

Textbook for Vocational Training – Technical Mechanics

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Preface

Technical mechanics is of great importance to engineering and production. It is in all machines and structures that the laws of technical mechanics become effective.

The findings of technical mechanics are not utilized by scientists and engineers exclusively. Skilled workers must dispose of knowledge of technical mechanics, too, in order to achieve a better understanding of the operation of tools and machines.

This theoretical knowledge supports the efforts of skilled workers to perform good work at expedient expenditure of energy.

To conclude with, basic knowledge of technical mechanics is necessary to detect sources of accidents. This contributes to avoid accidents.

This book contains selected fundamentals of technical mechanics. It is intended to support the theoretical and practical training of skilled workers of the metal working industry.

1. Units in engineering and science

1.1. Tasks of the units

The thickness of sheet metal is measured in millimetres. The distance between two towns is stated kilometres. Millimetre and kilometre are units. Why is millimetre used in one case and kilometre in the other?

The area of a workshop hall is calculated in square metres, and the capacity of a water tank in cubic metres.

Square metre and cubic metre are units, too. What are these units required for?

The task of these units is to be explained by an example. Three water tanks are partly filled. Which tank contains the largest quantity of water?

If the tanks are to be compared, they can be described, to begin with.

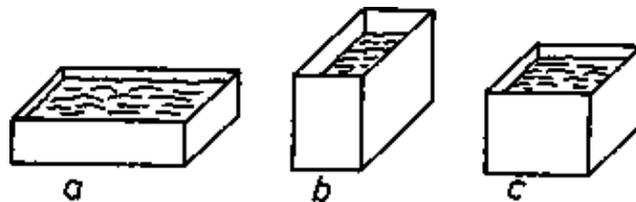


Figure 1.1. Which tank contains more water?

Tank a has a large water surface and a small depth. Tank c has a square bottom surface, and tank b is oblong.

Whereas the central tank is partly filled only, the two outer tanks are filled with water almost to the brim. So far, it has not been ascertained, however, which tank contains the largest quantity of water.

It is expedient to use a comparative value. As comparative value, the volume of 1 cubic metre, is chosen. In order to render a precise understanding between skilled workers, engineers and scientists possible, it was fixed:

A cubic metre is the volume of a cube at an edge length of 1 metre.

The abbreviation for cubic metre is m^3 .

Now it is easily possible to compare the quantity of water in the three tanks with one another. It has to be ascertained, how many cubic metres of water is contained in the individual tanks.

The volume is calculated:

Volume = surface • filling height

$$m^3 = m^2 \cdot m$$

Tank a: $V = 5m \cdot 3m \cdot 1m$

$$V = 15 m^3$$

Tank b: $V = 2m \cdot 4m \cdot 2m$

$$V = 16 m^3$$

Tank c: $V = 3m \cdot 3m \cdot 2m$

$$\underline{V} = 18 \text{ m}^3.$$

Now it can be stated that tank c contains the largest quantity of water.

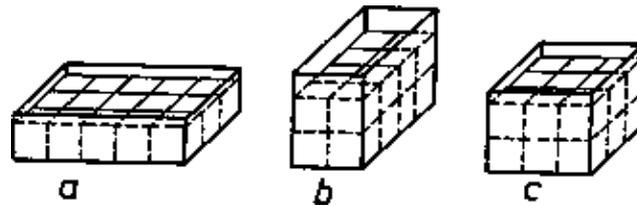


Figure 1.2. Volume

- tank a) $V = 15 \text{ m}^3$;
- tank b) $V = 16 \text{ m}^3$;
- tank c) $V = 18 \text{ m}^3$

The result $\underline{V} = 18 \text{ m}^3$

is to be examined more precisely. It applies

Volume = numerical value. unit of volume.

The cubic metre is designated as unit of volume. This unit permits the computation or measuring of volumes. Then the volumes can be compared to one another.

1.2. Scope of units

In order to permit an application of machines in many countries, and to render a good understanding of skilled workers, engineers and scientists of different countries possible, it was established at an international conference "Conférence Générale des Poids et Mesures" that equal units shall be applied in all participating countries. These units are compiled in the "Système International d'Unités".

The basic units include ¹⁾:

length metre m

weight kilogramme kg

time second s

¹⁾ See also survey of units in appendix,

From the unit of length can be derived:

surface square metre m^2

volume cubic metre m^3

The distance 1 m is uniform in all countries. Thus the volume of 1 m^3 is fixed invariably, too.

As in all enterprises the same units are used, one plant can produce the ball bearings, and another plant the shaft. Then it is also possible for a third plant to mount ball bearing and shaft in a machine.

1.3. Prefixes to units

The individual parts of a machine have to be worked very accurately. Sometimes, small component parts have a length of few millimetres only.

On the other hand, the distance between two towns is a length, too. If this distance is stated in metres, very large numeric values come to exist.

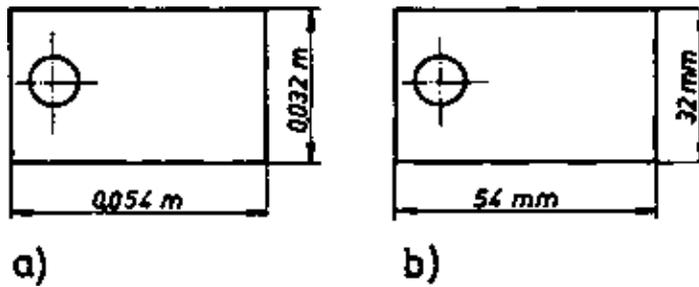


Figure 1.3. Workpiece with bore hole:

- a) circumstantial statement in m;
- b) expedient statement in mm.



Figure 1.4. Section from a map:

- a) circumstantial statement in m;
- b) expedient statement in km.

The numeric values will become very small or very large:

A workpiece is 0.032 m long, and the distance between two towns amounts to 124,000 m.

These numeric values render calculation difficult. Therefore the units are made smaller or larger by means of prefixes. On drawings of machine building, the length is stated in millimetres as a general principle.

One millimetre is 1/1,000 m.

The distance between two towns is stated in kilometres.

One kilometre are 1.000 m.

The following prefixes are used frequently ¹⁾:

Prefix	Sign of abbr.	Meaning	Example
Kilo	k	1,000 fold	1 km = 1,000 m
Dezi	d	0.1 fold	1 dm = 0.1 m
Zenti	c	0.01 fold	1 cm = 0.01 m
Milli	m	0.001 fold	1 mm = 0.001 m
Mikro	μ	0.000.001 fold	1 μm = 0.000,001 m

¹⁾ See also survey of prefixes in appendix.

1.4. Problems and questions

- 1.1. In which units can a length be stated?
- 1.2. In which units can a volume be stated?
- 1.3. In which unit is time measured?
- 1.4. In which unit is the speed of a motor lorry measured?
- 1.5. Name other units you have become acquainted with during work in our manufacturing plant!

2. Fundamentals of the mechanics of solids

2.1. Essence and fields of mechanics

Even in early history, men observed nature. They were wondering how the work of building a house could be made easier by using roll and rope, or how it could be made possible by simple machines to pump water from the river onto the fields.

The observations of many men and their experience formed the basis for the natural sciences.

Scientists have collected these observations and experience, and sought for laws of nature by means of experiments.

An especially important branch of natural sciences is mechanics.

Mechanics is the doctrine of the properties and motions of bodies and of the effects of forces.

In this connection, both solids and liquids and gases are investigated.

Mechanics is closely connected with production and engineering. The laws of mechanics are applied by tools and machines. Thus, a worker at a lifting jack can develop such a great force without difficulty that a whole vehicle is lifted (fig. 2.1.).

The fields of mechanics, which are of special importance to engineering and production, are comprised by technical mechanics.

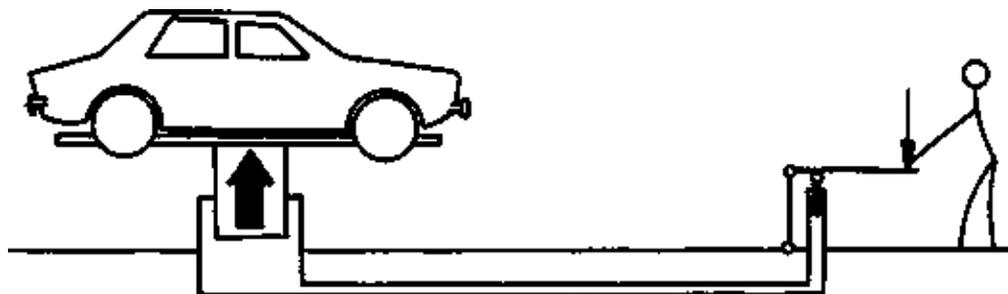


Figure 2.1. Application of the laws of mechanics.

A worker lifts a passenger car by himself, using a lifting jack

Technical mechanics investigates and describes the path of motion of solids,

- the motions of solids under the influence of forces,
- the effects of forces at rigid bodies,
- the effects of forces at elastic bodies,
- the effects of forces on liquids,
- the effects of bodies on gases.

The skilled worker must know the fundamental laws of technical mechanics so that he understands the operating mode of machines better, and is able to facilitate his work by expedient employment of tools.

2.2. Motion of bodies

2.2.1. Paths of motion

Everywhere in engineering and production, moved bodies can be observed. Motor vehicles are moving on the roads: buses, motor lorries, passenger cars.

In the factories, conveyor belts are operating and transport workpieces, chests and containers, building materials.

Axles and shafts, pulleys and gears are rotating at the machines. Tools or workpieces are moving.



Figure 2.2. Motor lorry on the road; path of motion rectilinear

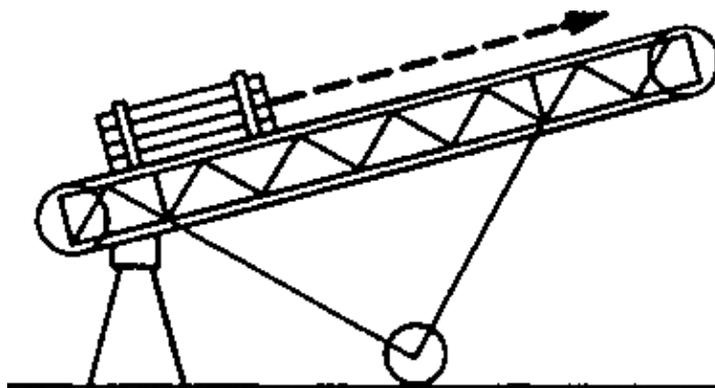


Figure 2.3. Chest on conveyor belt; path of motion rectilinear

Have these different motions anything in common?

The moved bodies all travel through a path of motion. The motor lorry (fig. 2.2.) moves, without changing its direction, on a road. Such a motion is called a rectilinear motion. With a rectilinear motion, the direction of the motion is not changed.

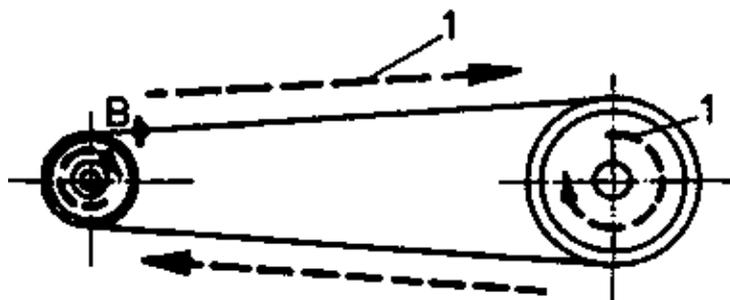


Figure 2.4. Belt drive; path of motion rectilinear and circular

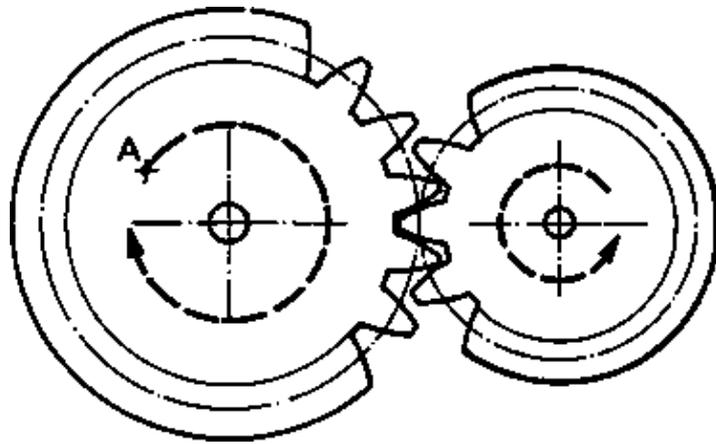


Figure 2.5. Toothed-gear drive; path of motion circular

With a rectilinear motion, the body moves on a straight line.

A chest on a conveyor belt moves in a similar way. The conveyor belt moves the chest on a rectilinear path of motion (fig. 2.3.). Gears are permanently connected with a shaft, and these shafts are held in bearings. When the gears rotate (fig. 2.5.), then the individual points of the gear move on a circular path. They rotate round the central axis of the shaft. On the left gear in fig. 2.5., a point is marked by a small colour spot. If you now watch the path of motion of this point A, you will recognize: Point A moves on a circular path. It changes its position continuously. The distance to the centre, the distance to the centre of rotation always remains constant, however. This motion is called circular motion or rotation.

In many machines, belt drives are moving. The belts enclose two circular belt-pulleys, which can rotate round their axes only.

The belt-pulleys (fig. 2.4) perform circular motions, i.e. each point of the belt-pulley moves on a circular path. In order to enable a description of the motion of the belt, point B is to be observed in its motion. Point B is marked by a colour spot on the belt. Point B moves rectilinear from the left to the right belt-pulley. There its motion changes into a circular path. On the bottom side the belt gets off the belt-pulley again, and point B is returned to the left belt-pulley on a straight line. There the motion changes into a circular path again, and point B arrives at its initial position again. Hence, point B always moves in continuous change on a rectilinear and circular path.

Summary

We distinguish between:

Rectilinear motions: The body moves on a straight path of motion.

Circular motions: The body rotates round an axis, and the individual points of the body travel through a circular path.
The circular motion is also called rotation.

With complicated motions, rectilinear and circular motions can be performed successively or at the same time.

2.2.2. Speeds

On a road, vehicles travel at different speeds (fig. 2.6). A passenger car travels more quickly than a motor lorry, and this travels more quickly than a semitrailer again. When they travel at different speeds, they cover different distances within the same time.

The distance receives formula sign s .

Fig. 2.6. shows: The passenger car covers the distance s_1 . In the same time, the motor lorry covers the distance s_2 , and the semitrailer the distance s_3 .

The quotient from covered distance and required time is termed speed.

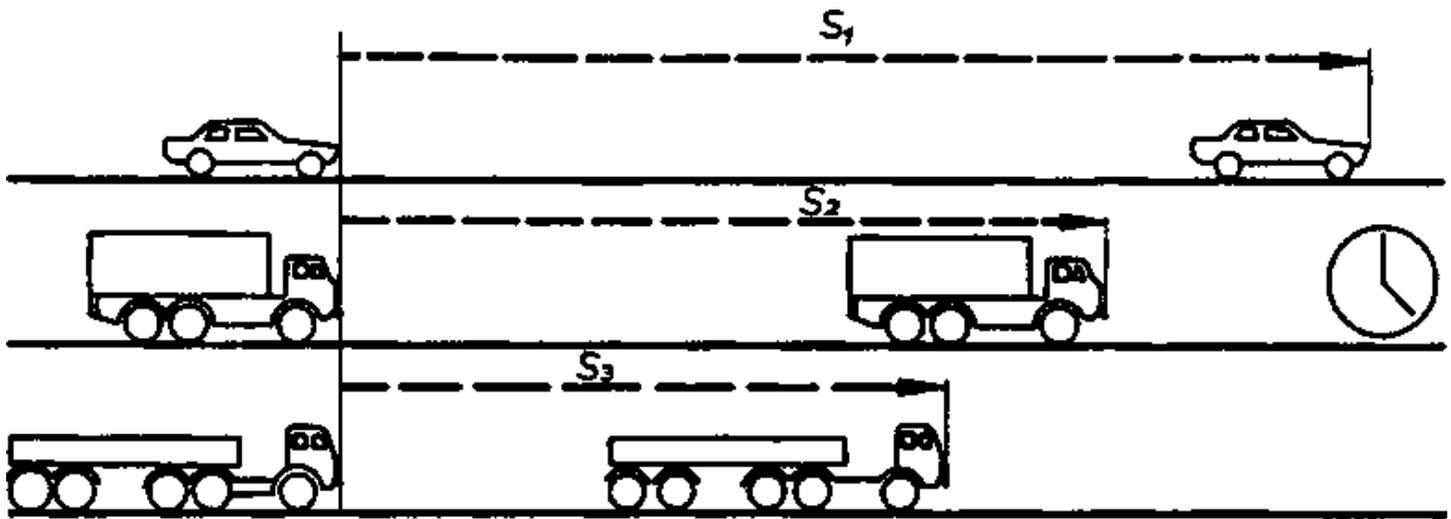


Figure 2.6. Vehicles at different speeds cover different distances s within the same time

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

Or written briefly with formula sign:

$$v = \frac{s}{t} \quad (2.1.)$$

v: speed
s: covered distance
t: required time

The distance is measured in metres or kilometres, and the time is stated in seconds (s), minutes (min) or hours (h).

Accordingly, the unit of speed becomes $\frac{\text{m}}{\text{s}}$; $\frac{\text{m}}{\text{min}}$; $\frac{\text{km}}{\text{h}}$.

The speed of vehicles is stated in $\frac{\text{km}}{\text{h}}$.

The cutting speeds of machine tools are stated in $\frac{\text{m}}{\text{min}}$ or $\frac{\text{m}}{\text{s}}$.

Example

A motor lorry covers the distance of 120 km in 180 minutes. What does its speed amount to?

Given:

covered distance $s = 120$
km

required time $t = 180$
min

Required:

speed v

Solution:

$$t = 180 \text{ min} = 3 \text{ h}$$

$$v = \frac{s}{t}; \quad v = \frac{120 \text{ km}}{3 \text{ h}}$$

$$V = 40 \frac{\text{km}}{\text{h}}$$

Result:

The average speed of the motor lorry amounts to $v = 40 \text{ km/h}$.

With circular speeds, it also applies

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

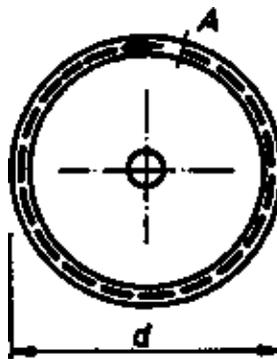


Figure 2.7. Length of distance on a circular path

For point A from fig. 2.7., the speed can be determined as follows.

When the belt-pulley rotates round its own axis once, point A travels once through the circumference of the circle with diameter d.

The length of a circular path amounts to

$$s_1 = \pi \cdot d.$$

When the belt-pulley rotates round its own axis U times in the time t, the U-fold distance is covered.

$$s_U = \pi \cdot d \cdot U$$

Hence the speed of point A amounts to

$$v = \frac{\pi \cdot d \cdot U}{t}$$

If the number of revolutions in the unit of time (s or min) is called speed n, it applies

$$n = \frac{U}{t}$$

and

$$v = \pi \cdot d \cdot n \quad (2.2.)$$

3.14

d: diameter of the circular path
 U: number of revolutions in period of time t
 n: number of revolutions
 t: period of time
 v: speed

Example

A belt-pulley with an outside diameter of 400 mm rotates at 480 revolutions per minute.

What is the speed at the periphery of the belt-pulley?

Given:

outside diameter $d = 400 \text{ mm}$
 revolutions $n = 480 \text{ 1/min}$

Required:

speed v

Solutions:

$$v = \frac{3.14 \cdot 0.4\text{m} \cdot 480}{1\text{min}}$$

$$v = 60288 \frac{\text{m}}{\text{min}}$$

Result:

The speed at the periphery of the belt-pulley amounts to appr. 603 m/min.

Examples of some speeds:

Speed at the edge of a grinding wheel	15 to 35 $\frac{\text{m}}{\text{s}}$
Cutting speed with turning with high-speed steel	6 to 66 $\frac{\text{km}}{\text{h}}$
Pedestrian	4 to 6 $\frac{\text{km}}{\text{h}}$
Passenger car	80 to 120 $\frac{\text{km}}{\text{h}}$
Airliner	500 to 850 $\frac{\text{km}}{\text{h}}$

Summary

Speed is the quotient from covered distance and required time

$$v = \frac{s}{t}$$

The required time amounts to

$$t = \frac{s}{v}$$

The covered distance becomes

$$s = vt.$$

with circular motions, the covered distance amounts to

$$s = ? dU$$

and the speed is

$$v = ? dn.$$

The units of speed are $\frac{m}{s}$; $\frac{m}{min}$; $\frac{km}{h}$.

v: speed

s: distance

t: period of time

d: diameter of circular path

n: number of revolutions per minute or per second; number of revolutions

? ? 3.14

2.2.3. Change of speed

A bus departs from the stop. Its speed increases. When it has reached the desired speed, it maintains this speed over a longer distance. At the next stop, the bus brakes and decelerates its speed till it stops.

The tachometer (speedometer) indicates the speed to the driver.

Accelerated motion

On starting of the bus, the following speeds were indicated (example 1):

Time after starting 0 1 2 3 4 5 10 s

Speed 0 5 10 15 20 25 50 km/h

When the speeds are converted into m/s, the following table comes to exist.

$$1 \frac{km}{h} = \frac{1000m}{60min} = \frac{1000m}{60 \cdot 60s} = 0.2778 \frac{m}{s}$$

$$\text{hence } 5 \frac{km}{h} = \frac{5 \cdot 1000m}{60min} = \frac{5 \cdot 1000m}{60 \cdot 60s} = 1.39m/s$$

Time after starting 0 1 2 3 4 5 10 s

Speed 0 1.39 2.78 4.17 5.56 6.94 13.89 m/s

For the selected example, it can be stated:

- The speed increases with time.
- The speed becomes larger continuously.

It is a uniformly accelerated speed.

On starting of a passenger car, the following speeds are read at the tachometer (speedometer) (example 2):

Time after starting	0	1	2	3	4	5	10	s
Speed	0	8	16	24	32	40	80	km/h
	0	2.22	4.44	6.67	8.89	11.11	22.22	m/s

The passenger car attains a higher speed more quickly, it was accelerated more strongly.

$$\text{Acceleration} = \frac{\text{change of speed}}{\text{required time}}$$

or as formula

$$a = \frac{v_1 - v_0}{t} \quad (2.3.)$$

a: acceleration

v_1 : final speed

v_0 : initial speed

t: required speed

The unit of acceleration is $\frac{m}{s^2}$

What is the speed when the body was accelerated by acceleration a in period of time t? The formula for acceleration is inverted and solved to v_1

$$v_1 = v_0 + at \quad (2.4.)$$

What distance has the body covered by then? The mean speed is

$$v_m = \frac{v_0 + v_1}{2}$$

With $s = v_m t$, s becomes

$$s = \frac{v_0 + v_1}{2} t$$

According to equation (2.4.), v_1 was

$$v_1 = v_0 + at.$$

Consequently, s becomes

$$s = \frac{1}{2}(v_0 + v_0 + at)t$$

$$s = v_0 t + \frac{a}{2} t^2 \quad (2.5.)$$

Example

What distances are covered by the aforementioned vehicles on starting within the first 10 seconds?

Given:

Bus initial speed $v_0 = 0$

speed after 10 seconds $v_{10} = 13.9 \frac{\text{m}}{\text{s}}$

required time $t = 10 \text{ s}$

Passenger car

initial speed $v_0 = 0$

speed after 10 seconds $v_{10} = 22.2 \frac{\text{m}}{\text{s}}$

required time $t = 10 \text{ s}$

Required:

Covered distance s

Solution:

According to equation (2.3.), the acceleration can be calculated:

$$a = \frac{v_{10} - v_0}{t}$$

Bus:

$$a = \frac{13.9 \frac{\text{m}}{\text{s}} - 0}{10\text{s}}$$

$$a = 1.39 \frac{\text{m}}{\text{s}^2}$$

Passenger car: $a = \frac{22.2\text{m/s} - 0}{10\text{s}}$ $a = 2.22 \frac{\text{m}}{\text{s}^2}$

According to equation (2.5.), for the covered distance applies

Bus: $s = 0 \cdot 10 \text{ s}$

$$s = v_0 t + \frac{a}{2} t^2 + \frac{1.39 \frac{\text{m}}{\text{s}^2}}{2} (10\text{s})^2$$

$$S = 69.5 \text{ m}$$

Passenger car: $s = 0 \cdot 10 \text{ s}$

$$S = 69.5\text{m} + \frac{2.22 \frac{\text{m}}{\text{s}^2}}{2} (10\text{s})^2$$

$$s = 111 \text{ m}$$

Result:

On starting, the bus covers a distance of 69.5 m, and the passenger car a distance of 111 m within the first 10 seconds.

A special acceleration occurs if a body falls down without hindrance. This motion is called free fall.

