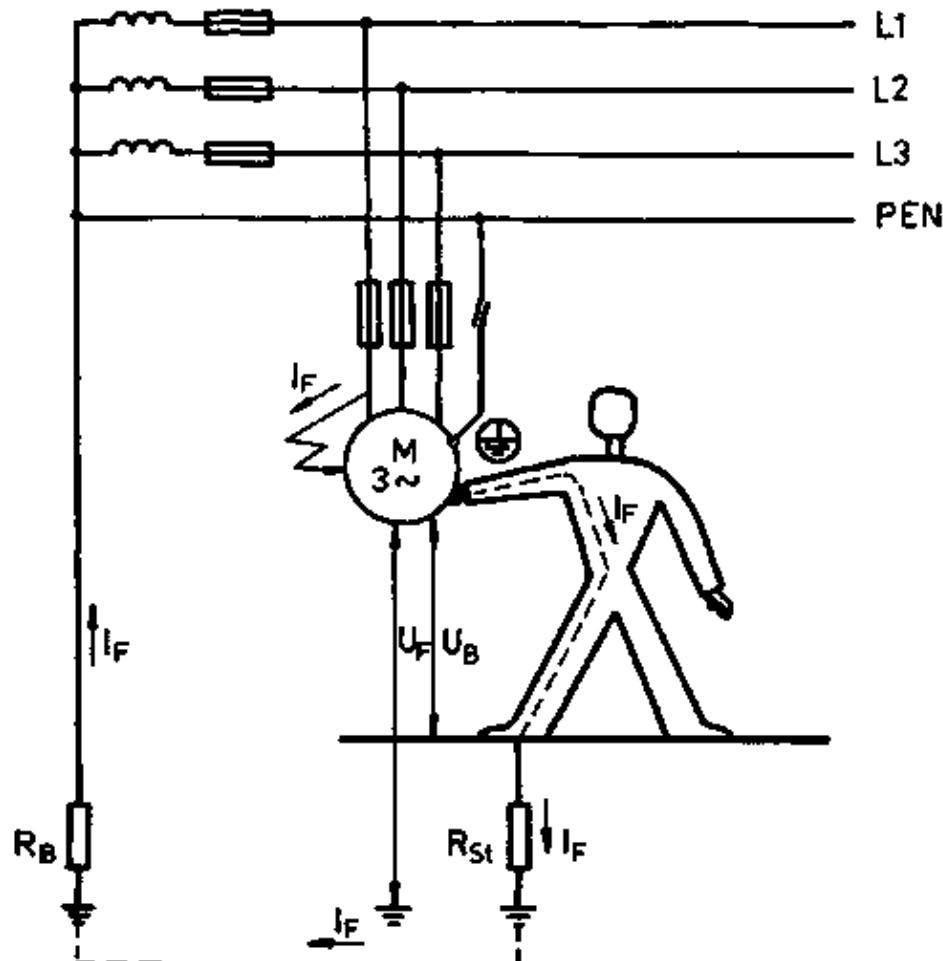


Fig. 9.1. Example of the representation of contact voltage, failure voltage and fault current (leakage current)

$I_F$  = fault current

$U_B$  = contact voltage

$U_F$  = failure voltage

$R_B$  = operational earthing resistance

$R_{St}$  = position transition resistance

As has been mentioned above, the detrimental effects is dependent on the fault current and the time of its action. The fault current is determined by the contact voltage  $U_B$  and the resistance of the human body. This resistance depends, among other things, on the skin surface and perspiration and may vary within wide limits. In the case of slight injuries of the skin surface, the resistance is considerably reduced. In order to acquire the effectiveness of protective measures by measuring techniques irrespective of the largely varying resistance of the human body, in regulations the maximum permissible contact voltage (e.g. 65 V, for alternating current the effective value is decisive) is specified. Another possibility is the statement of the maximum permissible contact voltage at a maximum permissible disconnection time. Table 9.1. contains a few values of an IEC Publication as an abstract.

Table 9.1. Maximum permissible contact voltage in dependence of the disconnection time

Contact voltage in V	Maximum permissible disconnection time in s
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< 50	?
50	5
75	1
110	0.2
220	0.05
280	0.03

In the past many different measures for the protection of man from electrical shock have been developed. Of the great variety of protective measures, one or several have to be selected in accordance with the given concrete conditions. For electrical devices, three protective classes have been specified. Devices with protective conductor connection belong to the protective class 1, devices with protective insulation belong to protective class 2 and devices for extra-low protective voltage to class 3.

For the protection of man from electric shock, protective measures in electrical devices and installations are necessary. The maximum permissible contact voltage continuously applied or occurring only during a specified period of time must not exceed the specified maximum value.

Questions:

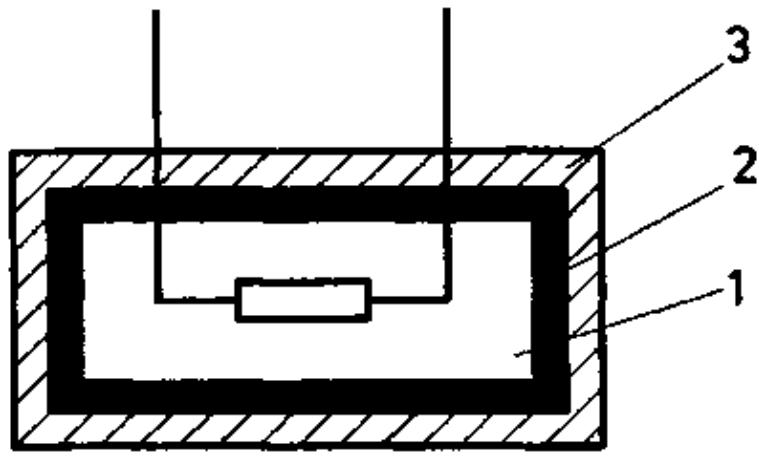
1. Gather information about "first aid" in cases of accidents due to electrical current!
2. Which are the conditions under which the danger due to an electric shock is particularly high?
3. Explain the terms contact voltage, failure voltage and fault current!

## 9.2. Measures for the Protection of Man from Electric Shock

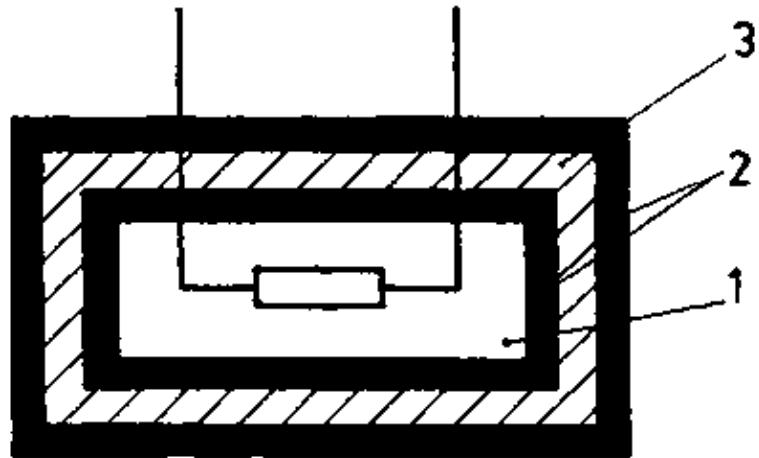
### 9.2.1. Protective Insulation

For electrical household appliances and portable tools (e.g. electric hand-drilling appliances), the electrical protective insulation is a very effective protection from electric shock. The emergence of a contact voltage is practically avoided by the special arrangement of the insulation. Fig. 9.2. shows two methods of arranging a protective insulation. For example, an electric hand-drilling appliance can be provided with a plastic enclosure (protective insulating covering) and the metallic gear and drill chuck can be connected with the motor by means of a clutch of insulating material (protective intermediate insulation). For this purpose, special requirements must be specified for the mechanical, thermal, electrical and chemical stability of the insulating materials to be used. At accessible points of live parts, which do not belong to the service circuit, a protective conductor must not be connected. Devices with protective insulation may be marked with two squares, an outer one and an inner one. (□)

**Fig. 9.2. Protective insulation**



a) protective insulating covering



b) protective intermediate insulation

- 1 Operational insulation
- 2 Metal enclosure
- 3 Protective insulation

#### 9.2.2. Extra-low Protective Voltage

If a safe protection against accidental contacts of the live parts in operation is not possible or if tools have to be used in an environment where particular hazards are given (e.g. in narrow boilers), an expedient protective measure is the use of extra-low protective voltage. In the no-load state, it must not be greater than 50 V. For electrical toys for children lower values are specified.