Exact measurements proved that each body is accelerated by $g = 9.81 \text{ m/s}^2$ at falling down.

This acceleration is called gravitational acceleration and receives the formula sign g .

The gravitational acceleration is approximately alike in all countries of the world. At the equator

$g = 9.78 \text{ m/s}^2$ and at the pole,
 $g = 9.83 \text{ m/s}^2$.

In this book, calculations are made uniformly with $g = 9.81 \text{ m/s}^2$.

Retarded motion.

In the example investigated before, two vehicles were observed on starting. Their speeds were increasingly continuously.

When stopping, the vehicles reduce their speeds and come to standstill.

When the bus brakes at the stop, then the following speeds can be read at the speedometer (tachometer), e.g.:

Time after beginning of braking	0	1	2	3	4	5	s
Speed	50	40	30	20	10	5	km/h

For the selected example, it can be stated:

The speed decreases with time.
 The speed becomes smaller continuously.
 It is a uniformly retarded motion.

Summary:

At the beginning of an acceleration, vehicles, workpieces and other bodies are accelerated.

$$\text{Acceleration} = \frac{\text{change of speed}}{\text{required time}}$$

The unit of acceleration is m/s^2 .

With a uniformly accelerated speed, the following equations are applicable:

Acceleration	Final speed	covered distance
$a = \frac{v_1 - v_0}{t}$	$v_1 = v_0 + at$	$s = v_0 t + \frac{a}{2} t^2$

a: acceleration
 v_0 : initial speed
 v_1 : final speed
 t: required time
 s: covered distance

With free fall, $g = 9.81 \text{ m/s}^2$.

With a uniformly retarded motion, the speed decreases uniformly.

2.3. Masses and forces

2.3.1. Masses

In the previous chapter, the motion of bodies was described. It was entirely neglected how large the bodies are. The volume is measured in cubic metres, see para' 1. Our experience says, however, that it is not

sufficient to know how large a body is to predict its motion. An empty chest can easily be moved by 1 man. If the same chest is filled with metal parts, however, 2 men are frequently not sufficient for lifting the chest.

The volume of the chest is equal in both cases.

The chest and the metal parts contained in it, have together a larger mass, however.

The mass is an important physical basic quantity.

Each body has a certain mass.

The mass has the formula sign m .

In order to render a comparison between the masses of different bodies, the kilogramme was fixed as unit of mass. Further units of mass are the gramme (g) and the ton (t).

$$1 \text{ t} = 1,000 \text{ kg}$$

$$1 \text{ kg} = 1,000 \text{ g}$$

Kilogramme pieces and gramme pieces were made. These by means of a lever scale, these mass pieces (weights) permit a comparison of masses.

The mass of bodies can be determined by means of gramme pieces and kilogramme pieces.



Figure 2.8. Comparison of masses on a lever scale

First it is checked whether the scale is in balance. In balance, both scale pans are equally high. Then the mass to be measured is placed in the left scale pan. Now the scale is no longer in balance. Then as many kilogramme pieces and gramme pieces are placed on the right scale pan, till balance sets in again. Then the masses in the two scale pans are of equal size.

There are balances for special tasks. The pharmacist requires a balance by which tiny masses can be measured. The mass of a vehicle, however, is determined by large special balances.

2.3.2. Forces

A bus is to be observed again, which departs from a stop. The bus has a mass. Its speed increases. It is – as found out in para' 2.2.3. – accelerated. What is the cause for this acceleration? Whereby does the body receive the higher speed?

The motion is forced by the motor. The motor sets the wheels in motion, and the bus is accelerated.

The same effect can be achieved if the bus – in case of engine trouble – is towed off by another vehicle. Then the speed increases on starting, too. Hence, the bus is accelerated.

The same effect – acceleration of a body – is achieved by its own motor, on the one hand, or by the towing vehicle.

In both cases, forces take effect.

In the first case, the motor produces the driving force. In the second case, the towing rod transmits the force from the towing vehicle to the bus.

Cause Effect

force acceleration of masses

In engineering and production, other effects of forces can be detected.

The wheels of goods wagons are connected with the wagon by leaf-springs.

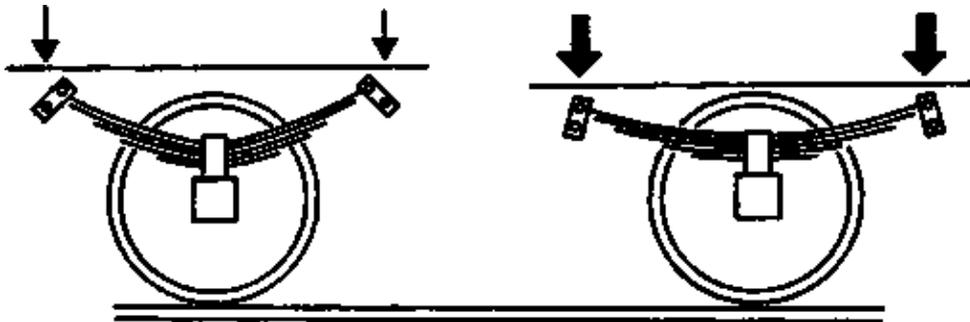


Figure 2.9. Leaf-springs at a goods wagon. Elastic deformation by action of a force

When the goods wagon is loaded, the leaf-springs are bent downwards. On emptying of the wagon, they assume their original form again.

Such a deformation is termed elastic deformation. With a plastic deformation, however, a body is deformed by forces and retains the new form even when the force is not effective any longer.

This can be distinctly recognized with the rolling of sheet metal (fig. 2.10.). The sheets are pressed by two rolls and thereby receive a new form: they are getting longer or wider and thinner.

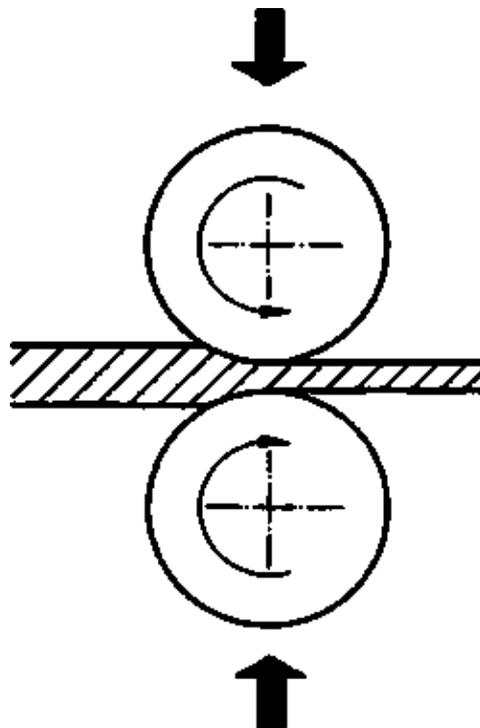


Figure 2.10. Rolling of sheet metal. Plastic deformation by action of a force

Cause Effect

force elastic deformation
plastic deformation

The bus receives a higher speed. The leaf–spring of the goods wagon is subject to elastic, and the sheet metal to plastic deformation.

The causes are acting forces.

The effects of these forces can be recognized (acceleration, deformation). The forces themselves cannot be seen, however.

Forces are not visible. They can be recognized by their effects.

The force has the formula sign F.

2.3.3. Inertia

All bodies (solid, liquid and gaseous) have a mass.

Natural scientists have occupied themselves much with force and mass. In thorough experiments, they found out that a mass will change its speed only if a force acts upon it. It is said that each mass has an inertia.

A mass remains in the state of rest if no force acts upon it, and is accelerated by a force only. Moved masses have an inertia, too.

If a mass moves on a rectilinear path at constant speed, then the mass will change its speed or path only when compelled by a force.

Each mass remains in the state of rest or of constant rectilinear motion unless it is compelled by a force to change its speed or path of motion.

This law of nature is called law of inertia. It was set up in the 17th century already by the great physicist Isaac Newton. This physicist established the law of acceleration, too: The acceleration of a mass grows with the acting force. Moreover, the acceleration becomes smaller if the same force acts upon a larger mass.

It applies

$$a = \frac{F}{m}$$

a: acceleration

F: force

m: mass

If this equation is inverted, the correlation, between force and mass becomes obvious:

$$F = m \cdot a$$

$$\text{force} = \text{mass} \cdot \text{acceleration}$$

From that, the unit of force F is derived.

The unit of force is kg • m/s². This is the force which conveys to a mass of 1 kg the acceleration of 1 m/s². To honour the memory of the physicist Isaac Newton, the unit of force received the name of Newton N.

$$1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$$

One Newton (N) is the force, which conveys to a mass of 1 kg the acceleration of 1 m/s².

In engineering and production, multiples of a Newton are used above all.

$$1 \text{ kN} = 1,000 \text{ N}$$

$$1 \text{ MN} = 1,000 \text{ kN} = 1,000,000 \text{ N}$$

(kN read kilonewton; MN read meganewton)

2.3.4. Weight

All bodies have a weight. They press with their weight on their supports.

Each machine loads the base with their weight. Each workpiece has a weight.

Small workpieces can be lifted by the hands. Large and heavy workpieces are transported by cranes.

The weight is a force which acts upon all masses. The weight always points to the earth's centre.

The earth pulls all masses with the gravitational acceleration, see para' 2.2.3.

$g = 9.81 \text{ m/s}^2$.

The weight is a force and is therefore measured in newtons, too.

The mass of 1 kilogramme is attracted by the earth with the force

$$G = 1 \text{ kg} \cdot 9.81 \text{ m/s}^2.$$

Hence, the weight presses on the support with the force

$$G = 9.81 \text{ kg} \cdot \text{m/s}^2$$
$$G = 9.81 \text{ N}.$$

We get conscious of this force when we hold a mass of 1 kilogramme in our outstretched hand.

In general, it applies

$$G = m \cdot g \quad (2.7.)$$

g: weight

m: mass

g: gravitational acceleration

Frequently it is rounded off, and approximate calculation is applied

$$9.81 \text{ N} \approx 10 \text{ N}$$

(Read 9.81 newton are approx. 10 newton).

$$9.81 \text{ kN} \approx 10 \text{ kN}$$

(Read 9.81 kilonewton are approx. 10 kilonewton).

2.3.5. Measurement of forces

Forces cannot be seen. Only the effects of forces can be observed.

As forces cannot be seen directly, they are measured in their effects. To this end, the elastic deformation of springs is suited best.

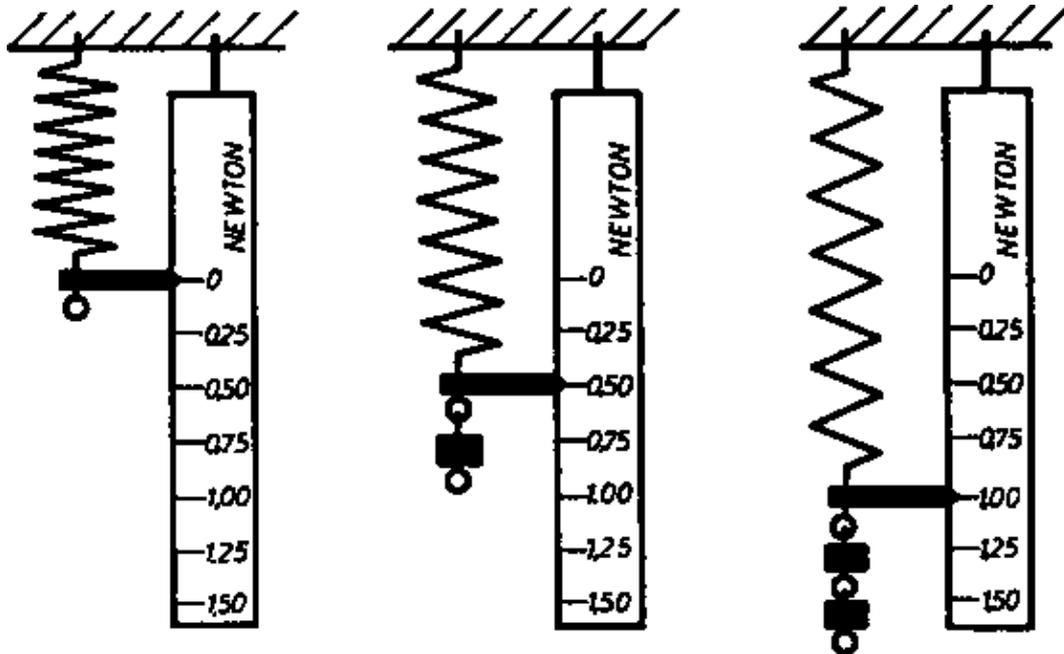


Figure 2.11. Simple spring tension meter (spring balance)

Fig. 2.11. shows the operating mode of a simple spring tension meter. A helical spring and a scale are firmly suspended side by side. In unloaded state, the spring is not expanded. The spring is firmly connected with a pointer. In unloaded state, the pointer points to zero value on the scale. If a mass of $50\text{ g} = 0.050\text{ kg}$ is hung on the spring, then the weight acts according to eq. (2.7.)

$$G = 9.81\text{ m/s}^2 \cdot 0.050\text{ kg}$$

$$G = 0.49\text{ kg} \cdot \text{m/s}^2$$

$$G = 0.49\text{ N.}$$

This value is indicated by the pointer on the scale. $100\text{ g} = 0.1\text{ kg}$ mass has a weight of

$$G = 9.81\text{ m/s}^2 \cdot 0.1\text{ kg}$$

$$G = 0.981\text{ kg} \cdot \text{m/s}^2$$

$$G = 0.981\text{ N}$$

This value can be read on the scale, too.

Larger forces are measured by means of stronger spring tension meters. With these, stronger springs are used.

2.3.6. Density

By means of a lever balance, the mass of a body can be determined. Its weight can be measured by the aid of a spring tension meter. Large machines are difficult to measure by using balance and spring.

Is there any possibility of ascertaining mass and weight of these bodies?

To this end, mass and weight of 1 cubic decimeter of the respective matter are determined.

It is stated:

1 dm ³	steel	lead	water
has the mass	7.85 kg	11.34 kg	1.0 kg

The quotient from mass and volume is termed density (?.)

$$\rho = \frac{m}{V}$$

m: mass
V: volume
?: density

If this equation is inverted, the mass can be stated immediately provided that density and volume are known:

$$m = ? \cdot V.$$

Table 2.1. contains a survey of the density of important matters.

The weight can be ascertained by using the density, too.

According to equation (2.7.), the weight is

$$G = mg.$$

With eq. (2.8.)

$$m = ?V$$

the weight becomes

Table 2.1. Density of selected substances

Substance	Density ? in kg/dm ³	Factor ? g in N/dm ³
Aluminium	2.70	26.5
Lead	11.34	111.2
Steel	7.85	77.0
Gold	19.3	189.3
Copper	8.92	87.5
Platinum	21.45	210.4
Silver	10.50	103.0
Zinc	7.13	69.9
Tin	7.28	71.4
Water	1.0	9.81
Concrete	1.9 to 2.8.	18.6 to 27.5
Glass	2.4 to 2.6	23.5 to 25.5
		$g = 9.81 \frac{m}{s^2}$ The values ? g were calculated at

$$G = ? \cdot V \cdot g$$

G: weight
?: density
V: volume
g: gravitational acceleration

Factor ? · g is listed in table 2.1.

2.3.7. Graphic representation of forces

Apart from the possible effects of forces, further properties of forces are important.

Forces always act on a line of action and in one direction.

In order to illustrate this relation, the action of forces on a wagon is to be examined more closely (fig. 2.12.).

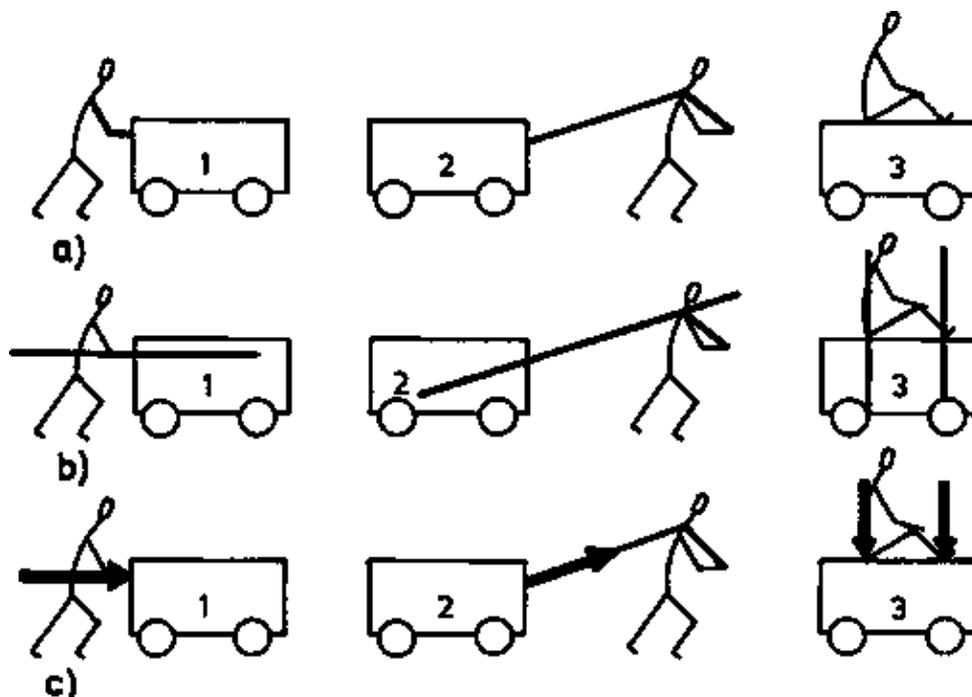


Figure 2.12. Forces at a wagon. Above: load. Centre: lines of action of the forces. Below: forces represented graphically

In fig. 2.12, the wagon is pushed in case 1, it is drawn in case 2 and additionally loaded in case 3. In fig. 2.12, centre, the lines of action of the forces are marked in.

In case 1, the force acts on a straight line running parallel with the base. It acts parallel with the path of motion of the wagon. In case 2 it acts at an acute angle to the path of motion, and in case 3 it is perpendicular to the path of motion of the wagon. In addition, the direction of the force has to be stated. In case 1, the force can act on the stated line of action to the right, i.e. toward the wagon, or be directed to the left, i.e. away from the wagon. The direction of force is represented by an arrow.

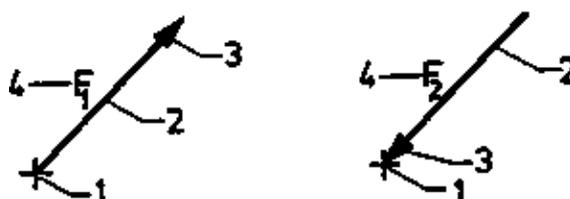


Figure 2.13. Representation of forces:

- 1 point of application, 2 line of action;
- 3 direction, 4 designation

To describe a force, the following statements have to be made:

- point of application of a force
- line of action of a force
- direction of a force
- quantity of a force

Occasionally, the quantity of a force is represented graphically; see para' 5.2..

2.4. Tasks and questions

2.1. Name 3 examples for rectilinear motions!

2.2. Name 3 examples for circular motions!

2.3. Describe the path of motion passed by a body when falling down!

2.4. Fig. 2.14 shows the path of motion passed by a chest when lifted. On the chest, two points A and B were marked by colour spots.

In fig. 2.15., a gear was represented, rotating about its own axis. On the gear, points C and D were marked by colour spots, too.

Compare the paths of motion of points A and B! Compare the paths of motion of points C and D! Compare the paths of motion of points A and D! Pay attention to form and length of the path of motion!

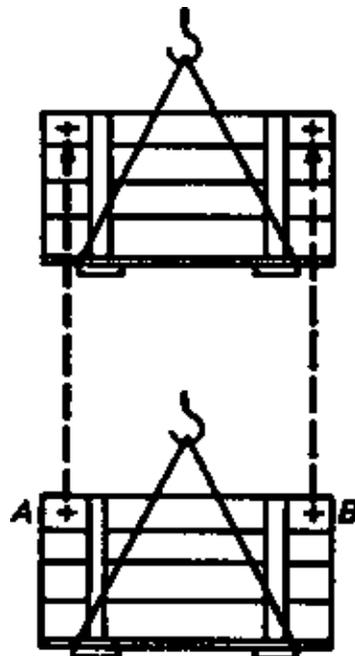


Fig. 2.14. A chest is lifted; task 2.4.

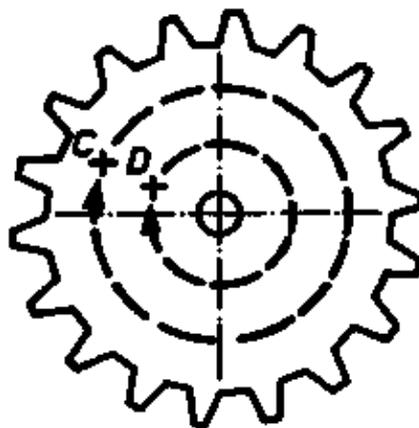


Figure 2.15. A gear rotates about its own axis; task 2.4.

2.5. A motor lorry covers a distance of 110 km in 2 hours. What is its average speed?

2.6. A conveyor belt (fig. 2.3.) transports a chest at a speed of 40 m/min. Within what time does the conveyor belt transport the chest over a distance of 10 m?

2.7. A belt-pulley of an outside diameter $d = 200$ mm rotates 600 times a minute. What is the speed at the periphery of the pulley?

2.8. A belt-pulley of an outside diameter $d = 100$ mm rotates 600 times a minute. What is the speed at the periphery of the pulley?

2.9. Why is the speed of the peripheral points of the belt-pulleys in tasks 2.7. and 2.8. different although their rotational speed is equal?

2.10 Complete the following table:

0.3 t 1.2 t 0.05 t t t

300 kg kg kg 250 kg 60 kg

2.11. Complete the following table:

1,200 g 50g g g 600 g

1.2 kg kg 11.0 kg 0.3 kg kg

2.12. Name 3 examples from production for the action of forces!

Describe the action of forces!

2.13. Complete the following table!

322 N 4.300 N N N 350
N

12.1 kN 0.08 kN kN

2.14. Complete the following table:

1,200 kN 800 kN 11,000 kN kN kN

1.2 MN MN MN 0.3 MN 2.5 MN

2.15. What is the weight of a mass of 12 kg?

2.16. what is the weight of a mass of 3.2 t?

2.17. what is the weight of a mass of 422 g?

2.18. The carrying capacity of a ceiling is stated to be 5 kN/m².

By this is understood that on an area of 1 m², a force (weight) may act of max. 5 kN.

What mass per 1 m² may be piled on this ceiling?

2.19. The carrying capacity (lifting force) of a crane is stated to be 20 kN.

What mass may the crane lift maximally?

2.20. The carrying capacity of a fork–lift truck is limited at 50 kN.

What mass may the fork–lift truck carry maximally?

2.21. Name the mass and the weight of

- a) 1 cm³
 - b) 1 dm³
 - c) 1 m³
- of water!

2.22. Name the mass and the weight of

- 7.5 dm³ of steel and of
- 7.5 dm³ of aluminium!

3. Simple force–converting devices

3.1. Levers

3.1.1. Lever principle

Before the lever principle is explained, an observation from practical work is to be evaluated.

At putting down a heavy chest by the crane, the wire rope, by which the chest was suspended at the crane, got jammed. It cannot be drawn from under the chest as the chest partly stands on it.

A worker tries to lift the chest. However much he exerts himself, he does not succeed in lifting the chest even few centimetres only.

Another worker comes up, and puts a crowbar under the chest (fig. 3.1.). By that, he is able to remove the chest so far from the ground that the rope can be drawn forth.

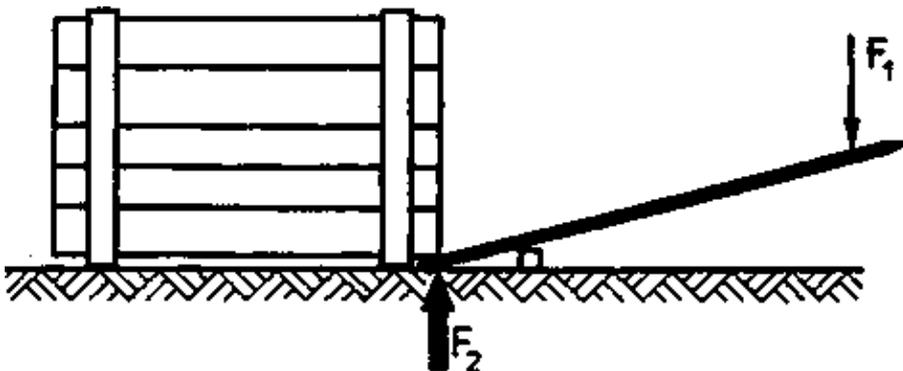


Figure 3.1. A lever can increase forces

Was the second worker stronger? No, he knew the lever principle and made use of his knowledge at work.

What is a lever? How does the lever principle run?

In order to answer these questions, some simple experiments are to be described.