When extra-low protective voltage is used, no live conductor must be earthed in operation. The extra-low protective voltage is generated in special transformers or by motor generators. The circuits of extra-low protective voltage have to be installed in such a way that they are safely separated from other circuits. For this purpose, only such plug-type connections or connectors have to be used which safely prevent an inadvertent connection of the tools and the like to other voltage sources. Like with protective insulation, conductive parts of the tools and the like, which do not belong to the service circuit, must not be connected with a protective conductor.

#### 9.2.3. Protective Isolation

Like the extra-low protective voltage, the protective isolation is a galvanic separation from the feeding supply network but the output voltage may be greater than 50 V. This protective measure is applicable, for instance in television repair shops, in work to be done in the open or in boiler houses.

In this case only one load may be connected to an output winding of a transformer. Fig. 9.3. shows an otherwise dangerous situation in the case of double body contact and connection of two devices to one winding. The using plug-type connectors, switches and a tool or the like with protective conductor connections (protective class 1), all protective conductor connections must be connected with each other. If work is to be done in narrow space on metallic floors and the like, then the isolating transformer should be provided with protective insulation and arranged outside of this space. In addition a potential equalisation line must be arranged between protective conductor connection and tool. In this way any danger due to a damaged lead is safely avoided.

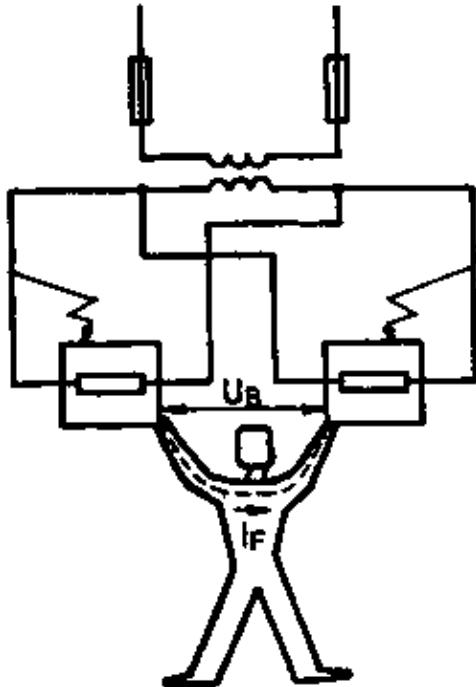


Fig. 9.3. Impermissible connection of two devices to one winding of a protective isolating transformer

$I_F$  = fault current

$U_B$  = contact voltage

#### 9.2.4. Protective Wire System

The measures described in 9.2.2. and 9.2.3. ensure a high protective effect against electric shock but they are very expensive. When in a room many electrical appliances are put into operation at the same time and when, for reasons of safety, it should be avoided that, in the case of body contact of one of the devices, the whole installation is switched off, then the protective wire system is the adequate protective measure. These conditions may occur, for example, in an operation theatre.

No conductor of the service circuit must be earthed, not even the neutral conductor—that is a condition of the protective wire system. On the other hand, all of the conductive parts not belonging to the service circuit and all conductive parts of the building (water pipes, other pipe lines, metallic structures of the building) must be connected together in any case. There is a puncture cut-out between neutral conductor (N) and earth. In the case of a simple body contact, there is no danger of an occurrence of a dangerous contact voltage. However, in the case of a double body contact, there is the danger of a dangerous occurrence of contact voltage.

If in an installation – as has been mentioned above –, with an suddenly occurring body contact no danger to man shall occur and no switching off is to be effected, then a switchgear monitoring continuously the isolation

$$25 \frac{\Omega}{V}$$

resistance to earth of the service circuit (e.g.  $25 \frac{\Omega}{V}$ ) is employed, releasing a fault signal (Fig. 9.4.). This type of monitoring will reliably indicate a first body contact. When signalling is not desired, care must be taken that, in case of a double body contact, the breaking current  $I_a$  will flow and the installation is reliably and quickly switched off. The conditions for the breaking current  $I_a$  will be explained in Section 9.2.5.