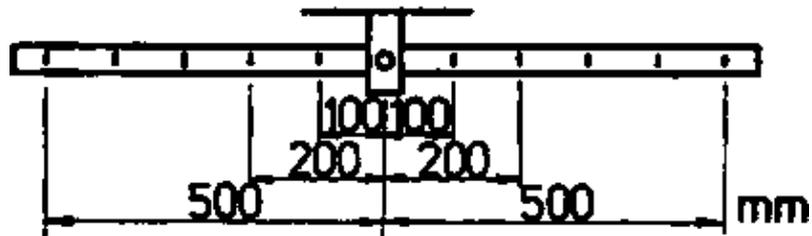


Figure 3.2. Two-sided lever

Fig. 3.2. shows a two-sided lever. A firm bar is suspended swivelling in the middle. It is in balanced condition, i.e. it remains at rest.

A firm bar, supported rotatable in one point, is called lever.

The two-sided lever, represented in fig. 3.2. has graduation marks at a distance of 100 mm each, to render an easier evaluation of experiments possible.

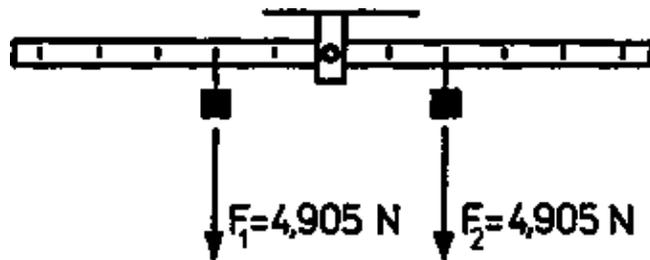


Figure 3.3. Experiment 1. Equilibrium

Experiment 1: The lever represented in fig. 3.3. is now loaded by two mass pieces of 0.5 kg each. Each mass has a weight of 9.905 N, see also para' 2.3.4. Both masses act at a distance of 200 mm = 0.2 m from the pivot. Hence, it can be said: The lever is loaded by a force $F_1 = 4.905$ N and by a force $F_2 = 4.905$ N. Both forces act to the left and right of the pivot at a distance of 0.2 m each. If F_1 did exist only, a rotation in clockwise direction would ensue. By counterforce F_2 , this rotation is prevented.

Under these conditions, there is equilibrium.

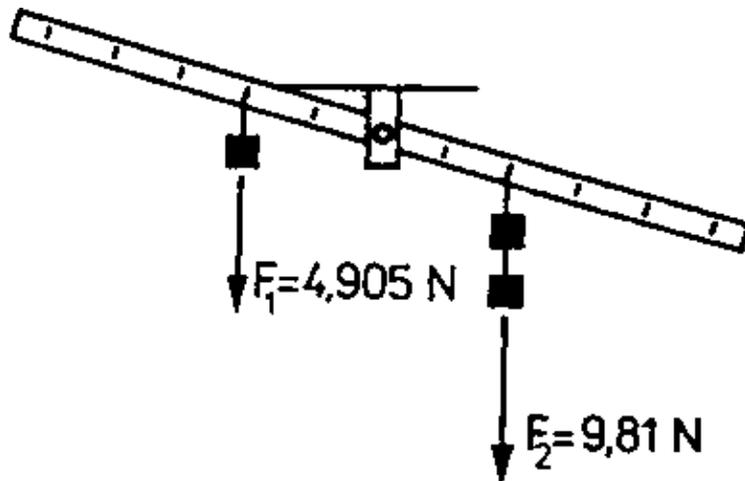


Figure 3.4. Experiment 2. No equilibrium

Experiment 2: Fig. 3.4. shows that 2 forces are acting on the lever. The distance of the forces from the pivot is equal, but the forces are different in quantity.

The lever swivels, there is no equilibrium.

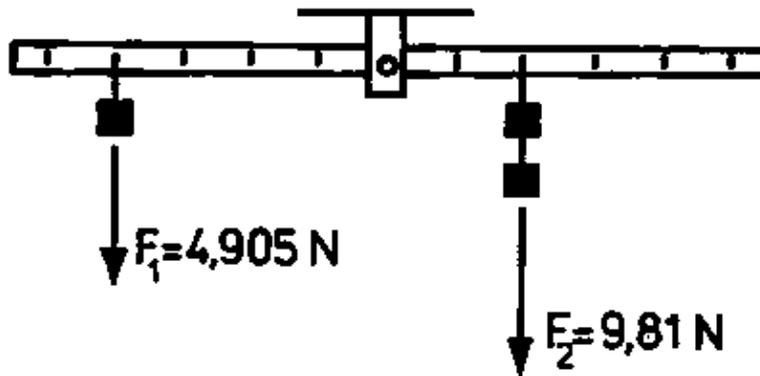


Figure 3.5. Experiment 3. Equilibrium

Experiment 3: In fig. 3.5. it is represented how 2 unequally large forces can be in balanced state if the distances from the pivot are different.

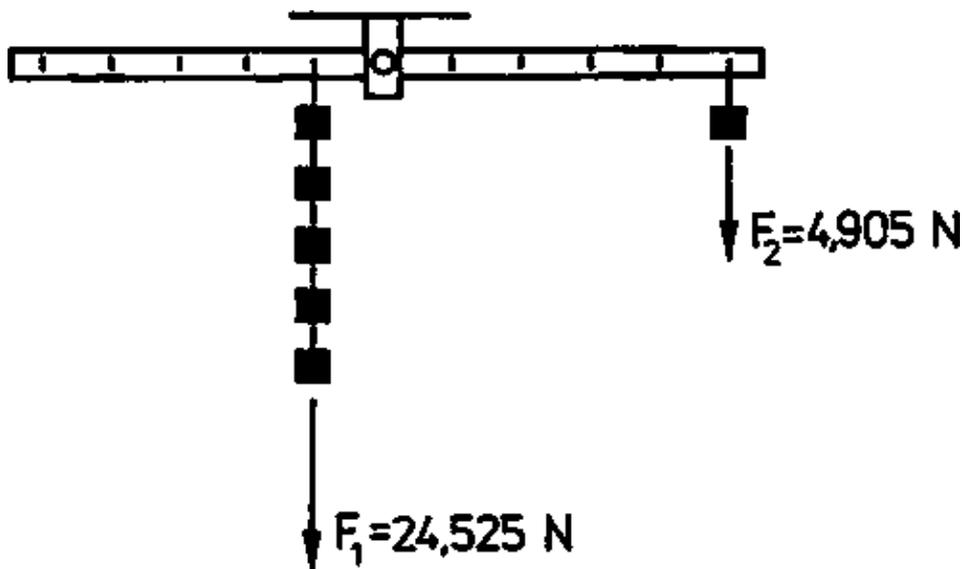


Figure 3.6. Experiment 4. Equilibrium

Experiment 4: The arrangement in fig. 3.6. shows that the force $f_2 = 4.905 \text{ N}$ can keep the balance to force $F_1 = 24.525 \text{ N}$ if F_2 acts sufficiently distant from the pivot.

Experiments 1 to 4 show the equilibrium at the lever is dependent on

- size of the acting forces
- vertical distance of the lines of action of these forces from the pivot of the lever.

This correlation is evaluated more closely in table 3.1.

Table 3.1. shows: There is equilibrium at the lever if

$$F_1 l_1 = F_2 l_2$$

F_1 : force that would turn the lever in anticlockwise direction if a counterforce did not exist

l_1 : vertical distance of the line of action of F_1 from the pivot

F_2 : force that would turn the lever in clockwise direction if a counterforce did not exist

l_2 : vertical distance of the line of action of F_2 from the pivot

Apart from the two-sided lever there are also one-sided levers and bent levers.

With one-sided and bent levers, the lever principle

$$F_1 l_1 = F_2 l_2$$

is applicable, too.

Fig. 3.7. shows the form of the levers.

With l_1 and l_2 , the vertical distance of the line of action of the force from the pivot is to be selected in all cases.

Table 3.1. Evaluation of the experiments at the two-sided lever

Experiment	Left side of lever			Right side of lever			Result
	Force F_1	Spacing l_1	$F_1 l_1$	Force F_2	Spacing l_2	$F_2 l_2$	
1	4.905 N	0.2 m	0.981 N	4.905 N	0.2 m	0.981 Nm	equilibrium
2	4.905 N	0.2 m	0.981 N	9.81 N	0.2 m	1.962 NM	no equilibrium
3	4.905 N	0.4 m	1.962 Nm	9.81 N	0.2 m	1.962 Nm	equilibrium
4	24.525 N	0.1 m	2.4525 Nm	4.905 N	0.5 m	2.4525 Nm	equilibrium

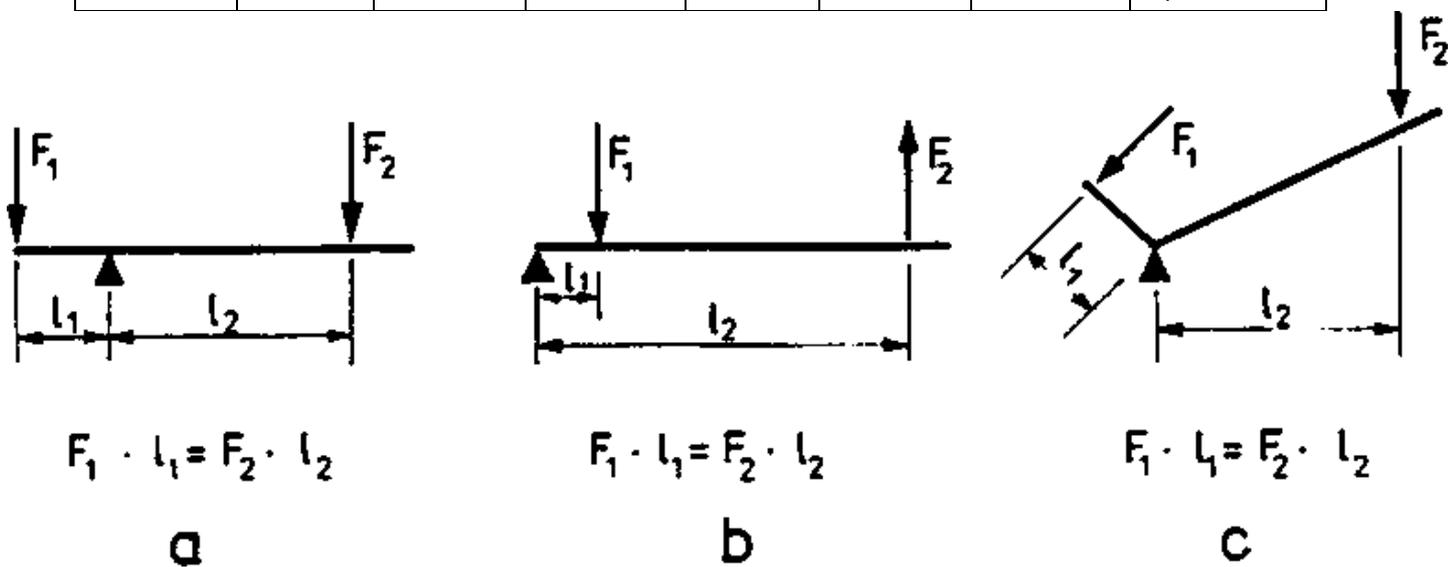


Figure 3.7. Equilibrium at two-sided lever, one-sided lever and bent lever

3.1.2. Application of the lever principle

Levers act in all machines and tools. It is not always easy to recognize the acting levers.

In the following, some examples are shown to explain how levers act in tools and machines.

Fig. 3.1. shows the employment of a crowbar for lifting larger loads. By means of a crowbar, heavy loads can be lifted few centimetres.

Levers act in all tongs. The force of hands is, in part, increased considerably. Fig. 3.8. shows a pair of pincers.

The force of hands is increased to such an extent that the 4 to 8-fold force of hands acts at the cutting edge. This value is increased by the wedge effect, concerning this see fig. 5.24 and fig. 5.26.

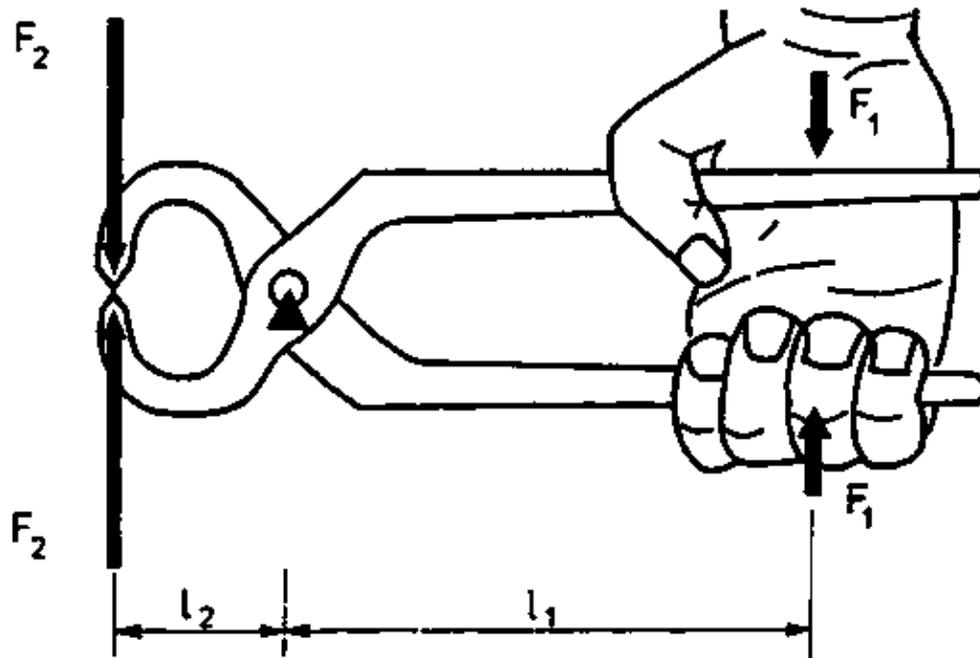


Figure 3.8. Pincers. The acting forces and levers were marked in

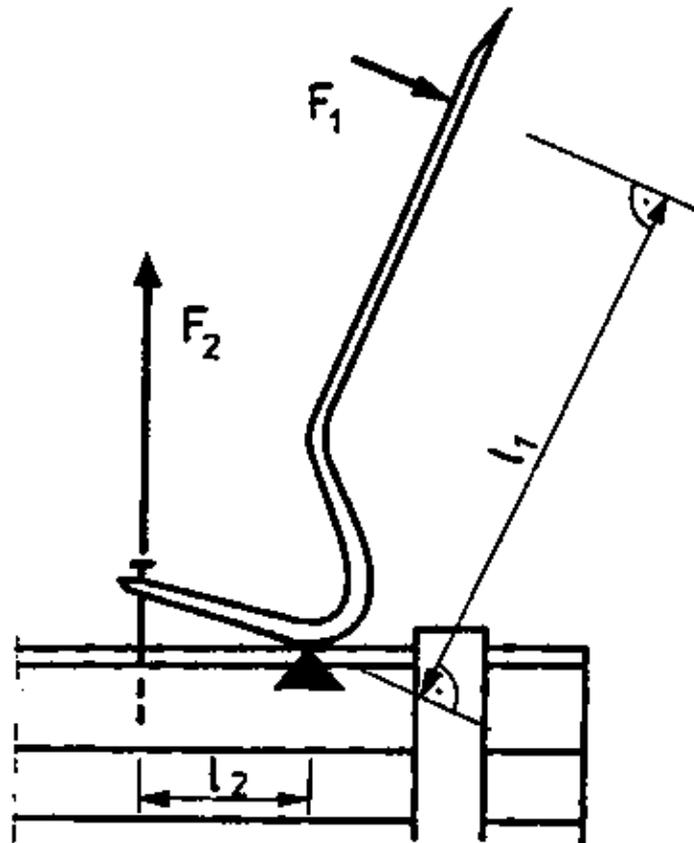


Figure 3.9. Nail-lifter. The acting levers were marked in

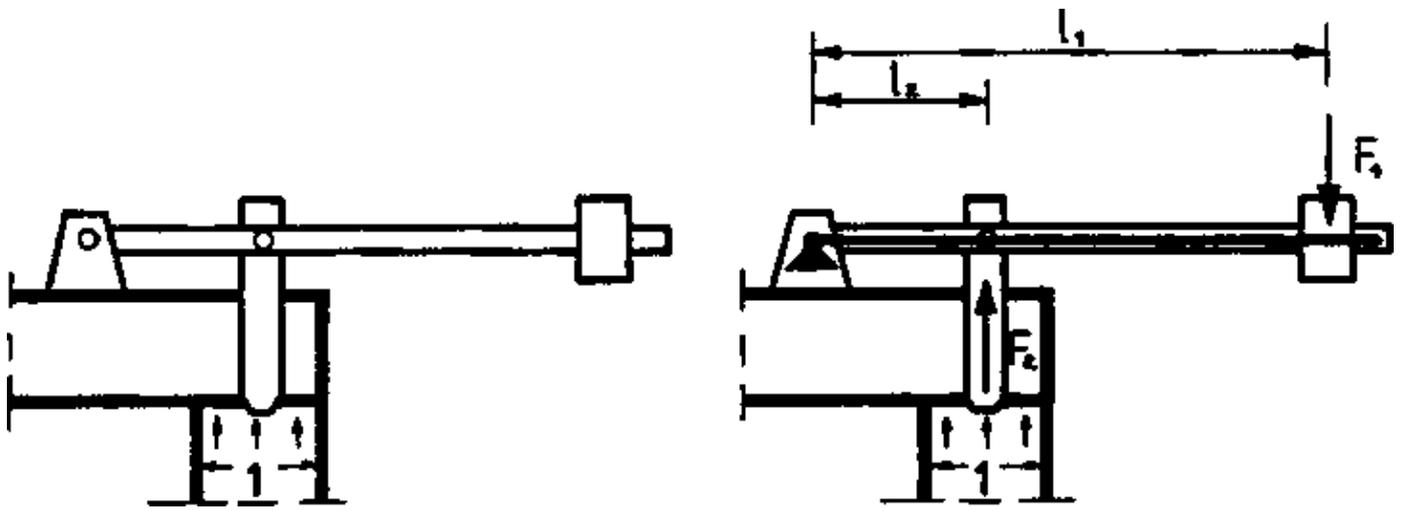


Figure 3.10. Safety valve. Representation highly simplified. In the picture on the right, the acting levers were marked in. 1 space filled with steam

Fig. 3.9. shows a nail–lifter. It serves for pulling out nails. The nail–lifter is an angle lever. It is to be observed that the vertical distance of the lines of action of the force from the pivot is to be applied as lever arm.

Simple safety valves can be constructed as one–sided levers. A safety valve is to prevent the pressure in a room filled with steam from exceeding a given value. The mass presses on the lever with force F_1 , and this transmits the force on the valve cone, and thereby closes the opening. When the pressure in the steam space becomes larger than scheduled, force F_2 becomes too large. It overcomes the equilibrium at the lever, and presses the lever upwards. Thereby the valve gets opened. Part of the steam escapes, and the pressure drops. Now the valve gets closed again.

3.2. Pulleys

3.2.1. General

Pulleys are circular disks with a groove, a fluted recess, at the periphery. The pulley can rotate about its central axis. The rope is guided in the groove.

Ropes can transmit tensile force. They do not change the quantity of the force.

By means of pulley and rope, forces can be diverted and partly changed with regard to quantity, too (loose pulley).

By means of pulley and rope, large effects can be achieved with small forces. Therefore pulley and rope are applied in almost all hoisting gear, e.g. with crane and elevator.

3.2.2. Fixed pulleys

The basic form of all pulleys is the fixed pulley. With the fixed pulley, a pulley is firmly mounted with its axle on a ceiling, a beam or machine frame.

Fig. 3.11. shows the operating mode of the fixed pulley. A mass of the weight G hangs on a rope, which is guided over a fixed pulley. Force F is in equilibrium with weight G . By the fixed pulley, the direction of force F is changed. It can be directed downwards, and lifts the mass upwards yet. In fig. 3.11. the acting levers are marked in.

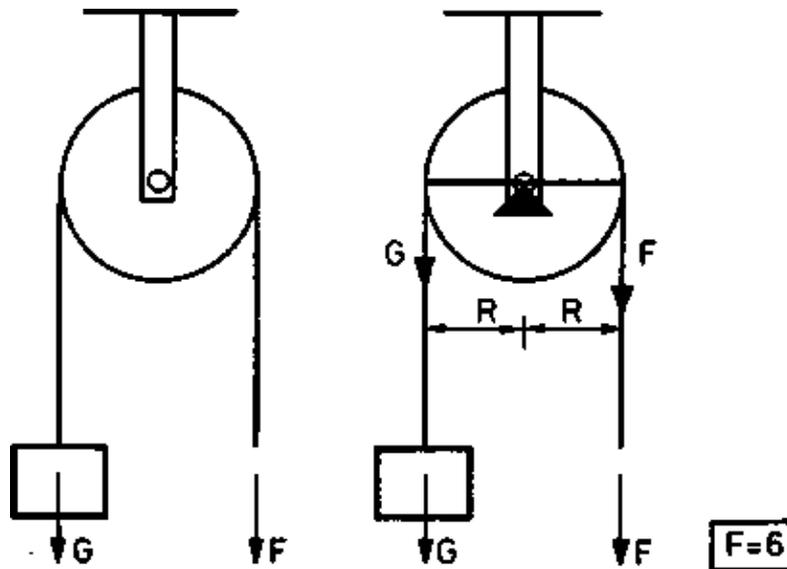


Fig. 3.11. Fixed pulley. In the picture on the right, the acting levers were marked in

According to the lever principle there is equilibrium if

$$FR = GR$$

$$F = G$$

By means of the fixed pulley, no force is saved. The force is diverted only.

3.2.3. Loose pulleys

Fig. 3.12. shows a loose pulley. The pulley lies in a rope coil, and moves up and down together with the mass.

When the pulley moves, then it rotates about point A. There is equilibrium at the lever marked in fig. 3.12. if

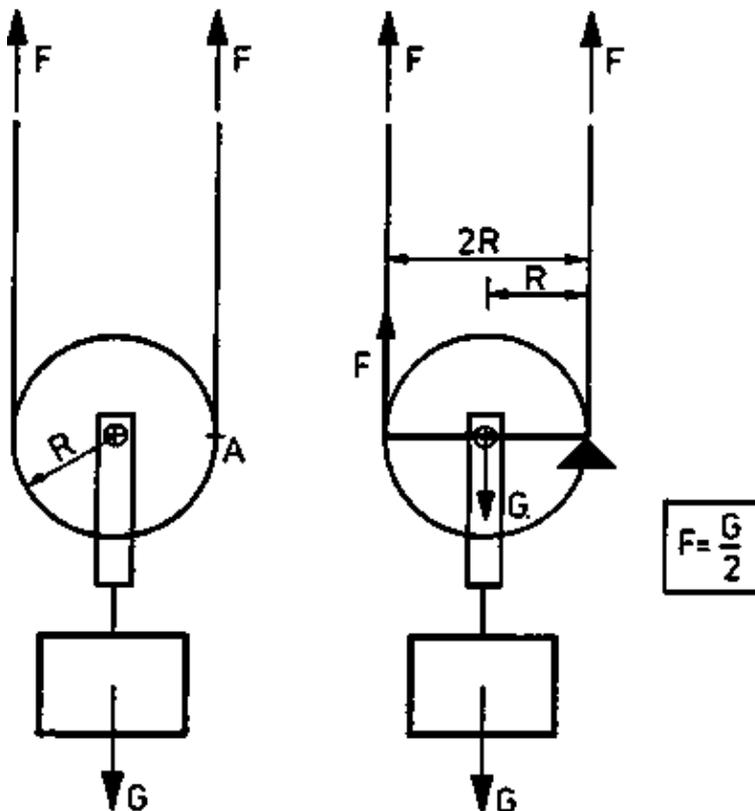


Fig. 3.12. Loose pulley. In the picture on the right, the acting levers were marked in

$$F2R = GR$$

$$F = \frac{G}{2} \quad (3.2.)$$

The same result is arrived at by the following consideration. Weight G is carried by 2 forces F . The rope transmits the tensile forces only without changing them. From that follows

$$2F = G$$

$$F = G/2.$$

The two ropes are called carrying ropes, too. A loose pulley has two carrying (supporting) ropes.

3.2.4. Factorial pulley block

The disadvantage of the loose pulley is that the force F has to act upwards. This disadvantage can be avoided if the force F is deviated again by means of a fixed pulley.