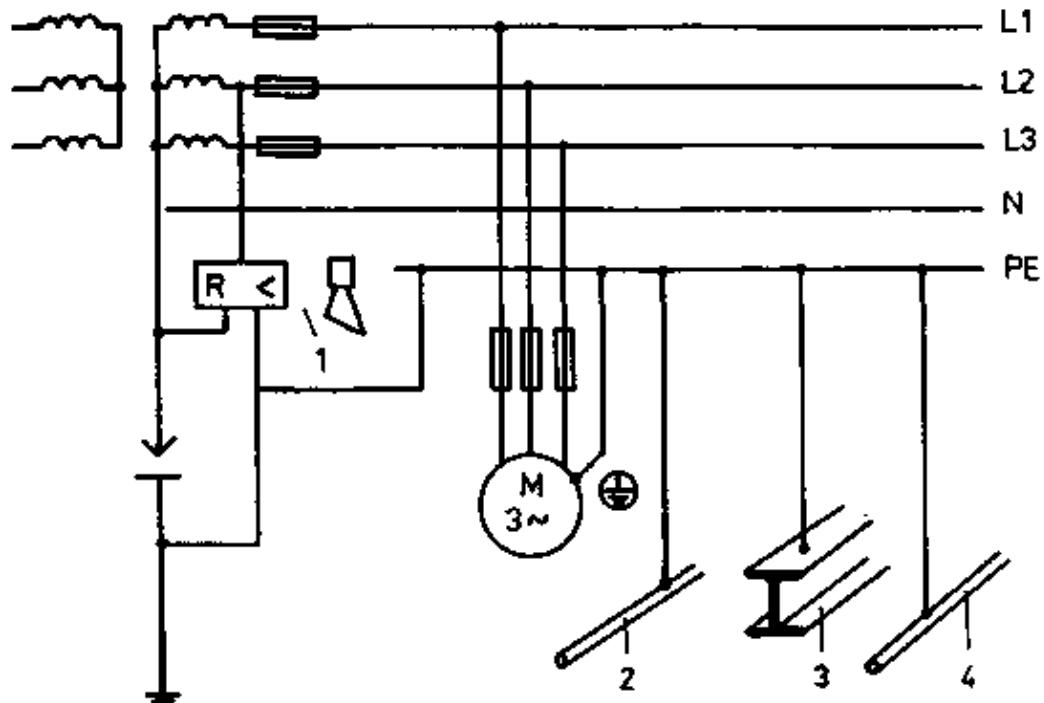


Fig. 9.4. Protective wire system with monitoring of the insulation resistance

- 1 Monitoring relay for insulation resistance to earth with signalling the fault by a horn
- 2 Water pipes
- 3 Conductive parts of the building
- 4 Other pipe lines

#### 9.2.5. Protective Earthing

In protective earthing, all of the conductive parts not belonging to the service circuit are connected with the protective earthing device by means of the protective conductor. Since the fault current is conducted through the earthing resistance, no impermissibly high contact voltage must occur in the form of voltage drop in this resistance until the breaking current is reached. Therefore, it is necessary to realise a sufficiently small earthing resistance (see formula 9.1.)

$$R_s \leq \frac{U_{B\text{ perm}}}{I_a}$$

where:

$U_{B\text{ perm}}$  maximum permissible contact voltage (e.g. 65 V)  
 $I_a$  breaking current  
 $R_s$  earthing resistance

Fig. 9.5. shows an example of a fault-current circuit

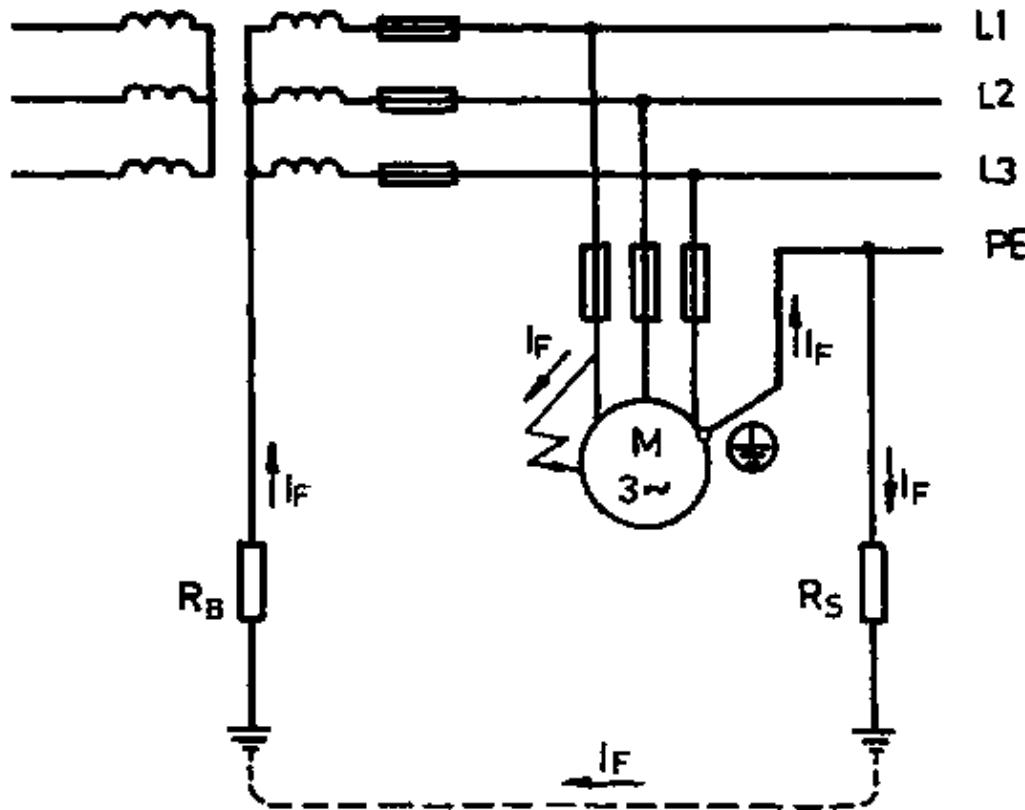


Fig. 9.5. Fault circuit involved in the protective measure called protective earthing

$I_F$  = fault current

$R_S$  = earthing resistance

$R_B$  = operational earthing resistance

Depending on the fuse used, the breaking current must be selected in such a way that it is higher by the factor  $k$  than the rated current of the fuse (formula 9.2.)

$$I_a = k I_n$$

where:

$I_a$  breaking current

$k$  switching off factor

$I_n$  rated current of the fuse

In this way it is to be achieved that, in case of a fault, the circuit is interrupted within a adequately short time. High values of  $k$  ensure an increased safety due to a quicker response of the fuse but in many cases they cannot be realised by an economically justifiable expense. The minimum values of  $k$  are specified in special regulations in dependence of the type of fuse connected.

For the return flow of the fault current, the water pipes may be used if permission is given. But this is rarely used today because no-metallic water pipes are increasingly employed.

#### 9.2.6. Connection to the Neutral

A protective measure which can be realised easily and at low expenditure is the connection to the neutral; it provides a good protective effect. All conductive parts and units not belonging to the service circuit are connected with the protective conductor (PE) which is connected with the earthed neutral conductor (N). Protective conductor and neutral conductor may in this case be formed by a common conductor (PEN) (protective conductor carrying current in operation) or they may be installed separately (protective conductor not carrying current in operation).

In the case of a breakage of the PEN conductor at the supply side, a high contact voltage may occur at the protective conductor under unfavourable conditions (danger of Life!). Therefore, additional comprehensive regulations must be observed for the installation of the protective conductor carrying current in operations, especially in overhead local transmission lines. Further, a connection of the PEN conductor to the earth bus at the feeding point and sometimes also in the network spurs is required. An effective potential equalisation must be provided in the customer installation.

If, for the connection of safety plugs, a two-core lead is used (protective conductor carrying current in operation), the lead (also known as supply line) must be connected to the protective contact first and then it must be brached off to the current carrying connection (see Fig. 9.6.). The fault-current circuit for body contact of L1 with the motor casing shows that the fault current flows through the fuse. For the breaking current the equation 9.2. also is applicable in this case.

Under certain conditions, monitoring of the voltage at PEN to reference earth may be required; in this case an all-pole cut-out is effected when a maximum permissible voltage is exceeded.