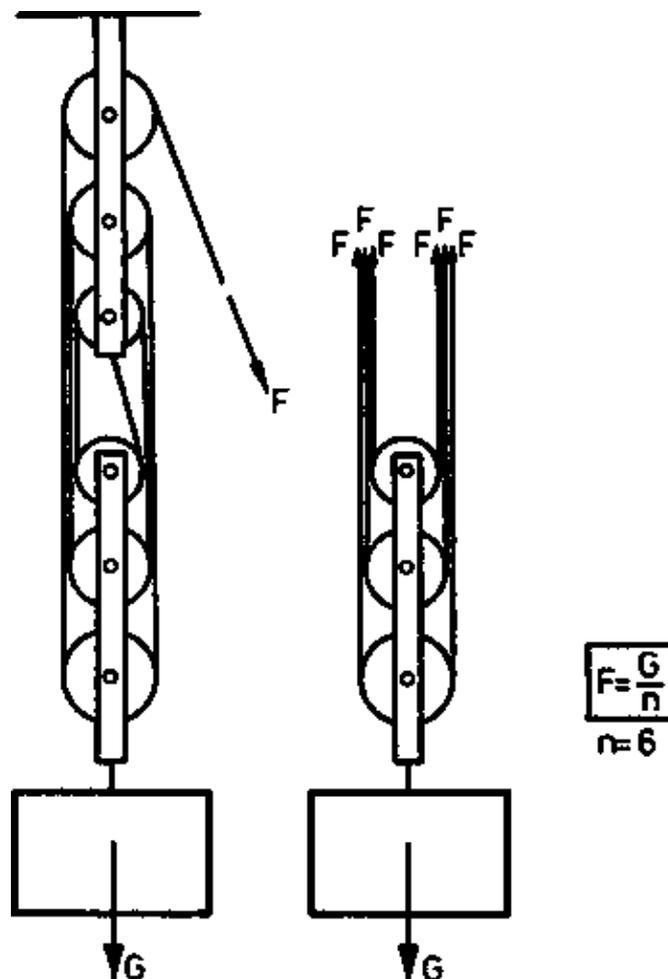


Fig. 3.13. Factorial pulley block. Arrangement of pulleys among one another

A larger effect can be attained by arranging several loose and fixed pulleys in succession. In fig. 3.13., the arrangement of pulleys and the rope guide are represented. The hoisting gear formed like that is termed factorial pulley block.

Due to the arrangement of pulleys among one another, much space is required. In fig. 3.17, a factorial pulley block is represented where the pulleys lie side by side in each case. In this form, the factorial pulley block is applied in engineering.

In fig. 3.13. it is shown that weight G has to be kept in equilibrium by the carrying (supporting) ropes. As the

force in the rope is equally large, each carrying rope carries an equal portion. It applies

$$nF = G$$

$$F = G/n \quad (3.4.)$$

G weight of the mass to be lifted

n number of carrying ropes

F force required for lifting

By means of a factorial pulley block, larger mass can be hoisted by smaller forces.

3.2.5. Winch and differential pulley block

A further increase of carrying capacity is attained with the differential pulley block. Component parts of the differential pulley block (fig. 3.15.) are a winch and a loose pulley.

With the winch, two pulleys of different diameter are firmly connected with each other and arranged in such a way that they can rotate about their common axis.

At the winch there is equilibrium if

$$FR = GR$$

$$F = \frac{r}{R} G \quad (3.5.)$$

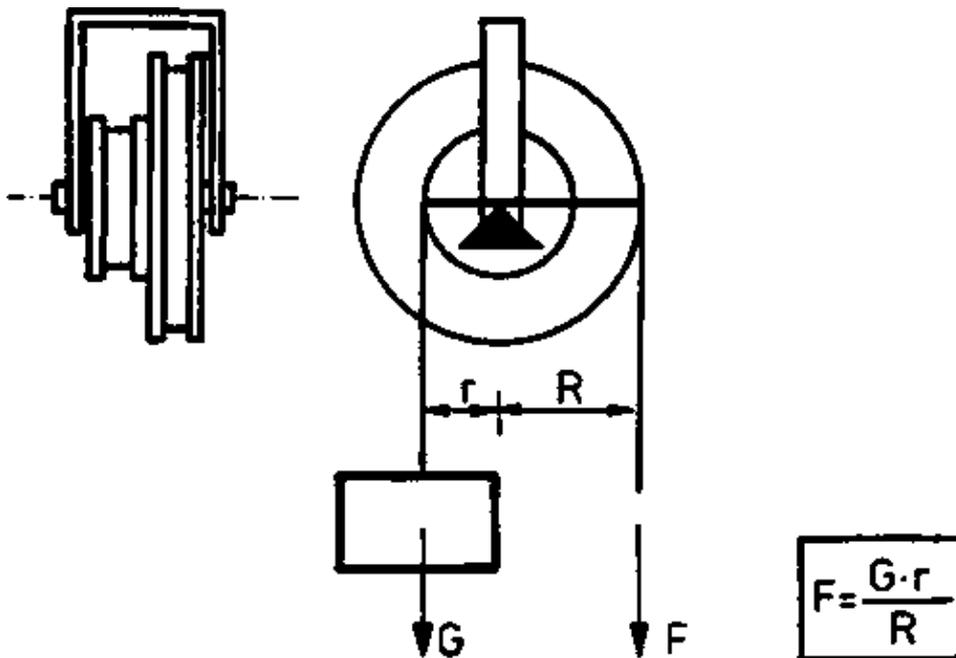


Fig. 3.14. Winch, In the picture on the right, the acting levers were marked in

If the winch is combined with a loose pulley, a pulley block with a very high carrying capacity comes to exist: the differential pulley block.

With the differential pulley block, the rope has to be replaced by an endless chain. A rope would slip through. To prevent the unloaded chain, contained in fig. 3.15., from slipping through, it lies in grooves in the winch. The individual chain links get hooked in the recesses.

The winch is to be turned anti-clockwise by force $G/2$ and F , and clockwise by force $G/2$.

$$FR + \frac{G}{2}r = \frac{G}{2}R$$

$$FR = \frac{G}{2}R - \frac{G}{2}r$$

$$F = \frac{G}{2} \cdot \frac{R-r}{R} \quad (3.6.)$$

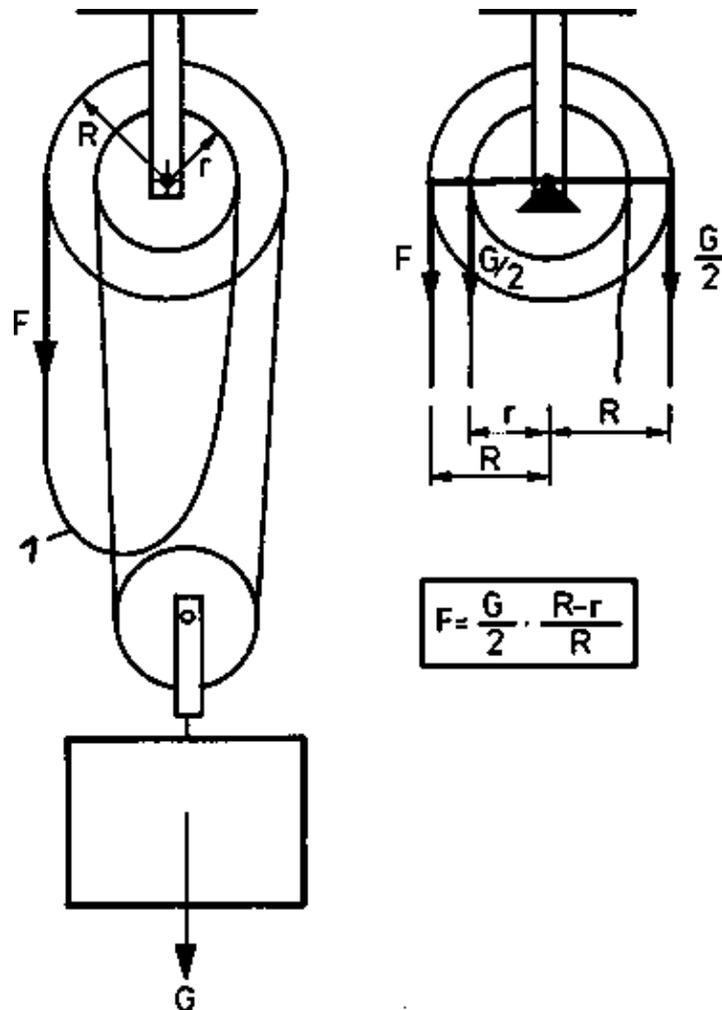


Fig. 3.15. Differential pulley block. In the picture on the right, the acting levers were marked in 1 unloading chain

3.2.6. Examples

Fixed pulley

By means of the fixed pulley, represented in fig. 3.16., a mass of 40 kg is to be lifted.

How large is the force, which has to be employed for lifting the mass?

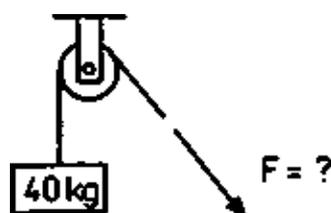


Fig. 3.16. Fixed pulley

Given:

$$\text{Mass } m = 40 \text{ kg}$$

Required:

Force F

Solution:

According to eq. (2.7.), the mass has the weight

$$\begin{aligned} G &= mg \\ G &? 40 \text{ kg} \cdot 10 \text{ m/s}^2 \\ G &? 400 \text{ N.} \end{aligned}$$

The calculation was performed with $g ? 10 \text{ m/s}^2$.

This approximation is sufficient.

According to eq. (3.1.), equilibrium exists at the fixed pulley if

$$F = G.$$

From this follows

$$F ? 400 \text{ N.}$$

Result:

A force $F ? 400 \text{ N}$ has to be employed for lifting mass $m = 40 \text{ kg}$ by means of the fixed pulley. The occurring friction was neglected. Concerning this, see also para' 4.2.4. Efficiency.

Factorial pulley block

By the aid of a factorial pulley block (fig. 3.17.), a mass of 150 kg is to be lifted. How large is the force required?

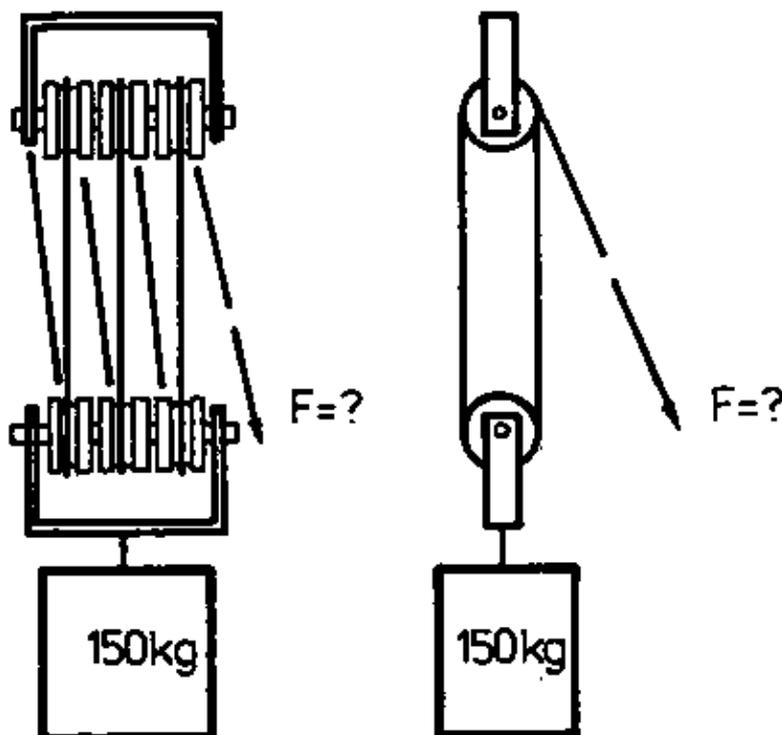


Fig. 3.17. Factorial pulley block. 1 three loose pulleys in file; 2 three fixed pulleys in file

Given:

$$\text{Mass } m = 150 \text{ kg}$$

Required:

$$\text{Force } F$$

Solution:

According to eq. (2.7.), the mass has the weight

$$\begin{aligned} G &= mg \\ G &\approx 150 \text{ kg} \cdot 10 \text{ m/s}^2 \\ G &\approx 1,500 \text{ N.} \end{aligned}$$

The calculation was performed with $g \approx 10 \text{ m/s}^2$.

This approximation is sufficient.

According to eq. (3.3.), equilibrium exists at the factorial pulley block if

$$F = \frac{G}{n}$$

In the pulley block represented in fig. 3.17, 3 carrying ropes lie side by side.

$$\begin{aligned} F &\approx \frac{1,500 \text{ N}}{3} \\ F &\approx 500 \text{ N.} \end{aligned}$$

Result:

The mass of 150 kg can be lifted by means of the factorial pulley with a force $F \approx 500 \text{ N}$.

The occurring friction was not considered. Concerning this, see para' 4.2.4. Efficiency.

Differential pulley block

By the aid of a differential pulley block, a mass of 500 kg is to be lifted. How large is the required force?

Given:

$$\text{Mass } m = 500 \text{ kg}$$

$$\text{Winch } R = 140 \text{ mm}$$

$$r = 120 \text{ mm}$$

Required:

$$\text{Force } F$$

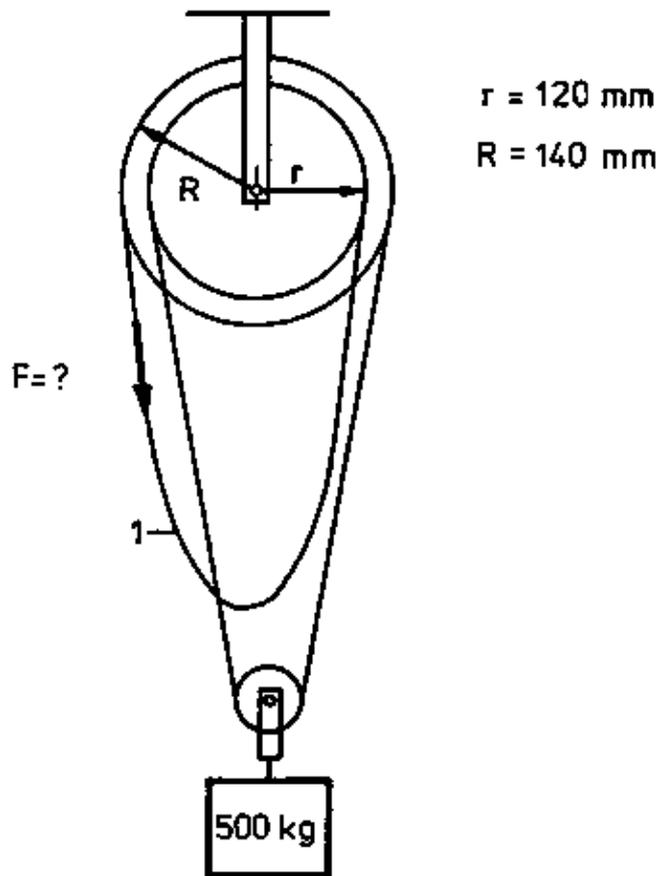


Fig. 3.18. Differential pulley block. 1 unloaded chain

Solution:

According to eq. (2.7.), the mass has the weight.

$$G = mg$$

$$G = 500 \text{ kg} \cdot 10 \text{ m/s}^2$$

$$G = 5.000 \text{ N.}$$

The calculation was performed with $g = 10 \text{ m/s}^2$.

This approximation is sufficient.

According to eq. (3.5), equilibrium exists at the differential pulley block if

$$F = \frac{G}{2} \cdot \frac{R-r}{R}$$

$$F \approx \frac{5,000 \text{ N}}{2} \cdot \frac{140 \text{ mm} - 120 \text{ mm}}{140 \text{ mm}}$$

$$F = 357 \text{ N}$$

Result:

A force of $F = 357 \text{ N}$ has to be employed for lifting a mass of 500 kg by means of the represented differential pulley block.

With this calculation, the occurring friction was neglected. Concerning this, see also para' 4.2.4. Efficiency.

3.3. Gearings

3.3.1. Construction and tasks of simple gearings

The most important motion of all machines is the rotary motion. All electric motors and internal-combustions engines deliver a rotary motion. This rotary motion of the motors must be changed in the speed, i.e. the number of revolutions per minute or second, and sometimes in the direction of rotation, to achieve the special tasks of the respective machine.

Thus the speed at the drilling machine or lathe has to be adapted to the respective work to be performed. These changes of the rotary motion are effected by means of gearings.

The most simple gearings consist of two belt pulleys each (fig. 3.19) or of two toothed gears (fig. 3.20).

These simple gearings are called belt drive and toothed-gear drive. To describe the operating mode of toothed-gear drives and belt drives it is not sufficient to state the number of revolutions only. The power transmission is of special interest, too.

The acting forces have a different distance from the axis of rotation. With the lever it was explained that the action of a force highly depends on the position to a pivot.

The product of force and vertical distance from rotary axis is termed torque.

$$\text{Torque} = \text{force} \cdot \text{vertical distance from rotary axis} \quad (3.6.)$$

The unit of the torque is newtonmeter ($\text{N} \cdot \text{m}$).

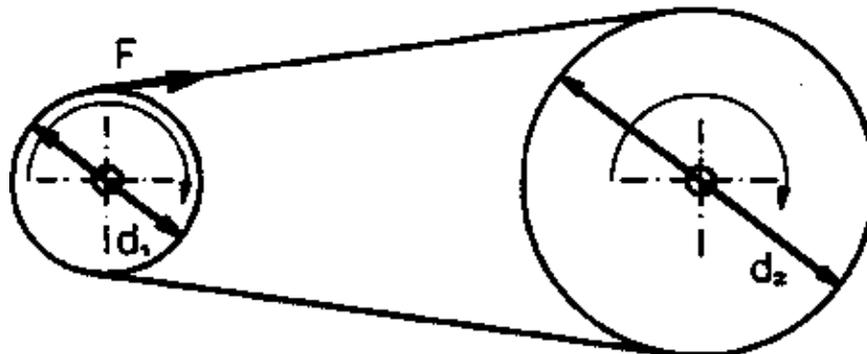


Fig. 3.19. Torque at the belt pulley

Fig. 3.19. shows the application of force at the belt pulley. The force acts at the distance $d/2$ from the rotary axis.

$$M = F \cdot \frac{d}{2} \quad (3.7.)$$

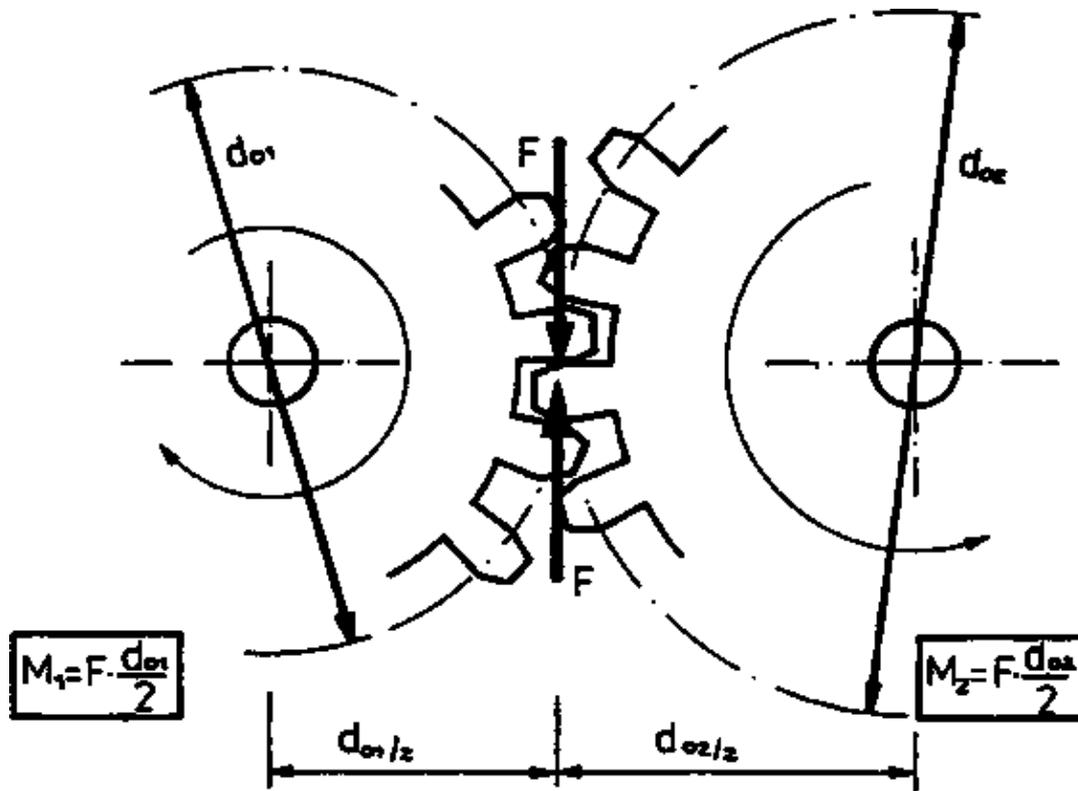


Fig. 3.20. Transfer of force between two spur gears. The representation of the form of teeth is simplified

In fig. 3.20, the application of force at a spur gear is represented. The teeth of the gears touch each other successively. The distance of the touching points from the axes of rotation is always equal. If the touching points are marked by a colour spot, they form a circle at rotation. The diameter of this circle is termed d_0 in machine construction. From this follows

$$M = F \cdot \frac{d_0}{2}$$

3.3.2. Belt drive

The belt drive consists of two belt pulleys of unequally large diameters (fig. 3.21.).

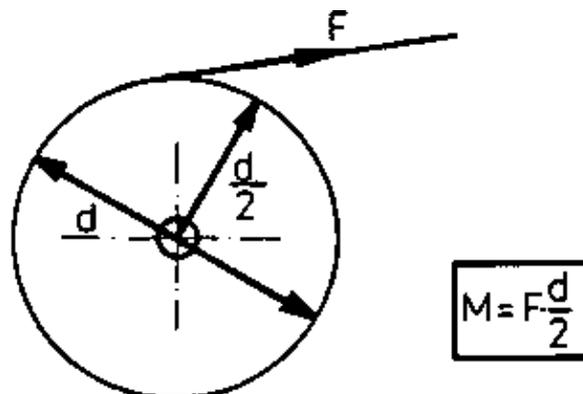


Fig. 3.21. Transfer of force at belt drive

The flat belt or vee belt encloses both belt pulleys. By the friction (see para' 4.1.3.) between belt and belt pulley, the force F is transmitted from the driving belt pulley to the belt, and at the driven belt pulley from the belt to the right belt pulley.

The speed of the belt equals the periphery speed of the belt pulleys. Hence, the speed at the periphery of the two belt pulleys is equal.

According to eq. (2.2) follows for the peripheral speed

$$v_1 = ? d_1 n_1 \text{ (driving belt pulley 1).}$$

$$v_2 = ? d_2 n_2 \text{ (driven belt pulley 2).}$$

From that, the ratio of the number of revolutions can be determined

$$v_1 = v_2$$

$$? \cdot d_1 \cdot n_1 = ? \cdot d_2 \cdot n_2$$

$$d_1 \cdot n_1 = d_2 \cdot n_2$$

$$i = \frac{n_1}{n_2} = \frac{d_2}{d_1} \quad (3.8.)$$

The ratio of the numbers of revolution is called transmission ratio.

The belt transmits the force without changing its quantity. The same force acts at the periphery of belt pulley 1 as at the periphery of belt pulley 2.

The following torques are applicable

$$M_1 = F \frac{d_1}{2} \text{ (driving belt pulley 1)}$$

$$M_2 = F \frac{d_2}{2} \text{ (driven belt pulley 2).}$$

The ratio of torques runs

$$\frac{M_1}{M_2} = \frac{F \frac{d_1}{2}}{F \frac{d_2}{2}}$$

$$\frac{M_1}{M_2} = \frac{d_1}{d_2} \quad (3.9.)$$

Example

A belt drive has the following values:

driving belt pulley 1
diameter 100 mm
no of revolutions 150 min^{-1}
torque $20 \text{ N} \cdot \text{m}$
driven belt pulley 2
diameter 400 mm

What is the number of revolutions of the driven belt pulley 2?

What is the torque at the driven belt pulley 2?

What is the transmission ratio?

Given:

diameter $d_1 = 100 \text{ mm}$

diameter $d_2 = 400 \text{ mm}$
 no of revolutions $n_1 = 150 \text{ min}^{-1}$
 torque $M_1 = 20 \text{ N} \cdot \text{m}$

Required:

no of revolutions n_2
 torque M_2
 transmission ratio i

Solution:

According to eq. (3.8.), for the ratio of numbers of revolution applies:

$$\frac{n_1}{n_2} = \frac{d_2}{d_1}$$

$$n_2 = n_1 \frac{d_1}{d_2}$$

$$n_2 = 150 \frac{1}{\text{min}} \cdot \frac{100\text{mm}}{400\text{mm}}$$

$$n_2 = 37.5 \text{ min}^{-1}$$

According to eq. (3.8), the torque M_2 amounts to:

$$\frac{M_1}{M_2} = \frac{d_1}{d_2}$$

$$M_2 = \frac{d_2}{d_1} M_1$$

$$M_2 = \frac{400\text{mm}}{100\text{mm}} \cdot 20\text{N} \cdot \text{m}$$

$$M_2 = 80 \text{ N} \cdot \text{m}$$

The transmission ratio is calculated according to eq. (3.7.):

$$i = \frac{n_1}{n_2}$$

$$i = \frac{150 \text{ min}^{-1}}{37.5 \text{ min}^{-1}}$$

$$i = 4$$

$$i = \frac{d_2}{d_1} = \frac{400\text{mm}}{100\text{mm}} = 4$$

Proof:

Result:

The transmission ratio of the examined belt drive amounts to $i = 4$.