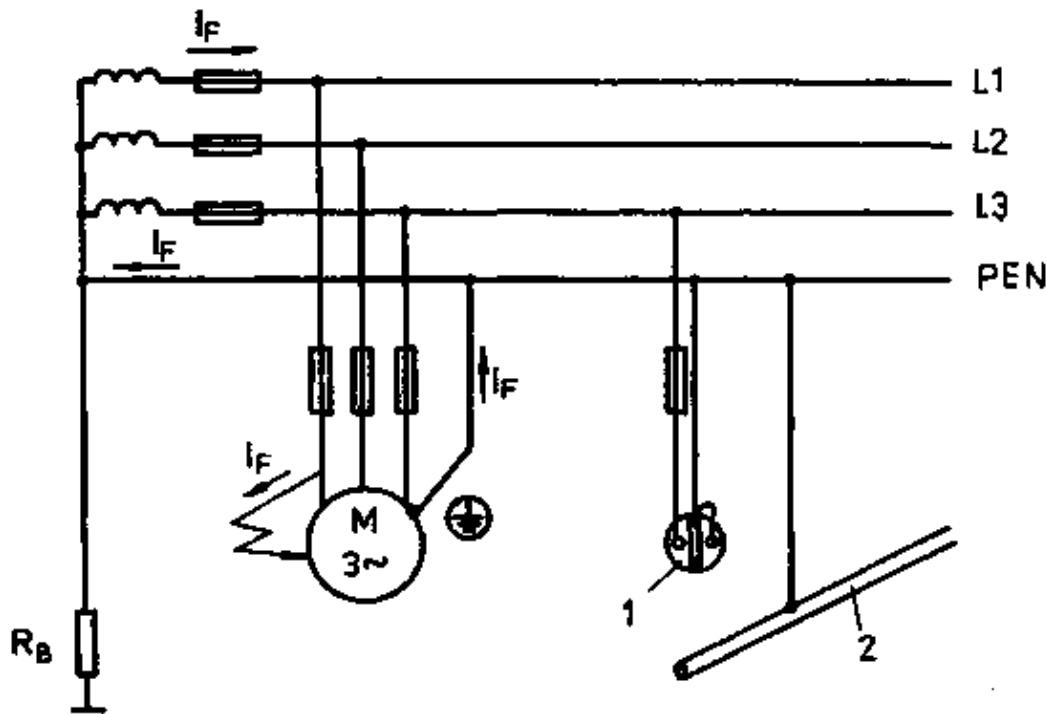


Fig. 9.6. Connection to the neutral (also known as multiple protective earthing)

- 1 Protective contact socket
- 2 Pipes and other conductive parts in buildings
- $I_F$  = fault current
- $R_B$  = operational earthing resistance

#### 9.2.7. Fault-current Protection

In the fault-current protection system, a special switching device – the FI protective switch or the FI relay – is connected in series with the loads. All accessible conductive parts must be earthed Fig. 9.7. shows the fault-current protection device.

A summation transformer is arranged in the FI protective switch which monitors the current flowing into and out of the installation. In the faultless condition, the sum of these currents is always equal to 0 according to the first Kirchhoff's law, even in the case of an unsymmetrical load.

The magnetic fields produced in the current transformer neutralise each other and the secondary of the transformer is not excited. If, in case of a fault, body contact occurs, the fault current does not flow via the summation transformer but via ground.