The driven belt pulley 2 rotates 37.5 times a minute. At the driven belt pulley 2. a torque acts of 80 Nm

For a belt drive according to fig. 3.19., generally applies:

	driving belt pulley 1	driven belt pulley 2
--	-----------------------	----------------------

diameter d	small	large
no of revolutions	high	low
direction of rotation	unidirectional	
torque	small	large

3.3.3. Toothed gear drive

Fig. 3.20. shows two spur gears. The transmission of force is effected by meshing of teeth.

The peripheral speed is equal with the two toothed gears. The direction of rotation is directed inverseley.

$$v_1 = v_2$$

According to eq. (2.2)

$$v_1 = ? d_{01} n_1 \text{ (driving toothed gear 1)}$$

$$v_2 = ? d_{02} n_2 \text{ (driven toothed gear 2)}$$

$$? \cdot d_{01} \cdot n_1 = ? \cdot d_{02} \cdot n_2$$

$$d_{01} \cdot n_1 = d_{02} \cdot n_2$$

Transmission ratio i

$$i = \frac{n_1}{n_2} = \frac{d_{02}}{d_{01}} \quad (3.10.)$$

The force F is equal with both toothed gears, too. From that follow, according to eq. (3.6.), the torques

$$M_1 = \frac{d_{01}}{2} F \quad \text{(driving toothed gear 1)}$$

$$M_2 = \frac{d_{02}}{2} F \quad \text{(driven toothed gear 2)}$$

$$\frac{M_1}{M_2} = \frac{\frac{d_{01}}{2}}{\frac{d_{02}}{2}}$$

$$\frac{M_1}{M_2} = \frac{d_{01}}{d_{02}} \quad (3.11.)$$

Example

A toothed-gear drive has the following values:

driving toothed gear 1 diameter 200 mm

no of revolutions 300 min⁻¹

torque 150 n • m

driven toothed gear 2

diameter 400 mm

What is the number of revolutions per minute with the driven toothed gear 2?

What is the torque at the driven toothed gear 2?

What is the transmission ratio i?

Given:

diameter d_0 $d_{01} = 200 \text{ mm}$
 $d_{02} = 400 \text{ mm}$
 no of revolutions $n_1 = 300 \text{ min}^{-1}$
 torque $M_1 = 150 \text{ N} \cdot \text{m}$

Required:

no of revolutions n_2
 torque M_2
 transmission ratio i

Solution:

According to eq. (3.10), for the ratio of the number of revolutions applies:

$$\frac{n_1}{n_2} = \frac{d_{02}}{d_{01}}$$

$$n_2 = n_1 \frac{d_{01}}{d_{02}}$$

$$n_2 = 300 \frac{1}{\text{min}} \cdot \frac{200\text{mm}}{400\text{mm}}$$

$$n_2 = 150 \text{ min}^{-1}$$

According to eq. (3.11), for the ratio of torques applies:

$$\frac{M_1}{M_2} = \frac{d_{01}}{d_{02}}$$

$$M_2 = \frac{d_{02}}{d_{01}} M_1$$

$$M_2 = \frac{400\text{mm}}{200\text{mm}} \cdot 150\text{N} \cdot \text{m}$$

$$M_2 = 300 \text{ Nm}$$

The transmission ratio is calculated according to eq. (3.10):

$$i = \frac{d_{02}}{d_{01}}$$

$$i = \frac{400\text{mm}}{200\text{mm}}$$

$$i = 2$$

Result: the transmission ratio of the observed toothed-gear drive amounts to $i = 2$.

The driven toothed gear 2 rotates 150 times a minute. At the driven toothed gear 2, a torque acts of 300 Nm

For a toothed-gear drive according to fig. 3.20. generally applies:

	driving toothed gear 1	driven toothed gear 2
diameter d_0	small	large

no of revolutions	high	low
direction of rotation	in opposite directions	
torque	small	large

3.4. Tasks and questions

3.1. In fig. 3.1., a crowbar was shown on lifting a heavy chest. How would the bar have to be applied, and how would the forces have to act to make the crowbar act as one-sided lever?

3.2. Why is it favourable to seize a pair of tongs at the extreme end?

3.3. Name five tools, at the use of which levers are acting.

3.4. Set up the formula for the equilibrium of forces

- with the safety valve (fig. 3.10.),
- with pincers (fig. 3.8.) and
- with the nil-lifter (fig. 3.9.).

Use the designations given in the illustrations.

3.5. Name five examples from production where pulleys are employed.

3.6. Name one example from production for the application of a factorial pulley block.

3.7. A differential pulley block (fig. 3.18) is to lift a mass of 600 kg. At the winch, $r = 150$ mm and $R = 170$ mm. By means of what force can the mass of 600 kg be lifted?

3.8. By what measures can the carrying capacity of the factorial pulley block, represented in fig. 3.15. or fig. 3.17., be increased?

3.9. In para' 3.2.5., the forces were calculated required by different pulley blocks for lifting masses of different quantities. Complete the following table:

	loose pulley	factorial pulley block	differential pulley block
lifted mass			
required force			

Compare the masses and forces to one another. Name possible fields of application for loose pulley, factorial pulley block and differential pulley block.

3.10. A spur gear of $d_0 = 160$ mm transmits a torque of $M = 40$ N • m

What is the force at the sides of the meshing gear teeth?

3.11. A belt drive has the following characteristic values:

$$d_1 = 180 \text{ mm}$$

$$d_2 = 90 \text{ mm}$$

$$n_1 = 600 \text{ min}^{-1}$$

$$M_1 = 36 \text{ N} \cdot \text{m}$$

Calculate the number of revolutions n_2 , torque M_2 and transmission ratio i .

4. Work, energy, power

4.1. Work

4.1.1. Calculation of mechanical work

In production, work is performed in manifold forms:

- workpieces are machined at machine tools,
- workpieces are transported,
- workpieces are assembled.

But also with quality control and with packing, work has to be performed.

Work requires force of men and machines. But it requires patience, diligence, knowledge and skill, too.

Mechanics examines but part of these processes. It examines only those operations, for the performance of which forces are required.

In mechanics applies

work = force • path

$$W = Fs \quad (4.1.)$$

This equation only applies if direction of force and direction of path coincide.

As unit of mechanical work, newtonmeter (Nm) comes to exist.

To explain the difference from the unit of torque, the unit of work receives a name of its own.

$$\begin{aligned} 1 \text{ joule} &= 1 \text{ newtonmeter} \\ 1 \text{ J} &= 1 \text{ Nm} \end{aligned}$$

The unit of work is called joule. Thereby the physicist James Prescott Joule (1818 to 1889) is remembered.

The equation $W = Fs$ is to be applied to a simple process from production.

Chests of transport are filled with metal parts. They have a mass of 10 kg, 15 kg and 20 kg. They are to be lifted by 800 mm or 1,000 mm. Fig. 4.1. shows the different heights.

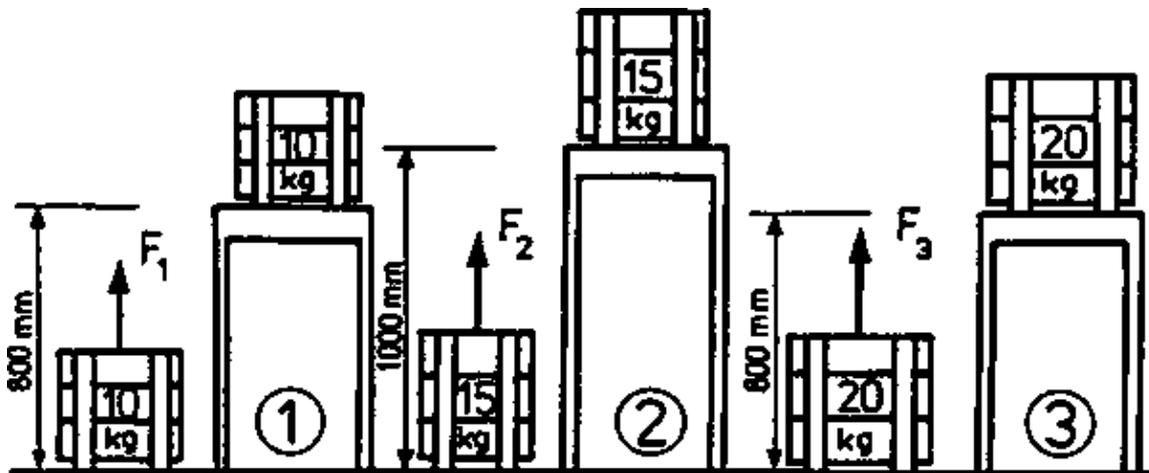


Fig. 4.1. Chests are lifted having a mass of 10 kg, 15 kg and 20 kg, respectively. In which case is the largest work performed?

In which case is the largest mechanical work performed? Does it require more mechanical work to lift 15 kg on to a table 1,000 mm high, or is more work to be employed to place the chest of 20 kg on the table 800 mm in height?

The mass of 10 kg has a weight $G = 100 \text{ N}$.

At lifting, force F_1 must overcome the weight.

The mechanical work with lifting is

$$\begin{aligned}W &= Fs \\W &= 100 \text{ N} \cdot 0,8 \text{ m} \\W &= 80 \text{ Nm} = 80 \text{ J}\end{aligned}$$

In the following survey, the individual working processes are compared.

Process	Mass	Force	Path	Work = force • path
1	10 kg	100 N	0.8 m	$100 \text{ N} \cdot 0.8 \text{ m} = 80 \text{ N} \cdot \text{m} = 80 \text{ J}$
2	15 kg	150 N	1.0 m	$150 \text{ N} \cdot 1.0 \text{ m} = 150 \text{ N} \cdot \text{m} = 150 \text{ J}$
3	20 kg	200 N	0.8 m	$200 \text{ N} \cdot 0.8 \text{ m} = 160 \text{ N} \cdot \text{m} = 160 \text{ J}$

The largest work is performed in case 3.

4.1.2. Golden rule of mechanics

In para' 3. Simple force–converting devices it was explained how forces can be saved by the aid of lever, pulley and rope.

When using these force–saving devices, is there work saved, too?

In order to answer this question, the work at the loose pulley is to be examined.

Fig. 4.2. shows a loose pulley lifting a mass. According to eq. (3.3.), $F = G/2$ is applicable. Force is saved. At the same time, the smaller force has to cover a longer distance.

Mass m is lifted by distance s_m ; the force must act along a path of double the length; $s_F = 2 s_m$.

The golden rule of mechanics is applicable:

What is saved in force, must be added in path.

The amount of work performed does not change. A smaller force has to act over a longer distance.

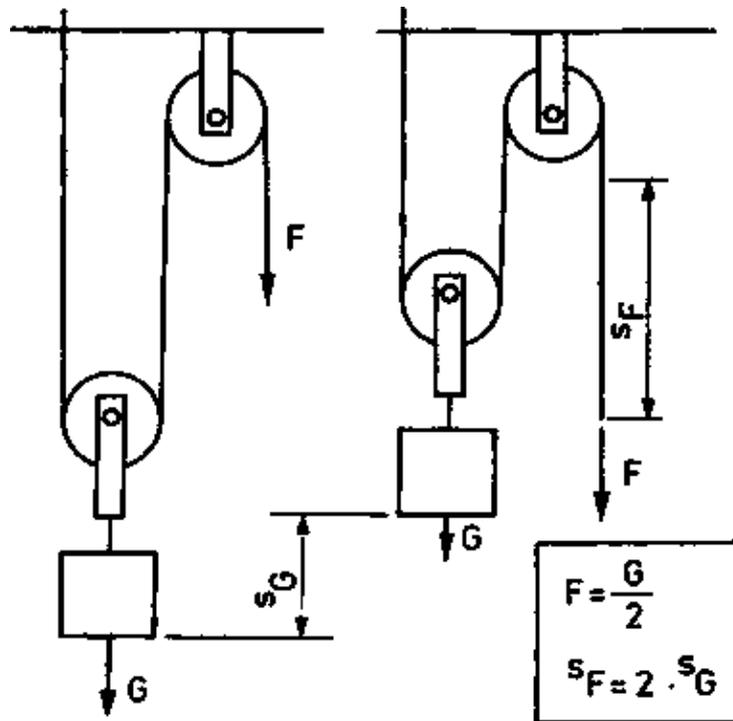


Fig. 4.2. The loose pulley saves force, but no work

With the working process represented in fig. 4.2., the work W_G is performed on lifting mass m of weight G .

$$W_G = G \cdot s_m.$$

The force acts along the distance s_F . The work W_F is performed.

$$W_F = F \cdot s_F.$$

The fixed pulley only deviates force F . At the loose pulley applies

$$F = G/2.$$

At lifting $s_F = 2 s_m$. From that follows for work W_F

$$W_F = \frac{G}{2} \cdot 2 \cdot s_m$$

$$W_F = G \cdot s_m.$$

$$\text{As } W_G = G \cdot s_m,$$

$$W_F = W_G.$$

The golden rule of mechanics applies to all force-converting devices, such as lever, pulley, pulley block.

By the aid of lever, pulley, pulley block, large effects can be attained with small forces, but no work can be saved.

4.1.3. Work of friction

When lifting a mass (fig. 4.1. and fig. 4.2.), work has to be performed. Is mechanical work required, too, if a mass is to be shifted horizontally?

Our experience says that mechanical work must be performed if a heavy transfer container is to be shifted aside a small distance. The container is pushed over the floor, and moves horizontally. A great resistance against the intended motion occurs. This resistance can be reduced if the transfer container is moved on rolls or driven on a wagon. But even then a resistance remains, the overcoming of which requires mechanical work,. What resistance is it that acts here?

All solid bodies have a rough surface. Sometimes, the roughness is so fine that it can be recognized under a microscope only. The rough surface comes to exist with the machining of materials, by corrosion (e.g. oxidizing), and by wear.

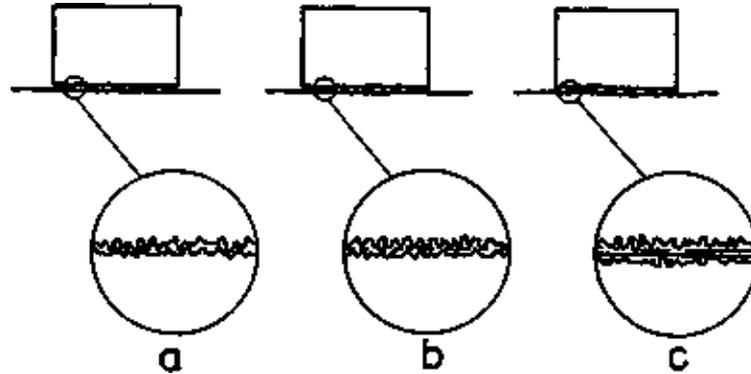


Fig. 4.3. a coulomb friction, b mixed friction and c floating friction.

Fig. 4.3. shows that the rough surfaces get indented and hooked. Thereby the bodies are difficult to displace against each other (coulomb friction). By means of a lubricant, the bodies are separated completely (floating friction) or partly (mixed friction) from each other. Under these conditions, the bodies can be displaced against each other by smaller forces.

Coulomb friction is very rare as traces of oil exist on each workpiece.

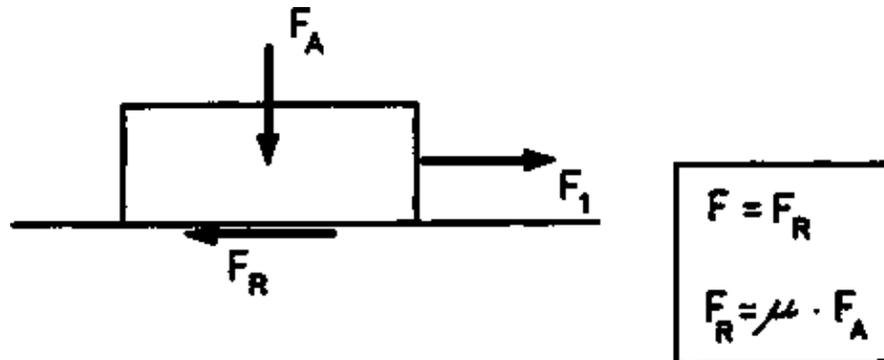


Fig. 4.4. Forces with sliding friction

In fig. 4.4., the forces are represented, which occur with the motion of a plane body over a flat surface. The body moves at uniform speed. The body is pressed on the base by contact force F_A . In the represented example, the contact force equals the weight.

The motion is caused by force F_1 . In the contact area, a resistance is formed: the friction force F_R . Lest the speed is not decreased, it is necessary that $F_1 = F_R$.

By experiments it was determined:

The friction force F_R is dependent

- on the materials of base and body,
- on the kind of the available lubricant,
- on the thickness of the lubricating film,
- on the amount of contact force F_A ,
- on the state of motion.

In the state of rest, the resistance of motion is largest. The friction force is not dependent on the size of the contact area.

The friction law was established by C.A. de Coulomb.

$$F_R = F_A \cdot \mu \text{ (sliding friction)} \quad (4.2.)$$

$$F_R = F_A \cdot \mu_0 \text{ (static friction)}$$

F_R = friction force

F_A = contact force

μ = friction coefficient of sliding motion

μ_0 = friction coefficient of rest

By experiments, the friction coefficients compiled in table 4.1., were ascertained. Table 4.1. shows:

Table 4.1.

Coefficients of friction

Materials	Coefficients of static friction μ_0		Coefficients of sliding friction μ	
	dry	lubricated	dry	lubricated
Steel on steel	0.15 ... 0.59	0.1	0.1	0.01 ... 0.05
Steel on bronze	0.2	–	0.18	0.07
Steel on grey cast iron	0.2	0.1	0.18	0.01
Wood on stone	0.7	0.4	0.3	–
Wood on wood	0.4 ... 0.65	0.16	0.2 ... 0.4	0.08
Leather belt on grey cast iron	0.55	0.22	0.28	0.12
Leather packing on metal	0.6	0.2	0.2 ... 0.25	0.12
Metal on wood	0.55 ... 0.60	0.1	0.35 ... 0.5	0.05

Friction in high-tensile screwed connections:

Steel on steel St. 38 0.45

Steel on steel St. 52 0.60

1. By lubrication, the friction force can be reduced considerably ($\mu_{\text{lubricated}} < \mu_{\text{dry}}$).
2. The friction force of motion is smaller than the friction force of rest ($\mu < \mu_0$).

A further reduction of friction forces can be attained by the use of rolls. when both bodies are separated from each other by rolls, it is easier to displace them against each other. This is made use of with all ball and roller bearings.

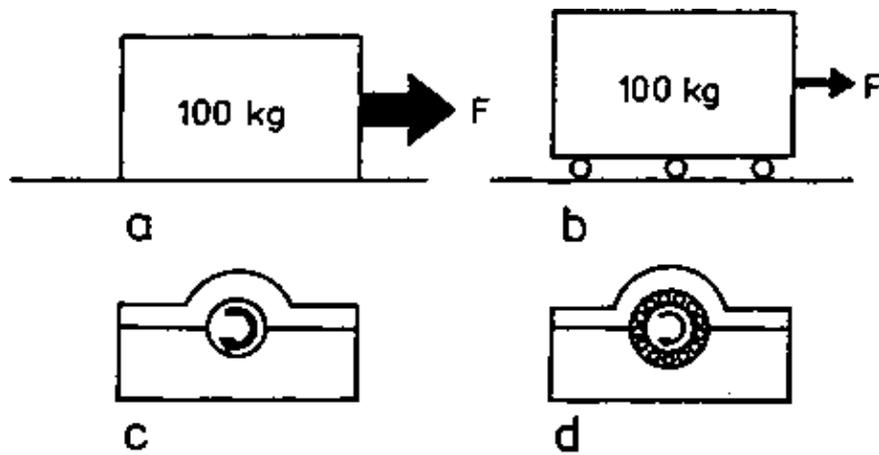


Fig. 4.5. By rolling, the resistance of motion can be reduced.

- a: mass glides on plane surface;
- b: mass rolls on plane surface;
- c: axle slides in a bearing;
- d: axle moves in a roller bearing

Fig. 4.5. shows how the resistance can be reduced. Rolls and wheels decrease the resistance with the motion over plane surfaces.

In the bearing, the axle or shaft also has to overcome a resistance of motion: the bearing friction.

The bearing friction decreases considerably if the axle or shaft moves on balls or rollers in the bearing. To this end, high-quality bearings contain balls or rollers.

The work of friction can be determined according to eq. (4.1.).

$$W = F s$$

With motion, the friction force has to be overcome. To attain this, a work has to be performed.

$$W_R = F_R s$$

$$W_R = F_A \mu s \quad (4.3.)$$

W_R : work of friction

F .: contact force

F_R : frictional force

μ : friction coefficient

s : distance covered

4.1.4. Forms of mechanical work

In the previous paragraphs it was pointed out that for lifting a mass, a work is required (lifting work).

By the work of friction, the friction is overcome. The acceleration of a mass also requires an acting force ($F = ma$, see para' 2.3.3.). With this, the work of acceleration is performed.

The elastic or plastic deformation of a body takes place under the influence of forces, see para' 2.3.2. A work of deformation has to be performed. The amounts of work can be calculated as follows:

Form of mechanical work	Acting force	Performed work
Lifting work	$F = G$	$W = Gs$

		with $s = h$
		$G = mg$
		$W = mgh$
Work of friction	$F_R = \mu F_A$	$W = \mu F_A s$
Work of acceleration	$F = ma$	$W = mas$

W: work μ : friction coefficient acc table 4.1.

G: weight m: mass

g: gravitational acceleration, $g = 9.81 \text{ m/s}^2$

h: height F_A : contact force

s: distance covered a: acceleration

The unit of work is joule.

$$1\text{J} = 1\text{Nm} = \frac{9.81\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \text{m}$$

4.2. Energy

4.2.1. Occurrence

An electric motor moves a crane, and lifts a body of large mass by the aid of pulley and rope. With this, a work is performed.

A motor lorry is driven by an internal-combustion engine. The motor performs the work, which is required for overcoming the friction and for the acceleration of the vehicle.

At forging, a heavy hammer powerfully falls on a red-hot workpiece, and causes a plastic deformation of this workpiece. With this, the hammer performs work.

In many workshop halls, a protective helmet has to be worn, as even small parts, falling down from a large height, may cause serious head injuries. A wrench, falling down from a height of 8 to 10 m, may cause serious accidents.

Energy is the ability to perform work.

Energy occurs in many forms. Table 4.2. lists the most important forms of energy.

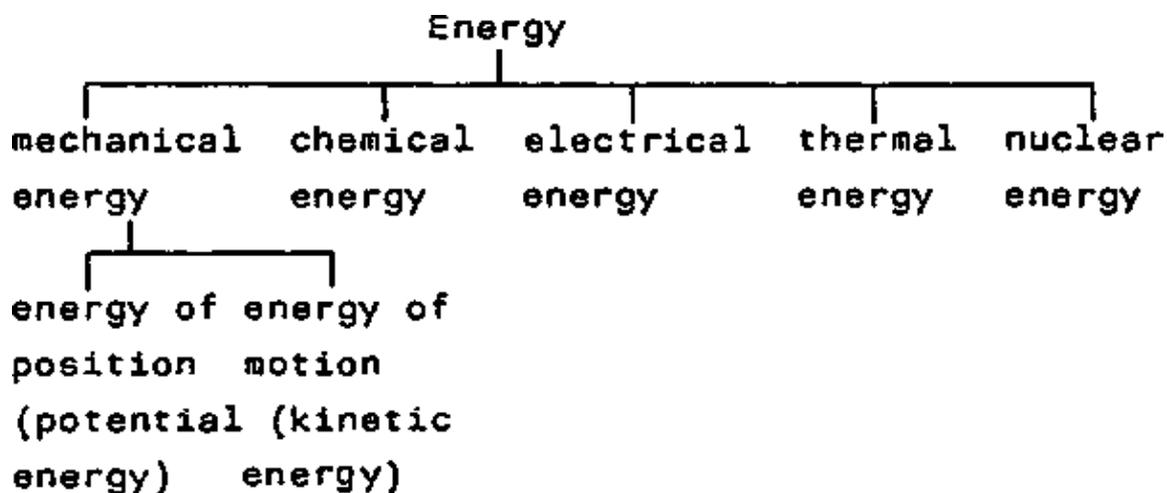


Table 4.2. Forms of energy

4.2.2. Mechanical energy

Mechanical energy occurs as energy of position (potential energy) and as energy of motion (kinetic energy).

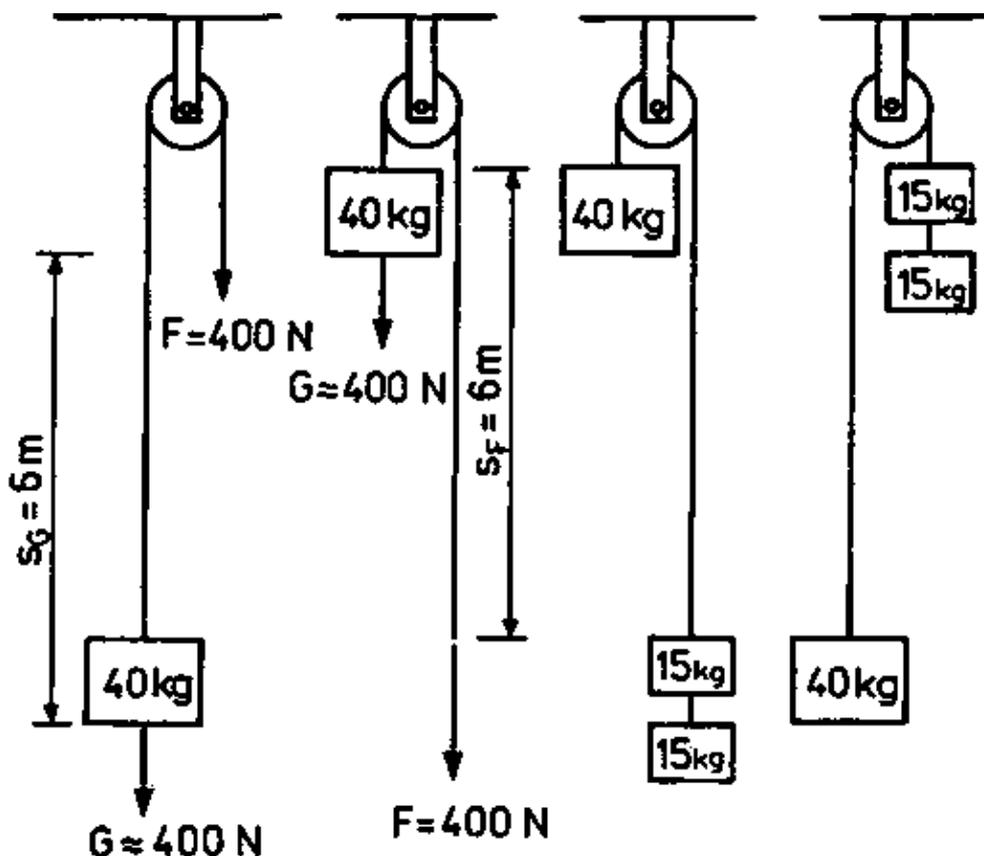


Fig. 4.6. Energy of position. If a mass loses in height, then it is able to perform work

Potential energy. Fig. 4.6. shows how a mass of 40 kg is lifted by 6 m by the aid of a fixed pulley. The mass has the weight $G \approx 400$ N. As at the fixed pulley $F = G$ is applicable, a force $F = 400$ N must act (figs. 4.6. a and b). The paths of force and mass are of equal quantity at the fixed pulley.

$$S_F = S_G$$

The performed lifting work amounts to

$$\begin{aligned} W_{Hub} &= G \cdot S_G \\ W_{Hub} &= 400 \text{ N} \cdot 6 \text{ m} \\ W_{Hub} &= 2,400 \text{ N} \cdot \text{m} \\ W_{Hub} &= 2,400 \text{ J} \end{aligned}$$

In the higher position, the mass is able to perform work. Figs. 4.6 c and d show that the mass is capable of lifting 2 smaller masses if it lowers itself.

The work required for lifting, was stored in the larger mass.

$$\begin{aligned} W_{pot} &= G \cdot h \\ W_{pot} &= m \cdot g \cdot h \end{aligned}$$

W_{pot} : energy of position
 G : weight
 m : mass
 h : difference of height at lowering
 g : gravitational acceleration

The unit of energy is joule (J), too. $1 \text{ J} = 1 \text{ Nm}$. The energy of position is also called potential energy.

If a mass loses in height, then it is able to perform work.

Kinetic energy is the energy of motion. With forging, a heavy hammer is lifted, and falls down on the red-hot workpiece from above, and performs work there by deforming the workpiece plastically. The larger the mass of the hammer, the more work it can perform. Its energy grows with the mass.

Its energy also increases with the speed, however, at which the hammer strikes the workpiece.

Accurate calculations and measurements proved that the kinetic energy amounts to

$$W_{\text{kin}} = \frac{m v^2}{2}$$

W_{kin} : kinetic energy

m: mass

v: speed

Moved masses can perform work. Thus the energy of wind and of flowing water is utilized.

Quickly moved masses may, however, also cause heavy damage.

If a quickly moved vehicle is abruptly slowed down by an impact, the high kinetic energy may destroy the vehicle. The kinetic energy increases by the square of speed.

Even very small masses may cause dangerous injuries if they are moved very quickly. At grinding, particles are parted off the grinding wheel. These particles have a very small mass, but a very high speed. Due to the high speed, they have a very high kinetic energy, and may cause dangerous eye injuries. Therefore, safety glasses have always to be worn with grinding.

A special form of kinetic energy is the energy of circular or rotary motion.

Energy of deformation. A special form of potential energy is the energy of elastic deformation. This energy is stored with the elastic deformation of a body. When loading a spring, e.g., work has to be performed. This work is stored in the spring as energy of deformation.

Use of that is made when winding up a wrist-watch. The spring of the watch is loaded at winding up – it stores a small amount of energy – and performs work in the course of the day by keeping the watch going.

4.2.3. Conversion of energy

The individual forms of mechanical energy can be converted into each other directly.

Fig. 4.7. shows a simple experiment, which permits recognizing the conversion of mechanical energy well.

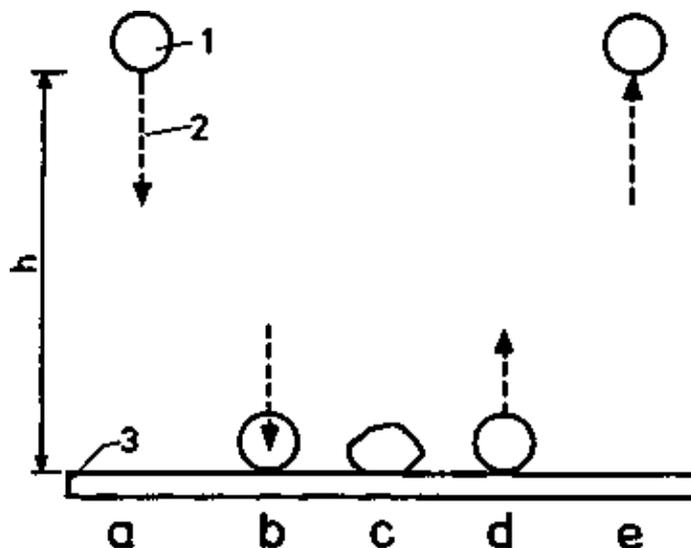


Fig. 4.7. Conversion of mechanical energy. Potential energy – kinetic energy – energy of deformation

A steel ball falls down on a steel plate from some height (4.7.a). The steel ball has the potential energy $W_{pot} = mgh$. At falling down, the ball is accelerated. It bounds against the plate at increased speed (4.7.b). Then it

$$W_{kin} = \frac{mv^2}{2}$$

has the kinetic energy. With this energy, the ball bounds against the plate and performs work there by deforming itself and the plate elastically (4.7.c). For this, the whole kinetic energy is consumed, and is now stored as energy of deformation. Plate and ball will get unloaded again and assume their original shape. The ball comes off the plate at high speed. It wins kinetic energy again (4.7.d). The motion is now directed upwards, however. The ball rises upwards vertically, and performs lifting work. Then the kinetic energy is converted into potential energy again. When the original height is reached again, the speed of the ball is reduced to zero. Its whole kinetic energy has been converted into potential energy. The ball remains in the state of rest for a short period, and falls down freely again (4.7.f). Thereby the conversion of the forms of energy anew.

State	potential energy	kinetic energy	energy of deformation
1	maximum value	zero	zero
2	zero	maximum value	zero
3	zero	zero	maximum value
4	zero	maximum value	zero
5	maximum value	zero	zero

The experiment shows:

1. The forms of mechanical energy can be converted into each other.
2. No energy gets lost.

Under these conditions, the ball should never come to rest.

After some time, the height becomes smaller, however, and finally the ball remains lying on the plate.

This is caused by friction. Frictional forces occur between ball and the air, and between ball and plate. To overcome these frictional forces, work has to be performed. To this end, energy is withdrawn from the mechanical energy.

Where does this energy remain?

It is converted into thermal energy. And thermal energy cannot be reconverted into mechanical energy immediately.

The individual forms of mechanical energy may change into one another without any further aids.

All other forms of energy (table 4.2) can be converted into one another by means of special machines only, however.

The conversion of electrical energy into mechanical energy is performed by the electric motor.

The conversion of chemical energy into mechanical energy may be performed, by way of example, by an internal combustion engine in a motor vehicle.

Principle of the conservation of energy:

Energy cannot be created anew, or get lost. With the conversion of individual forms of energy into one another, part of the energy is used for overcoming the friction. With that, thermal energy comes to exist.

Potential energy can be converted into one another immediately.
Kinetic energy
Energy of deformation

Chemical energy can be converted into mechanical energy by motors.
Electrical energy

4.2.4. Mechanical efficiency

Friction acts in all machines. By good lubrication and by roller bearings, friction can be reduced, but not eliminated. When mechanical work is performed, work of friction has always to be performed simultaneously. The performance of this work of friction requires energy.

Part of the energy supplied to a machine has to be used for performing the work of friction. This part of the energy is converted into thermal energy by friction.

Example

A crane is to perform a lifting work of 8,000 joules. For the performance of this lifting work, the machine consumes a larger energy, however.

Energy input	Effective work: lifting work	work of friction
10,000 joules	8,000 J	2,000 J

The machine operates to perform lifting work. This work is to be termed effective work.

The work of friction does not aid the actual effective work: lifting, drilling, milling etc. The work of friction has to be applied additionally. The smaller the portion of the work of friction, the more favourable is the efficiency of the machine.

$$\text{Efficiency} = \frac{\text{effective work performed}}{\text{energy input}}$$

$$\eta = \frac{W_1}{W_2} \quad (4.4.)$$

The mechanical efficiency η is always smaller than 1 as it is not possible to prevent the friction in a machine completely.

The mechanical efficiency for the above example amounts to:

$$\eta = \frac{8,000\text{J}}{10,000\text{J}} = 0.8$$

As the performance is frequently done by electric motors, the watt is applied as unit of power.

$$1 \text{ watt} = \frac{1 \text{ joule}}{1 \text{ second}}; \quad 1 \text{ W} = \frac{1 \text{ J}}{1 \text{ s}}$$

1 watt is the power, by which a work of
1 joule is performed in 1 second

The designation watt is to recall the memory of the inventor of the steam engine. James Watt (1736 to 1819).

What is the power of the two cranes mentioned?

Crane 1:

Performed work

Lifting work acc para' 4.1.4.

$$W = mgh$$

$$W = 1,000 \text{ kg} \cdot 9,81 \frac{\text{m}}{\text{s}^2} \cdot 10 \text{ m}$$

$$W = 98,000 \text{ kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot \text{m}$$

$$1 \text{ J} = 1 \text{ N} \cdot \text{m} = 9,81 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \text{m}$$

With $1 \text{ J} = 1 \text{ N} \cdot \text{m} = 9,81 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \text{m}$ becomes $W = 10,000 \text{ J}$.

The crane performs a work of 10,000 joule.

Required time $t = 20 \text{ s}$

Power acc eq. (4.5.)

$$P = \frac{W}{t}$$

$$P = \frac{10,000 \text{ J}}{20 \text{ s}}$$

$$P = 500 \frac{\text{J}}{\text{s}}$$

It applies $1 \text{ W} = \frac{\text{J}}{\text{s}}$.

Thus becomes

$$P = 500 \text{ W}$$

$$P = 0,5 \text{ kW}.$$

The power of crane 1 amounts to $P = 0,5 \text{ kW}$.

Crane 2.

Performed work

Lifting work like crane 1 $W = 10,000 \text{ J}$

Required time $t = 10 \text{ s}$

b) 50 kg is to be lifted by 2 m

c) 1 t 0 1 m?

4.2. Is it force or work that is saved at the tongs (fig. 3.8.) and the loose pulley (fig. 3.12.)?

Use the golden rule of mechanics when giving your explanation.

4.3. Name three examples from production where mechanical work is performed.

4.4. By what means can the bearing friction be reduced?

4.5. How does the efficiency of a machine change if the bearings are badly lubricated?

4.6. Name three examples from production where it is tried to reduce friction.

4.7. Name three examples from production and engineering where friction is necessary for performing the respective task.

4.8. An engine performs a work $W = 4,000$ joule in 2 seconds. What is its power? State the result in watts and kilowatts.

4.9. A crane lifts a mass of 1.5 tons by 3 metres in 5 seconds. What is its power? State the result in watts and kilowatts.

4.10 A screwdriver has a mass of 0.4 kg. Its weight amounts to $G = 4$ N. This small weight is hardly noticed in the state of rest. If the screwdriver falls down from a height of 10 metres, it may cause serious injuries. Where does this destructive effect come from?

5. Statics

5.1. Introduction

Forces are acting, "e.g. when" a workpiece is lifted, a machine stands on its foundation, a chip is removed from a workpiece by means of a lathe, a screw presses two workpiece together.

As pointed out in para' 2.3.2. already, forces are not visible. Forces can be recognized by their effects.

Effects of forces:

- acceleration of masses
- elastic deformation of solid bodies
- plastic deformation of solid bodies

Deformations are frequently so small that they can not be notice.

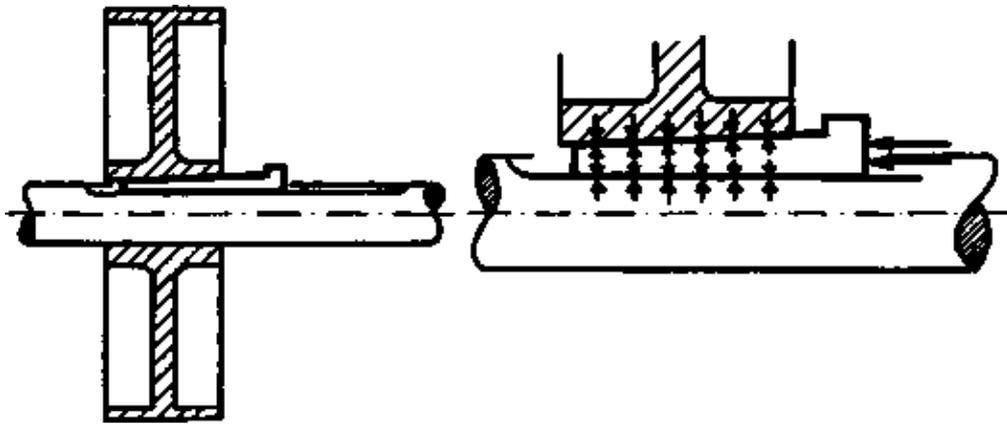


Fig. 5.1. Forces at a key.

Fig. 5.1. shows the acting forces at a key, which connects a shaft and a belt pulley with each other. The key is guided in a groove. When driving it in, the key presses against the shaft and the belt pulley, and thus braces the two machine parts firmly with each other. When the belt pulley is to be loosened from the shaft, then the key is loosened by driving it out, and shaft and belt pulley can be separated from each other again.

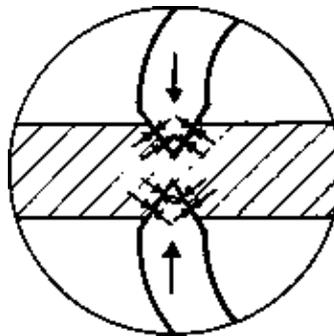


Fig. 5.2. Forces at the cutting edge of a pincers

The key transforms the driving-in forces, and thereby increases it considerably. With that, the action of this force is increased, too. Fig. 5.2. shows how the forces at a pair of pincers (see also fig. 3.8.) are transformed at the cutting edge once more. The force of hands is increased by the lever. At the edge, the wedge-shape of the edge causes another conversion of force. Thus it is possible to separate a fairly thick metal wire by mere force of hands.

When considering these examples and other processes from engineering and production, the questions arise:

- What forces occur?
- How are the forces transformed?
- When is there equilibrium between the acting forces?

These questions are dealt with in a section of technical mechanics: statics.

Statics is the theory of the equilibrium at solid bodies.

As the deformations are very small, statics neglects these deformations.

Statics examines rigid, i.e. not deformable bodies.

5.2.1. Graphic representation with scale of forces

In order to enable the description of a force, statements have to be made regarding

- its quantity,
- its line of action,
- its direction and
- its point of application.

In order to render a better imagination of acting forces possible, these are represented graphically. Fig. 5.3. shows the graphic representation of the line of action, direction and points of application of a force. This representation is applied if the quantity of a force is not known yet, or if only the position of forces to each other is to be shown.

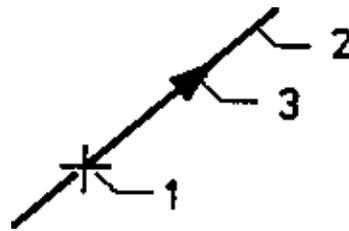


Fig. 5.3. Representation of a force in the layout plan: 1 point of application, 2 line of action, 3 direction

In order to render a graphic representation of the quantity of a force possible, too, a scale of forces has to be selected.

Examples for scales of forces:

- 1 cm $\hat{=}$ 10 N
 - 1 cm $\hat{=}$ 250 N
 - 1 cm $\hat{=}$ 2 kN
 - 1 cm $\hat{=}$ 5 MN
- (" $\hat{=}$ " read: "corresponds to").

By means of the scale of forces, the quantity of force can then be represented as length.

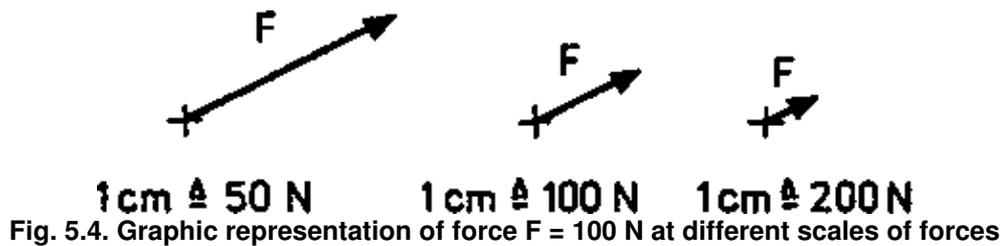


Fig. 5.4. Graphic representation of force F = 100 N at different scales of forces

Fig. 5.4. shows how the quantity of force F = 100 N can be represented at different scales of forces.

5.3. Composition of forces

5.3.1. Resulting force

In engineering and production, several forces frequently act simultaneously. To recognize the common action of these forces, they are comprised to a resulting force R. The resulting force is often briefly called resultant.

The resultant force F_{Res} has the same effect as the initial forces.

5.3.2. Forces of equal direction of action

Fig. 5.5. shows how a locomotive pulls a goods train. It develops a tractive force F_1 . If the goods train is too heavy or a steep gradient has to be overcome at a mountain, the train is some times pulled by two locomotives.

Which force is now acting on the goods train?

Locomotive 1 pulls with force F_1 .

Locomotive 2 pulls with force F_2 .

Both forces have the same line of action.

Both forces have the same direction.

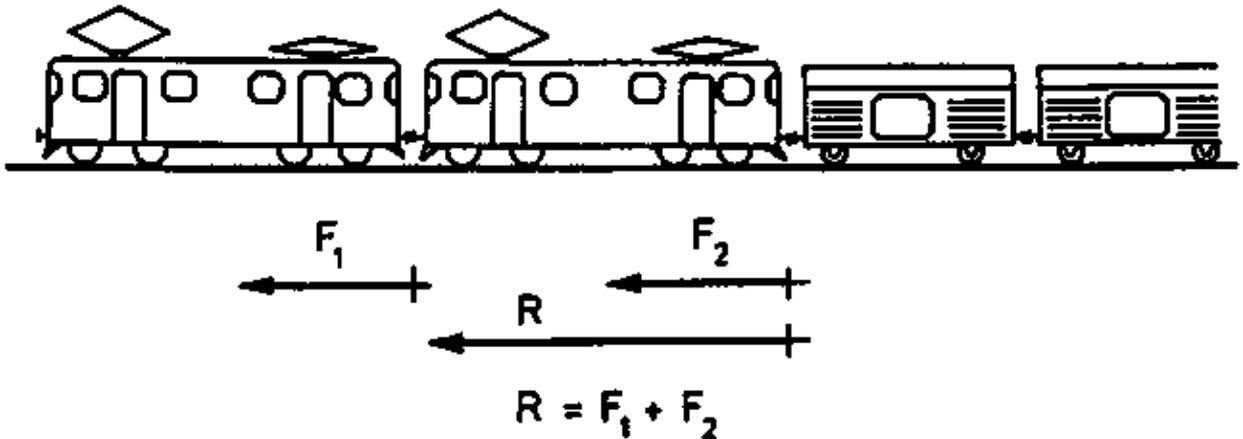


Fig. 5.5. Forces of equal direction can be added directly: $F_{Res} = F_1 + F_2$; ($R = F_{Res}$)

Our experience says that under these circumstances the forces can be added directly.

In fig. 5.6., the resulting force F_{Res} then becomes

$$F_{Res} = F_1 + F_2$$

As F_{Res} has the same effect like F_1 and F_2 together, F_{Res} can replace the forces F_1 and F_2 .

Further calculations may proceed from the fact that only F_{Res} is acting.

Unidirected forces, which lie on a common line of action, can be added immediately.

5.3.3. Forces of opposite line of action

The common action of two forces of equal line of action, but of opposite direction, can be recognized very well at a sporting competition.

At a sporting event, two teams of apprentices measure their strength in tug-of-war. Fig. 5.6. shows the situation.

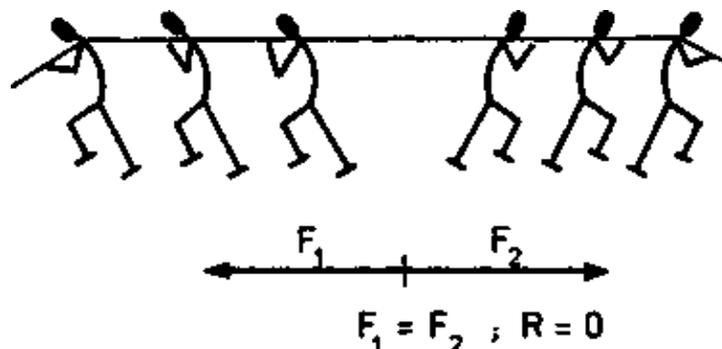


Fig. 5.6. Two teams are pulling at a rope Opposed forces in equilibrium

The team on the left pull to the left with all their might, and the team on the right tries to pull the rope to the right.

The rope transmits the forces without changing them. The line of action of the force proceeds in the rope.

Although both teams make every effort, the rope does not move. The forces neutralize each other. Their overall effect equals zero.

Only when one team attains a somewhat larger force, the rope will move to this side.

Opposed forces, proceeding on one line of action, can be combined to one resulting force (fig. 5.7.). To this end, the forces are plotted successively beside the layout plan in the scale of forces. The second force is marked in at the peak of the first force in optional sequence, fig. 5.7. representation on the right. The connection from the starting point of the first force to the arrow-head of the latter force is then the resulting force F_{Res}

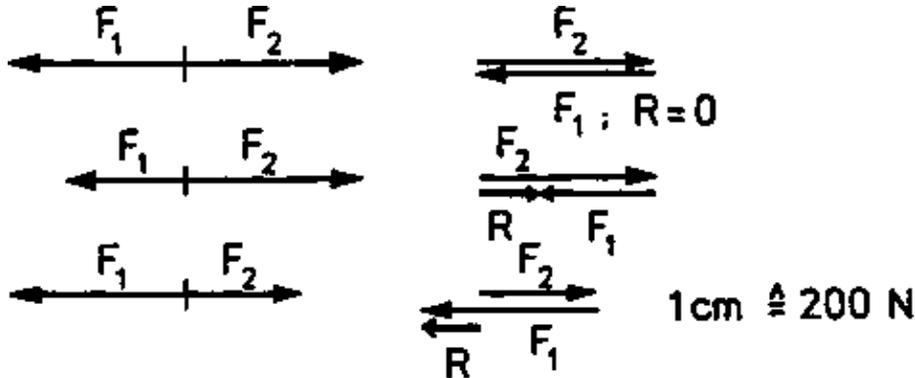


Fig. 5.7. Opposed forces. On the left, layout plan, on the right, graphic representation for ascertaining the resultant

Fig. 5.7. shows three possible cases. If the forces concerned are equal, they neutralize each other, and the resultant will amount to zero ($F_{Res} = 0$). The forces are in equilibrium. There is no outer action.