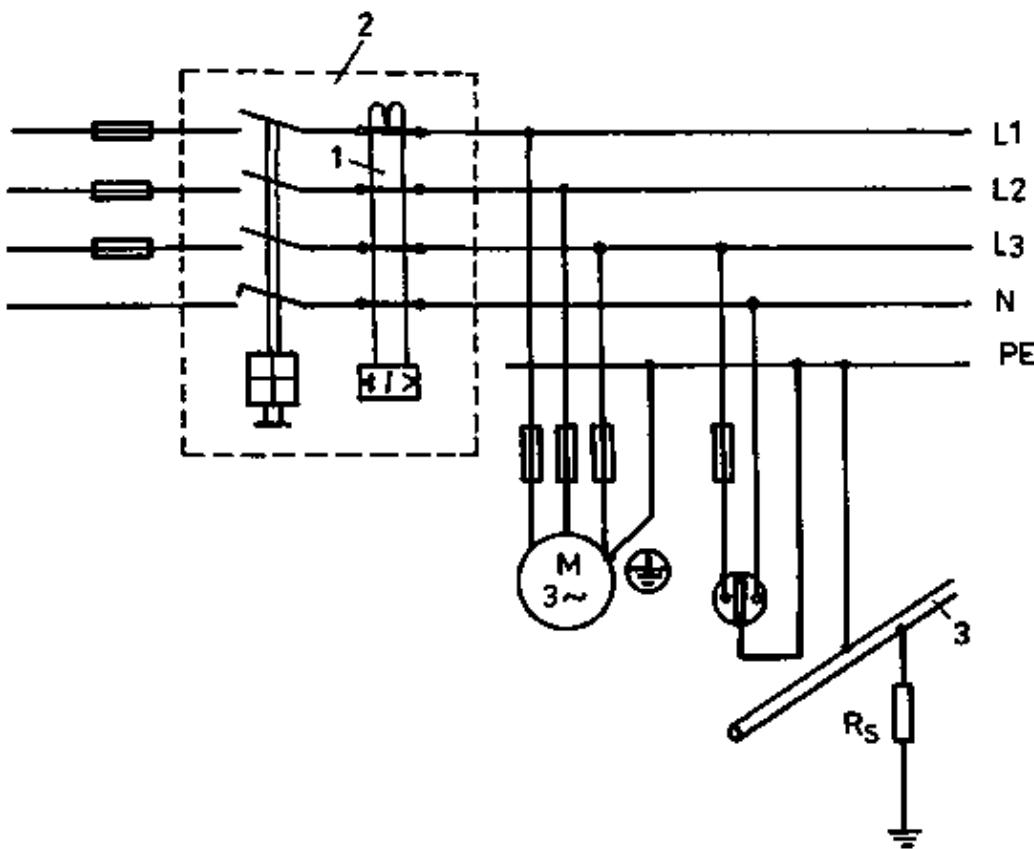


Fig. 9.7. Fault-current protection system with earthing the protective conductor connection

1 Summation transformer in fault-current protective switch

2 Fault-current protective switch

3 Pipe lines as earthing resistance

$R_s$  = earthing resistance

Consequently, the sum of the inflowing and out flowing currents in the transformer is unequal to 0. The voltage generated in the secondary of the summation transformer causes the triggering of the switch and the all-pole switching off of the installation within a very short time (about 20 ms).

The FI protective switches differ by the height of the tripping current (rated fault current  $I_{fn}$ ).  $I_{fn}$  should be designed in such a way that it is three times the leakage current to be expected. The earthing resistance must be so small that the rated fault current causes a maximum voltage drop of  $U_{B\ perm}$  at the most (formula 9.3.).

$$R_s = \frac{U_{B\ perm}}{I_{fn}} \quad (9.3.)$$

where:

$R_s$  earthing resistance

$U_{B\ perm}$  maximum permissible contact voltage

$I_{fn}$  rated fault current of the FI protective switch

For a protective switch with  $I_{fn} = 30 \text{ mA}$  and  $U_{B\ perm} = 65 \text{ V}$ , an  $R_s$  of  $2.15 \text{ k}\Omega$  is obtained. This value can be reached without great difficulties. Also, for a protective switch with  $I_{fn} = 500 \text{ mA}$ , an earthing resistance of  $130 \text{ }\Omega$  is sufficiently small. When a tool and the like is with necessity connected with an earth lead (water pump, electrical thermal storage water heater), this method of earthing will suffice when the required earthing resistance is ensured.

In connection with the protective measure known as connection to neutral, the FI protective system can be used to advantage according to an IEC recommendation (Fig. 9.8.). The advantage over the connection to the neutral consists in the fact that – in case of a relatively low fault current which is considerably lower than the rated current of the fuse connected in series – a quick switching off of the faulty installation is effected. The

problems of connection to the neutral associated with the realisation of the switching-off factor  $k$  (equation 9.2.) are avoided, the total switching-off time is shorter.