Fig. 5.7. centre, shows that force F_2 – directed to the right – is larger. The resultant R is then directed to the right, too.

Fig. 5.7. below shows two forces, the one directed to the left being larger. In this case, the resulting force is directed to the left, too.

The resulting force F_{Res} fully replaces the initial forces. F_{Res} has the same effect as F_1 and F_2 together. Thus, R can replace F_1 and F_2 . With further calculations it can be proceeded from the fact that only F_{Res} is effective.

5.3.4. Composition of forces of different lines of action

5.3.4.1. Determination of the resultant of two forces

In the previous paragraphs, forces were examined which lie on one line of action. Frequently, the forces have different lines of action, however. Then the lines of action cut each other in an angle (fig. 5.8.).

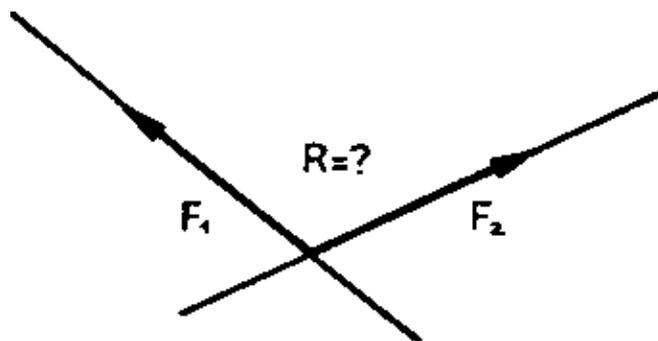


Fig. 5.8. How large is the resulting force? Which direction has the resulting force? ($F_{Res} = R$)

How can the combined action of two forces be determined, which have a common point of application, but different lines of action?

An experiment is to explain the graphic solution. A thin rope (thick thread) is guided over two fixed pulleys (fig. 5.9., above).

In para' 3.2. it was determined that

- a rope only transmits forces, and
- a fixed roller can reverse forces by means of a rope. Rope and fixed roller do not change the quantity of the force.

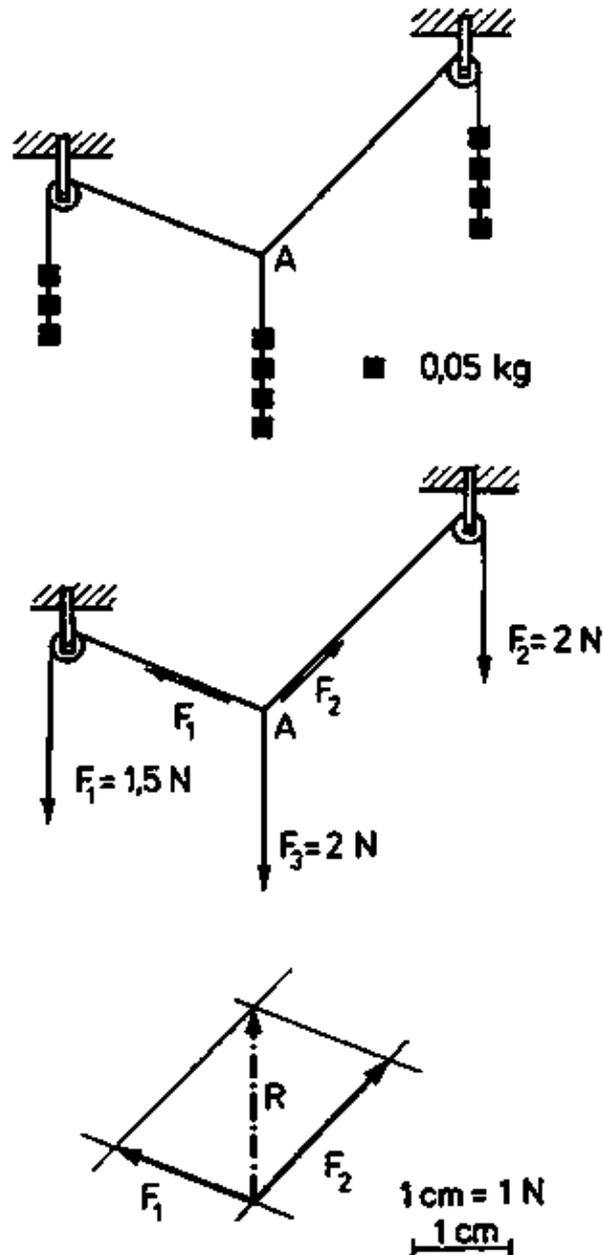


Fig. 5.9. Experiment for ascertaining the resultant with forces having different lines of action

Now, the following mass pieces (weights) are fastened to the rope:

- left rope end: 3 mass pieces of 0.05 kg each
- right rope end: 4 mass pieces of 0.05 kg each
- point A of rope: 4 mass pieces of 0.05 kg each

With this load, the rope assumes the position represented in fig. 5.9. from any different position. The forces are in equilibrium in this position only. Which forces act on point A?

Each mass piece has a weight of appr. 0.5 N. Consequently, 1.5 N act at the left rope end, and 2 N at the right rope end. The forces F_1 and F_2 are transmitted by pulley and rope and reversed, and thus also act on point A.

Fig. 5.9., centre, shows the application of force in point A. The forces acting on point A are

$$\begin{aligned} F_1 &= 1.5 \text{ N} \\ F_2 &= 2 \text{ N} \\ F_3 &= 2 \text{ N} \end{aligned}$$

In this position and with these lines of action, the forces F_1 and F_2 can carry the weight F_3 . Their resultant must have the line of action and the quantity of F_3 but be directed opposite to F_3 . This resultant of F_1 and F_2 is obtained by representing the forces F_1 and F_2 graphically, and by complementing this representation to a parallelogram. Fig. 5.9. below, shows the structure of a parallelogram.

A parallelogram is a quadrangle, the opposite sides of which are equally long and parallel.

The connecting line from the intersection point of forces F_1 and F_2 to the opposite corner is then the resulting force F_{Res}

From the parallelogram of forces, there can be measured out:

$$F_{\text{Res}} \hat{=} 2 \text{ cm}$$

From that follows after conversion, by the aid of the scale of forces:

$$F_{\text{Res}} = 2 \text{ N}$$

Thus, quantity, direction and line of action of the resulting force are established.

Force F_{Res} is the resulting force. It can fully replace the initial forces F_1 and F_2 . It has the same effect as F_1 and F_2 together.

This statement complies with the represented experiment. For carrying the mass pieces in point A ($F_3 = 2 \text{ N}$), a force is required, which also has the quantity 2 N, acts on point A, has the same line of action like F_3 (vertical), and is directed opposite to F_3 .

These properties has force R in the parallelogram of forces.

Principle of the parallelogram of forces

If two forces F_1 and F_2 of different lines of action act on one point, the resulting force is obtained by representing the forces F_1 and F_2 graphically by means of a scale of forces. The graphic representation is then complemented to a parallelogram. The diagonal, beginning in the starting point of forces F_1 and F_2 is then the resulting force F_{Res} . The resulting force F_{Res} has the same effect like the initial forces F_1 and F_2 .

Hence, F can replace F_1 and F_2 .

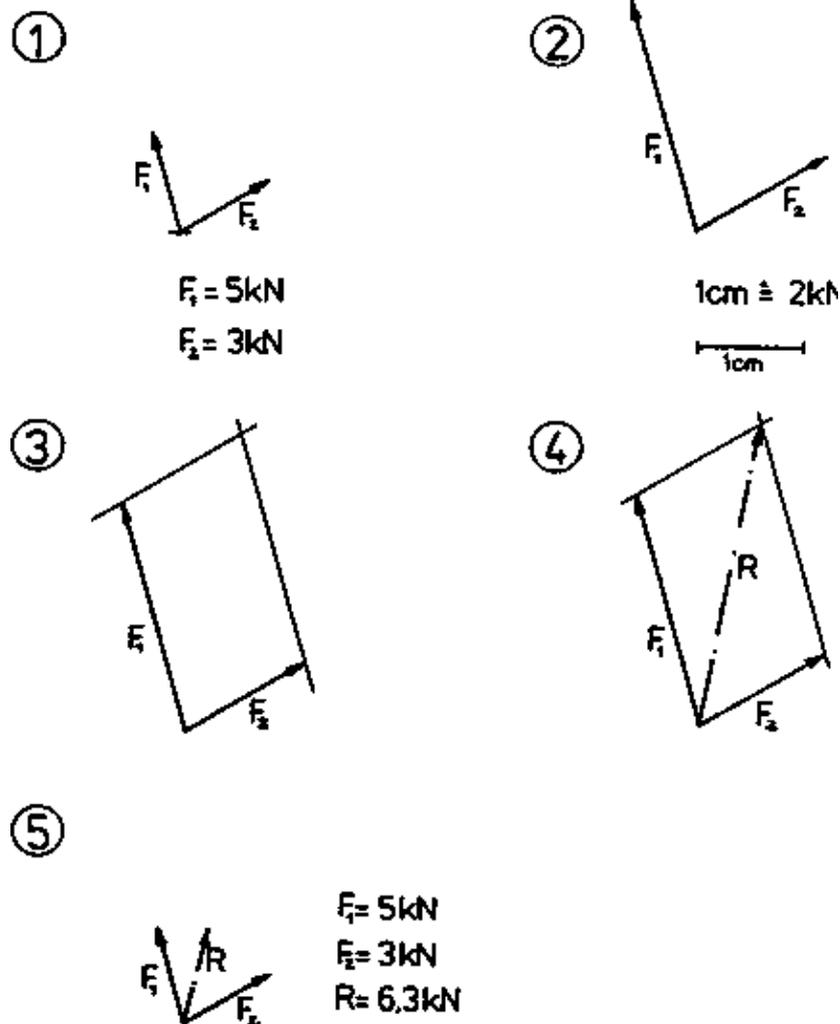


Fig. 5.10. Operations with applying the principle of the parallelogram of forces

Fig. 5.10. shows the operations with applying the principle of the parallelogram of forces:

1. In the layout plan, the common point of application of the forces and the line of action and direction of forces F_1 and F_2 are represented.
2. A scale of forces is selected. The size of the scale of forces is determined by the available paper size and by the accuracy demanded. The two forces are drawn by the aid of the scale of forces.
3. The graphic representation of forces F_1 and F_2 is complemented to a parallelogram. Through the arrow-head of F_1 , a parallel line is drawn to F_2 , and through the arrow-head of F_2 , a parallel line is drawn to F_1 .¹⁾
4. The connection of the intersection point of F_1 and F_2 to the opposite corner of the parallelogram is then the resulting force R .
5. A parallel to the line of action of R is drawn through the point of application of the layout plan. The direction (arrow-head) of R is learned from the parallelogram of forces, and the quantity of R is measured from the parallelogram by the aid of the scale of forces.

¹⁾ A straight line is displaced in parallel in the following way:

A triangle is set to the straight line (F_1) to which a parallel is to be constructed: position 1 in fig. 5.11.

A ruler or second triangle is set to this triangle. The ruler is firmly pressed to the support, and the triangle is moved along the ruler. When the triangle has reached the arrow-head of F_2 , it is held fast, and the required parallel to F_1 is

drawn at its top edge.

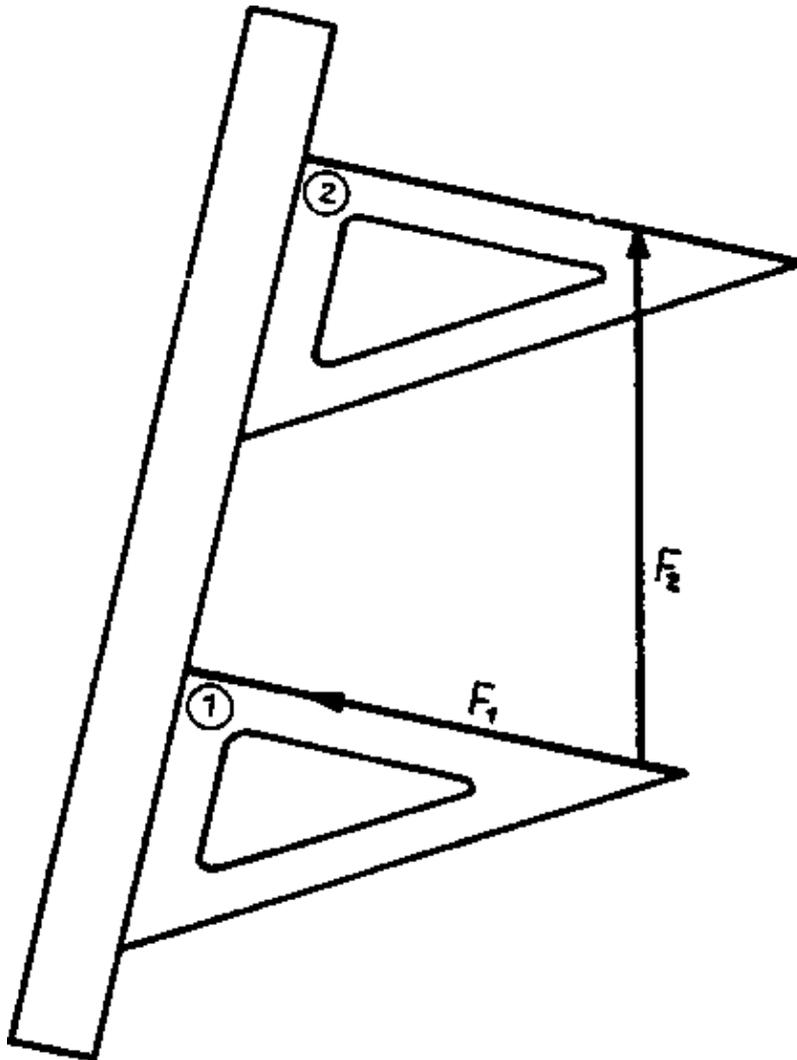


Fig. 5.11. Parallel shift of a straight line by the aid of ruler and trianlge

5.3.4.2. Determination of the resultant from three and more forces

By means of the principle of the parallelogram of forces, the resultant from three and more forces can be determined, too.

These forces may have different lines of action. They must have a common point of application, however.

The operations for ascertaining the resulting force of four initial forces are shown in fig. 5.13.

In the layout plan, fig. 5.13.(1), the acting forces are represented. First, the resulting, force of forces F_1 and F_2 is formed. The parallelogram of forces is drawn by the aid of a scale of forces, and then the resultant $R_{1;2}$ is determined, fig. 5.13.(2)

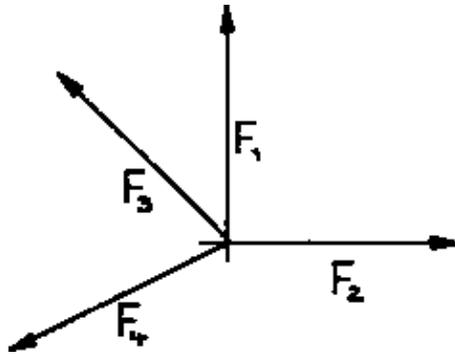


Fig. 5.12. How large is the resultant? Which direction has the resultant?

$F_{\text{Res}1;2}$ has the same effect as F_1 and F_2 together. Hence, $F_{\text{res};2}$ can replace the forces F_1 and F_2 . In the next operation (fig. 5.13.(3)), the resultant $F_{\text{Res}1;2}$ and force F_3 can be combined, and the resultant $F_{\text{Res}1;2;3}$ comes to exist. This resultant replaces the forces F_1 , F_2 and F_3 .

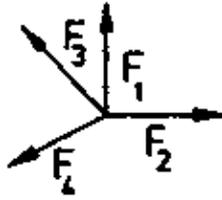
After that, the resultant $F_{\text{Res}1;2;3}$ is brought together with F_4 in a next parallelogram of forces, and the result is the final resultant F_{Res} (fig. 5.13.(4)).

The final resultant F_{Res} has the effect of F_1 , F_2 , F_3 and F_4 together, and hence replaces the initial forces.

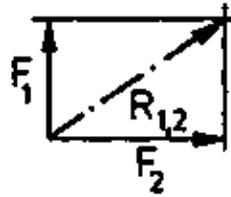
The quantity of F_{Res} is measured out by means of the scale of forces, and direction and line of action are entered into the layout plan (fig. 5.13.(5)).

The sequence, in which the forces are combined, does not influence the result.

①



②

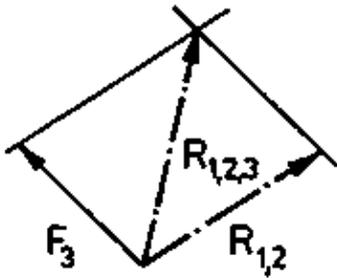


$F_1 = 2 \text{ kN}; F_2 = 3 \text{ kN}; F_3 = 3 \text{ kN}; F_4 = 5 \text{ kN}$

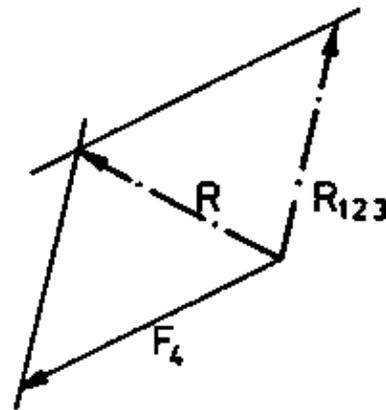
$1 \text{ cm} \hat{=} 2 \text{ kN}$



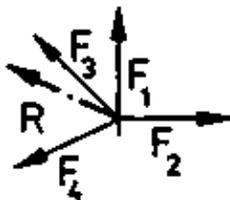
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⑤



$R = 4 \text{ kN}$

Fig. 5.13. Operations for ascertaining the resultant from several single forces

The sequence is optional.

It is not necessary to draw the individual parallelograms one after another, they can also be arranged one upon the other, i.e. in one drawing. Thereby, drawing work is saved, but the clearness is reduced.

5.4. Force and counterforce

In para' 5.3.3. it was stated that two forces neutralize each other if their quantity and line of action is equal, but their directions opposed. Then the two forces are in equilibrium, and have no outward effect.

This phenomenon can be observed wherever two bodies are in touch with each other, and one body exerts a force on the other.

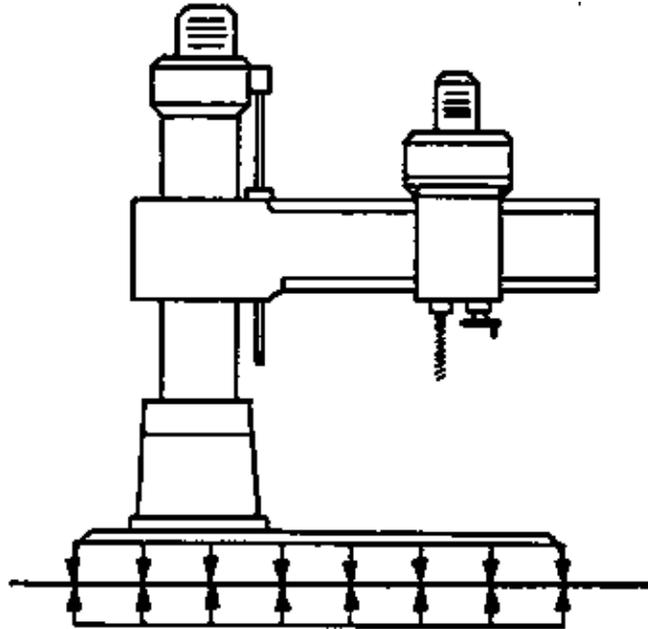


Fig. 5.14. Force and counterforce under a drilling machine

Fig. 5.14. shows a drilling machine. The drilling machine presses on the floor with its weight. In the contact joint between machine and floor, the weight is distributed over the area of contact. A surface force comes to exist. With its weight, the machine presses downwards vertically. Consequently, the surface force is directed downwards vertically, too.

The floor carries the machine. A counterforce is formed. As the machine stands firmly and quietly, there is equilibrium in the floor joint. According to the above consideration, the counterforce must have the same quantity and line of action, but an opposite direction.

In this case, the counterforce is directed upwards vertically.

Generally, it can be stated:

If a body exerts a force on another body, this body will form a counterforce.

Acting force and counterforce are

- equally large,
- have the same line of action,
- are of opposite direction.

Force and counterforce are in equilibrium:

5.5. Breaking a force up into two forces of given direction

The principle of the parallelogram of forces states that the resultant has the same effect as the two initial forces together.

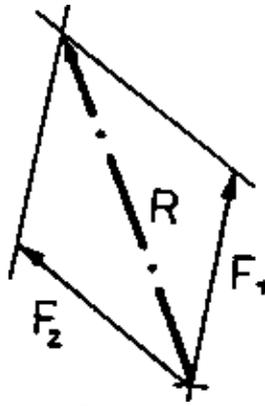


Fig. 5.15. Parallelogram of forces. R has the same effect as F_1 and F_2 together.

F_{Res} has the same effect as F_1 and F_2 together.

F_{res} can replace the forces F_1 and F_2 .

This interrelation can be defined reversely, too:

F_1 and F_2 together have the same effect as

F_{Res} • F_1 and F_2 together can replace F_{Res} .

With this statement it is possible to break up a given force into two forces whose lines of action are given. Fig. 5.16. shows the operations for breaking a force up into two forces of given direction.

In the layout plan in fig. 5,16 (1), a force $F = 4$ kN is given. In addition, the straight lines 1-1 and 2-2 are stated. Required are:

What is quantity and direction of two forces, having the following properties?

They act on the lines of action 1-1 and 2-2.

They have the same effect as F .

To find an answer to this question, the force $F = 4$ kN is represented graphically by the aid of a scale of forces (fig. 5.16.(2)), In addition, the parallel lines to 1-1 and 2-2 are drawn in such a way that they run through the starting point of F . It is on these straight lines where the sides of the paralelogram of forces lie. The lengths of the sides are still unknown, however.

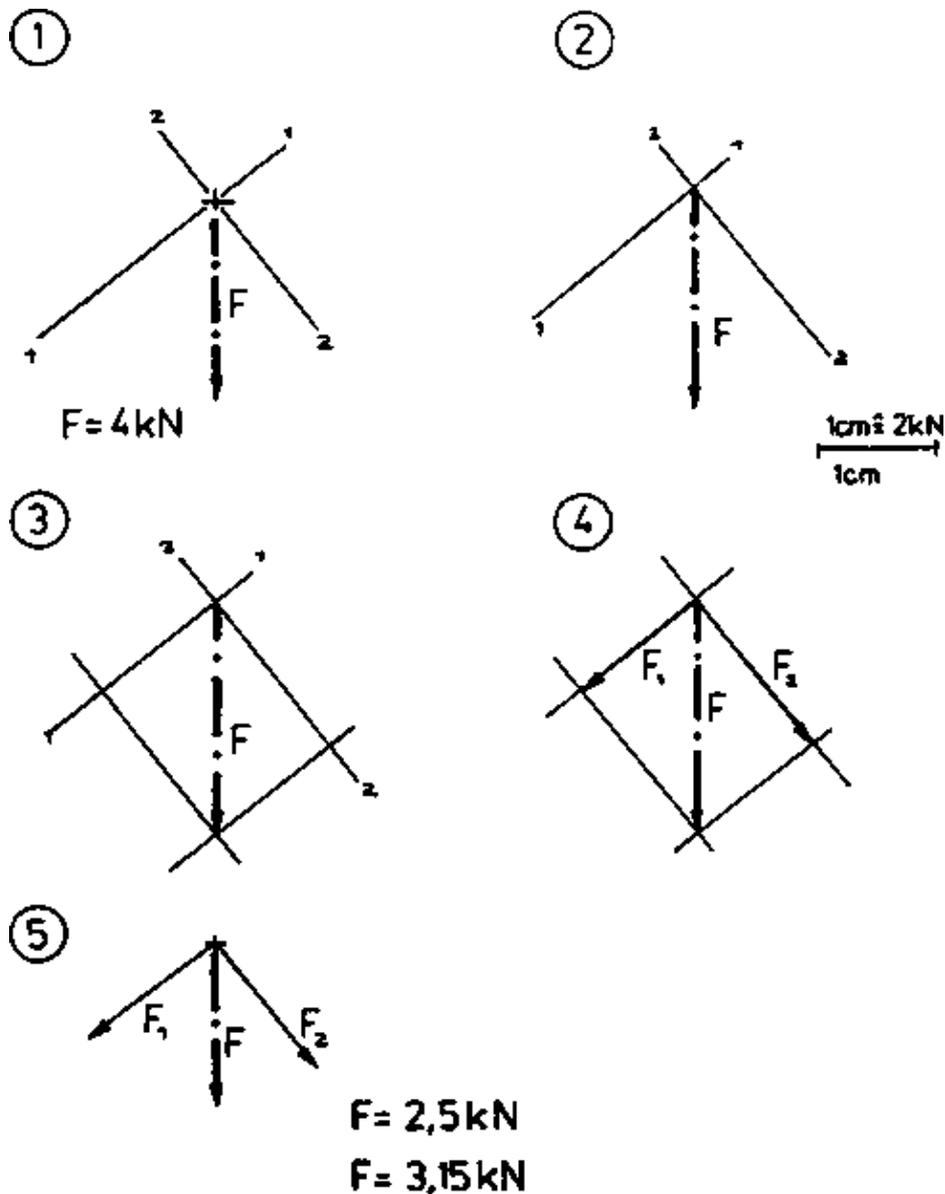


Fig. 5.16. Operations for breaking a force up into two forces of given direction

In order to complete the parallelogram of forces, fig. 5,16(3), the opposite sides of the parallelogram of forces are drawn. These sides again run parallel to the straight lines 1-1 and 2-2, and go through the arrow-head of force F . Therewith the parallelogram of forces is drawn completely. The sides of the parallelogram are the required forces F_1 and F_2 , fig. 5.16.(4). By means of the scale of forces, the quantity of these forces can now be measured out. To concluded with, the direction of forces F_1 and F_2 is transferred to the layout plan. The forces F_1 and F_2 together have the same effect as F . Hence, F_1 and F_2 can replace F .