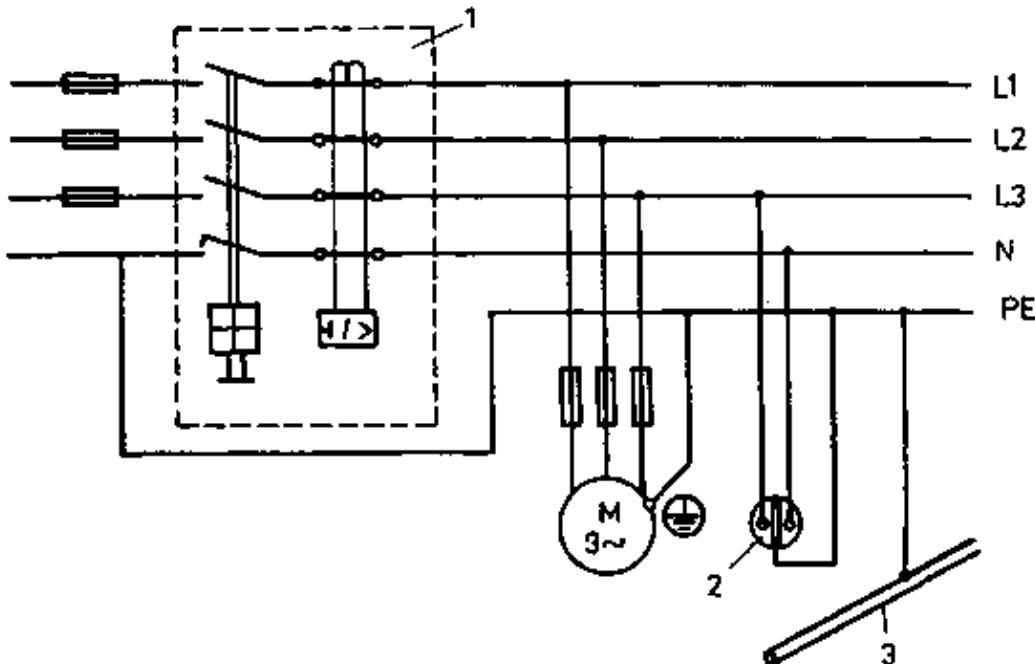


Fig. 9.8. Connection to neutral combined with FI protective switch

1 FI protective switch

2 Protective contact socket

3 Connection of PE to conductive parts in the building

In order to protect man from the dangerous effects of an electrical shock, various protective measures can be taken in dependence of the concrete conditions given. Besides the protective measures without protective conductor (extra-low voltage, protective insulation), there are protective measures with protective conductor (protective isolation, protective conductor system, protective earthing, connection to the neutral). The measures of the second group differ with respect to protective effect and costs. It is possible to apply several protective measures at the same time (connection to the neutral with FI protective system). The selection of the suitable protective measure is dependent on the type of three-phase network given and the dangers that may occur in the handling of electrical tools and the like. Further, it must be decided whether or not several tools and the like may be switched off in cause of a fault current (e.g. in case of connection to neutral) or only the defective tool (e.g. in case of separate FI protective switch for each tool and the like). Further, it must be decided whether in case of simple body contact it should only be signalled and the work can be finished without endangerment (e.g. protective conductor system with monitoring of the insulation resistance).

Questions:

1. Explain the protective effect of the various protective measures!
2. What protective measure is suitable for dwelling installation? Start from the consideration of the different three-phase current mains!
3. What are the advantages of the connection to neutral with FI protective switch over the connection to neutral?
4. What has to be observed when using the protective isolation?
5. Why have special connection to be used for the protective measure known as extra-low protective voltage?
6. Why should the earthing resistance not exceed a maximum value when the protective measure known as protective earthing with FI protective system is used?

7. Why should the breaking current value be higher than the rated current of the fuse connected in series?

### **9.3. Checking the Protective Measures**

Protective measures taken will only then offer sufficient protection against the horrible effects of an electrical shock when they are serviceable at any time. Therefore, checking the effectiveness of the protective measures is demanded by the legislator. This checking must include both the protective measure for portable tools and the like and those for fixed tools and plants.

For portable tools and the like, checking of the proper condition of the protective conductor is required every six months or at shorter intervals depending on the stress on these means. Care should be taken that the protective conductor connection at the terminals is always longer than that at the lines carrying current in operation in order that, in case of a failure of the cord attachment, the protective conductor will break as the last.

Inspections of the electrical installations have to be carried out every 1 to 3 years. The way how the inspections have to be carried out is specified in special legal regulations.