Further considerations can proceed on the assumption that only F_1 and F_2 exist, and F is no longer effective.

When breaking up one force into three or more forces acting on a common point of application, a definite solution cannot be attained.

5.6. Force-converting technical devices

5.6.1. Preliminary note

A skilled worker will generally not design any new technical equipment, tools and devices. He is to recognize the operating mode of available aids, however, to enable him, at his work,

- to employ his strength expediently, and
- to employ tools and devices in such a way that they do not get damaged and accidents are avoided.

Forces are not visible immediately. Consequently, the skilled worker must recognize the forces acting in equipment, tools and devices. The conversion of forces can be frequently found.

- at the one-sided wedge,
- at the two-sided wedge,
- at the inclined plane,
- at the thread and
- at the hoisting sling.

5.6.2. Hoisting slings

In most cases, the loads moved by a crane are hanging on ropes. Frequently, these ropes are not arranged vertically, they meet at the hook under an acute angle.

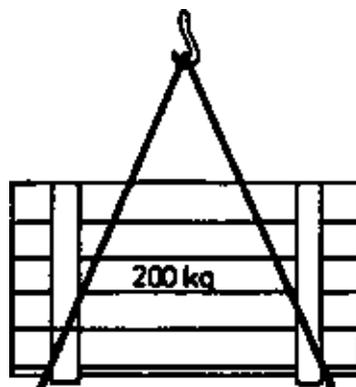


Fig. 5.17. How large is the force in the rope?

Fig. 5.17. shows how a load at a mass of 200 kg is carried by a hoisting sling.

Tensile forces are acting in the rope. These tensile loads have the same direction as the ropes themselves, i.e. the line of action of the rope forces lies on the centre line of the ropes. The initial force is the weight. In the example, the weight amounts to

$$G = 2,000 \text{ N} = 2 \text{ kN}$$

Hence, the force $G = 2 \text{ kN}$ has to be broken up into two forces F_1 and F_2 , which act in the centre lines of the ropes.

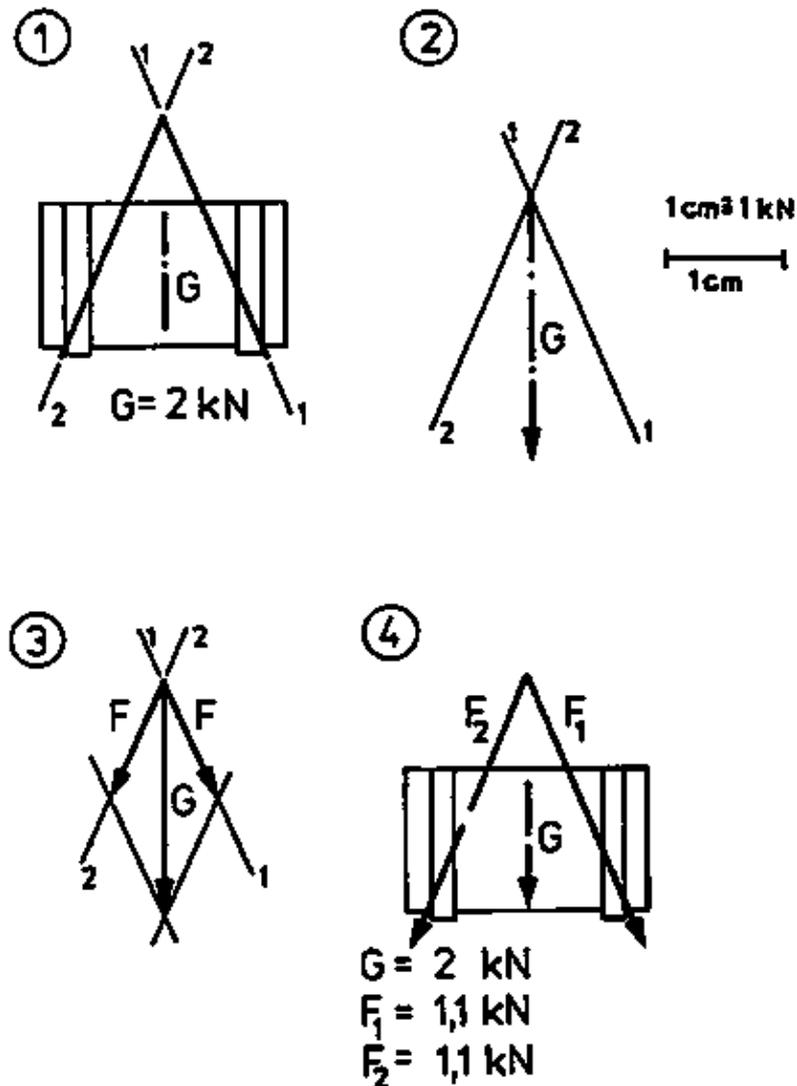


Fig. 5.18. Diagrammatic determination of the rope forces in a hoisting sling.

Fig. 5.18. shows the operations for ascertaining the rope forces. In the layout plan, the direction of the ropes is represented, (fig. 5.18.(1)). Then the force G is represented graphically by means of a scale of forces, (fig. 5.18.(2)). Through the starting point of G , the parallel lines to the ropes are drawn. Thereby the straight lines 1-1 and 2-2 come to exist. Subsequently, the parallelogram of forces is completed. The sides of the parallelogram of forces are the required forces F_1 and F_2 . The quantity of F_1 and F_2 is measured out by means of the scale of forces. The direction of F_1 and F_2 is transferred to the layout plan, (fig. 5.18.(4)).

In the selected example, a weight of 2 kN causes two rope forces of 1.1 kN each. Hence, the sum of the rope forces is larger than the weight itself.

How do the forces in the rope change if the angle between the ropes becomes larger?

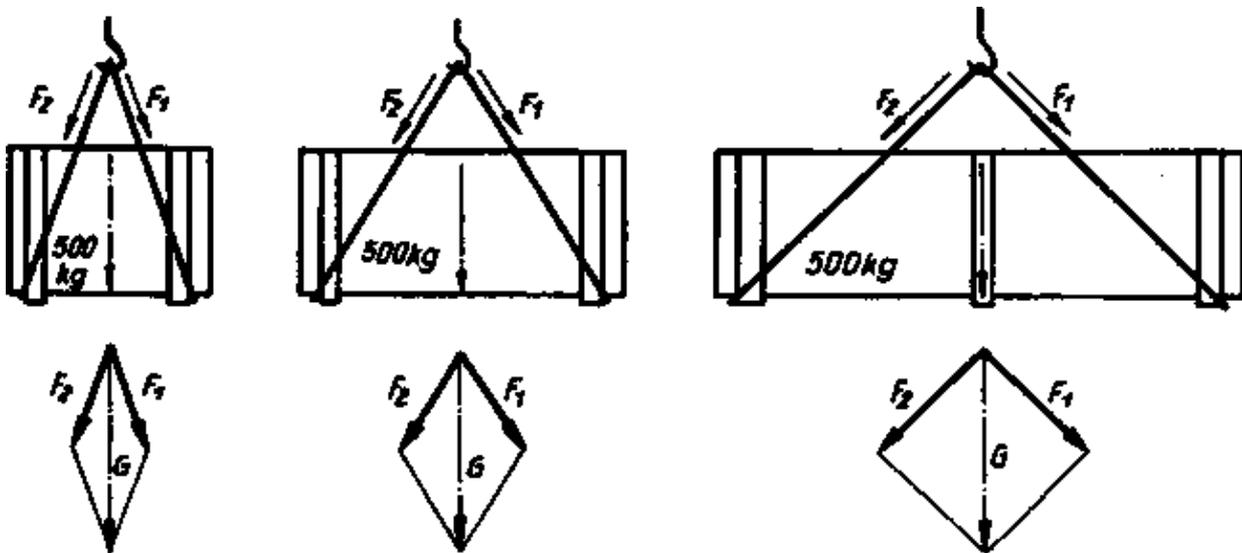


Fig. 5.19. The larger the angle between the ropes, the larger becomes the force in the rope.

Fig. 5.19. shows three loads, all of which have the same mass and thus the same weight. The ropes meet at the hook at different angles.

The resolution of forces at the parallelogram of forces shows:

The larger the angle between the ropes, the larger are the forces in the rope.

5.6.3. Inclined plane

In engineering and production, masses have frequently to be moved to a larger height by a minimum of force. In workshop halls, this work is often performed by a crane.

In many cases – especially in the open, a crane is not available, or the forces of a crane are too small for lifting very large masses.

Large masses are moved to a larger height, e.g. when

- a motor lorry travels uphill,
- a goods train surmounts a gradient,
- a fork-lift truck drives up a ramp in the works.

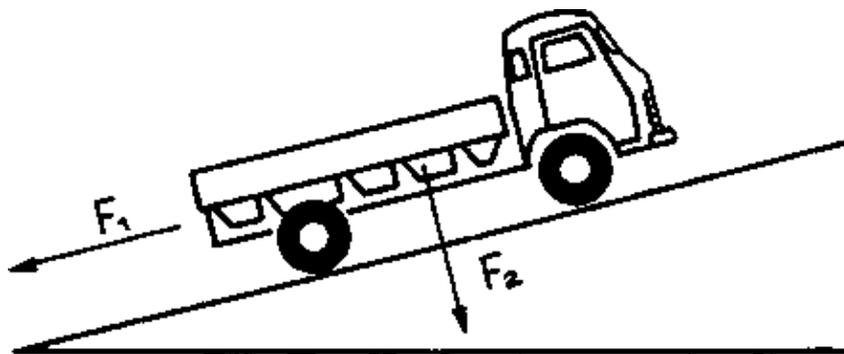


Fig. 5.20. How large is the slope output force F_1 and the normal force F_2 ?

Fig. 5.20. shows a motor lorry driving up an inclined plane. The rails, on which a goods train moves, or the ramp are inclined planes.

The weight of the motor lorry (fig. 5.20.) acts downward vertically. The physical behaviour of the motor lorry on an inclined plane is determined by two forces:

- by the slope descending force F_1 and
- by the normal force F_2 .

The slope descending force F_1 tries to roll the motor lorry down the inclined plane – i.e. the mountain, the gradient or the ramp. The slope descending force runs parallel to the inclined plane.

The normal force F_2 presses the motor lorry on to the inclined plane. The normal force is directed vertical to the inclined plane. In the area of contact between motor lorry and plane, it produces the frictional force.

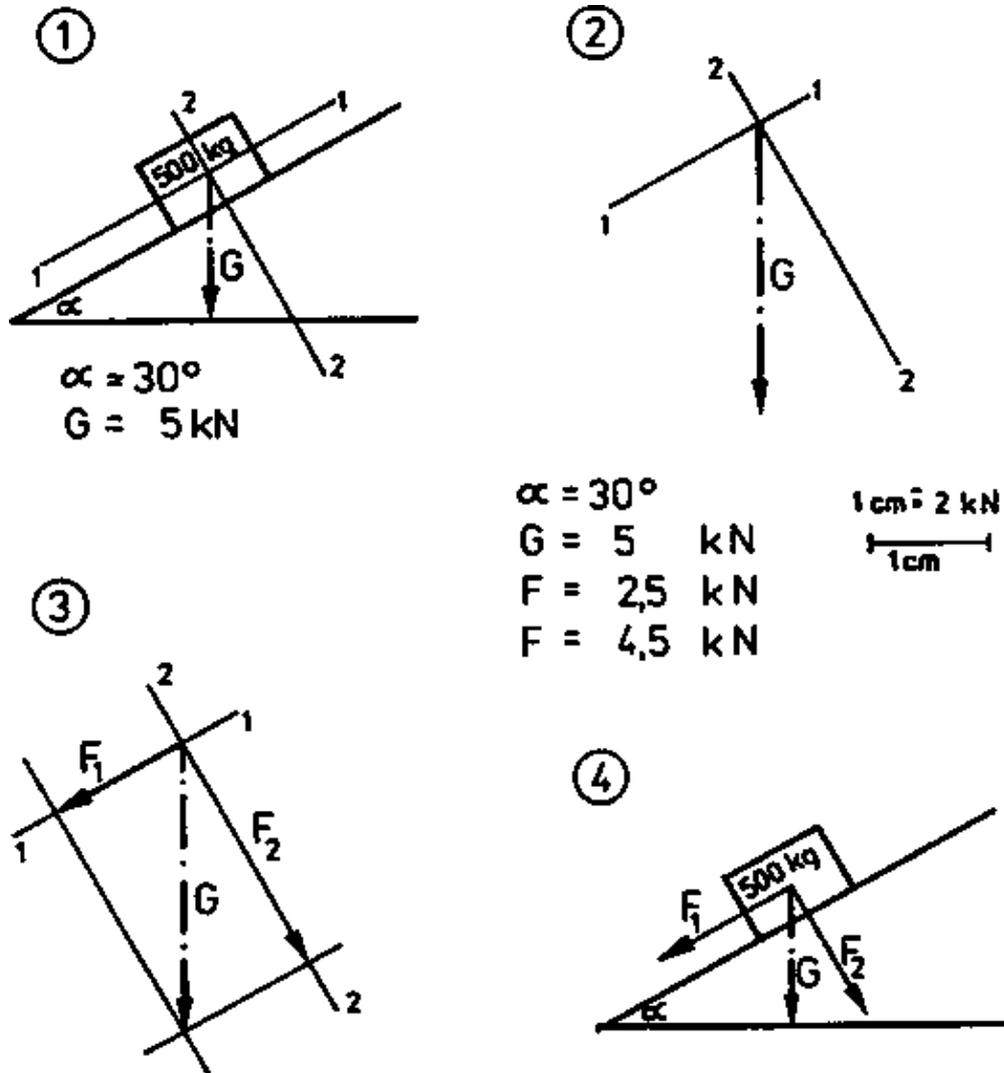


Fig. 5.21. Diagrammatic determination of slope descending force F_1 and normal force F_2 at the inclined plane.

The determination of the slope descending force F_1 and normal force F_2 is by means of resolution of forces.

In fig. 5.21. a mass of 500 kg is represented, which is unmoved on an inclined plane. The mass of 500 kg has a weight of $G = 5 \text{ kN}$. The weight acts vertically downwards.

How large are the slope descending force F_1 and the normal force F_2 ?

First G is represented by means of a scale of forces, fig. 5.21.(2). Then the lines of action 1–1 and 2–2 are entered at the starting point of G .

Now the parallelogram of forces is drawn, (fig. 5.21.(3)). The sides of the parallelogram are the required forces F_1 and F_2 . By means of the scale of forces, F_1 and F_2 are measured out from the parallelogram of forces. Their directions are transferred to the layout plan, (fig. 5.21.(4)).

In the selected example, the slope descending force F_1 amounts to 2.5 kN, and the normal force $F_2 = 4.33 \text{ kN}$. Hence, the slope descending force is smaller than the weight. If the mass is to be moved up an inclined plane,

only the slope descending force and the frictional force have to be overcome.

The mass is pressed on to the support at a force $F_2 = 4.33 \text{ kN}$. How do slope descending force and normal force change if the gradient becomes smaller, i.e. if the angle of inclination of the plane becomes smaller?

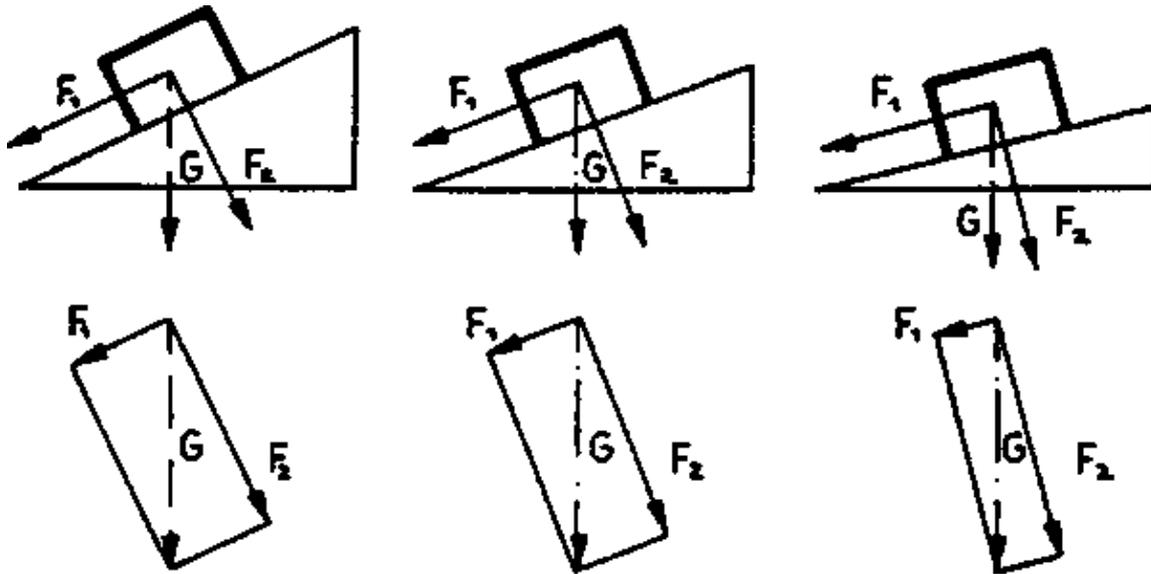


Fig. 5.22. The smaller the angle of inclination α , the smaller is the slope descending force F_1

In fig. 5.22. a mass of equal weight is represented on different inclined planes.

When the parallelograms of forces are analyzed, it follows:

The smaller the angle of inclination α , the smaller becomes the slope descending force F_1 , the larger becomes the normal force F_2 .

When analyzing the results, it further becomes conspicuous that the slope descending force is always smaller than the weight. When the frictional forces are kept small by rolls and wheels, it is possible, to move a large mass to a larger weight by means of a smaller force.

With the routing of roads and rail tracks, use is made of this principle.

Masses which are deposited on an inclined plane, have to be secured against gliding down, With vehicles, the hand brake has to be applied. Wagons or barrel-shaped bodies have to be secured by wedges, otherwise the slope descending force might set the body in motion.

5.6.4. Two-sided wedge

Separating and splitting processes are often achieved by tools of different wedge shapes.

The most simple form of the edge of a parting tool is the two-sided wedge.

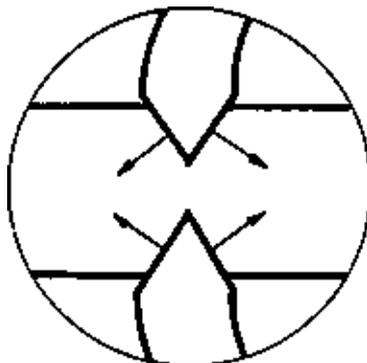


Fig. 5.23. Forces at the cutting edge of a pincers (representation scaled up).

Fig. 5.23. shows the forces at the cutting edge of a pincers. In para' 3.1.2. (fig. 3.8.) it was shown that, by utilization of the lever principles, a force acts at the cutting edge four to eight times larger than the force of hands by which the pincers are pressed together. In the cutting edge, another conversion of forces occurs. The edge penetrates the wire to be separated. At the cutting edge, surface forces are formed standing vertically on the contact areas (fig. 5.2.). For reasons of simplification, only the resultants of these surface forces are examined (fig. 5.23.). These cutting edge forces can be determined by resolution of forces. Fig. 5.24. shows the operations for ascertaining the cutting edge forces at the two-sides wedge.

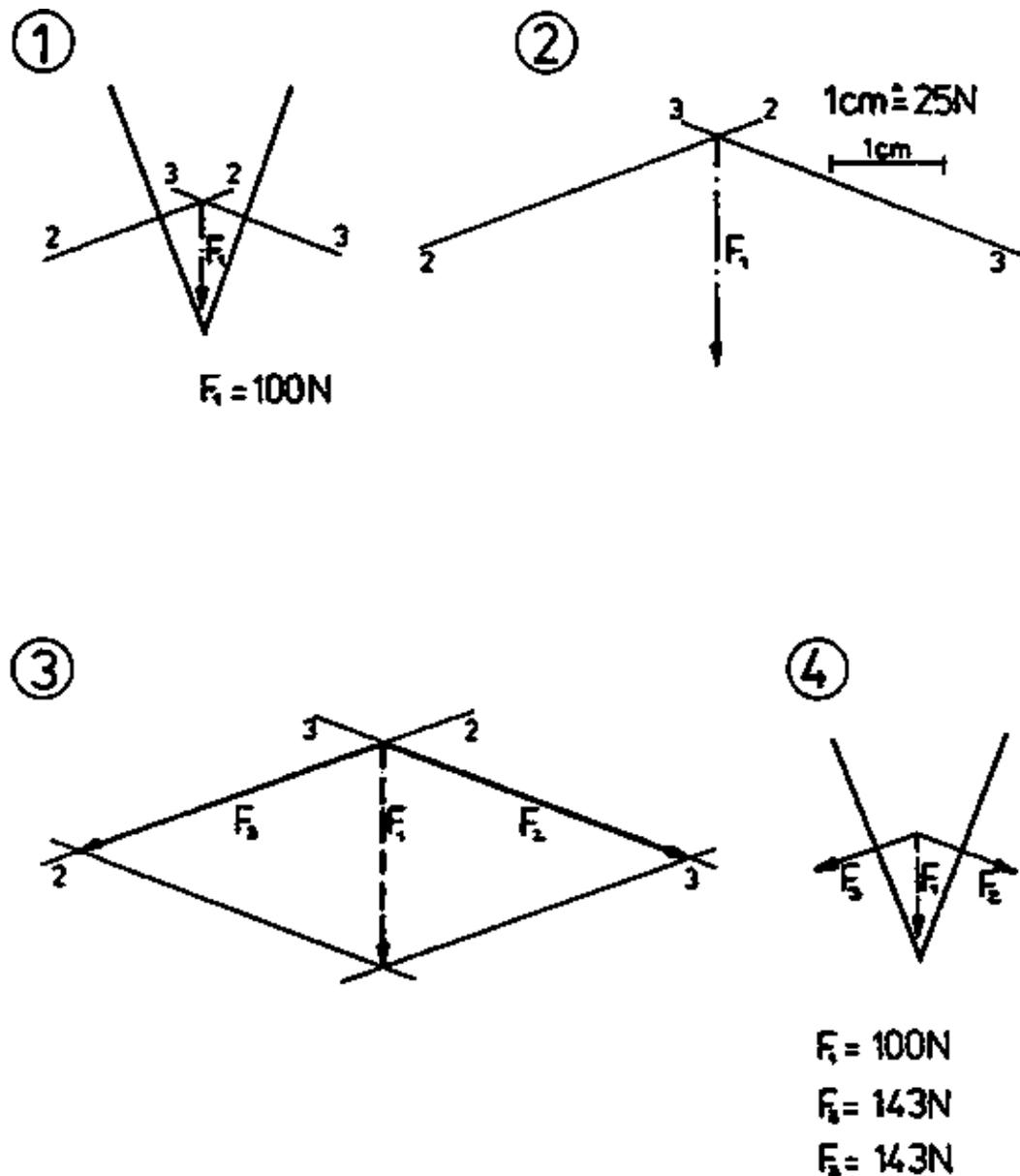


Fig. 5.24. Diagrammatic determination of the cutting edge forces at the two-sided wedge. F_1 : Driving-in force. F_2 and F_3 normal forces on the sides of the wedge.

The driving-in force F_1 has to be broken up into two forces the lines of action of which run vertically to the cutting edge surfaces; straight lines 1-1 and 2-2. Beside the layout plan (fig. 5.24.(1)), the driving-in force F_1 is drawn in the scale of forces (fig. 5.24.(2)). Through the starting point of F_1 , parallel lines are drawn to the straight lines 1-1 and 2-2.

Then the parallelogram of forces is completed (fig. 5.25(2)). The sides of the parallelogram of forces are then the required cutting edge forces F_2 and F_3 . They are measured out from the parallelogram of forces by means of the scale of forces. To conclude with, the directions of F_2 and F_3 are transferred to the layout plan, fig. 5.25.(4). In the selected example, a driving-in force of $F_1 = 100$ N takes effect. This force causes two cutting edge forces of $F_2 = 143$ N and $F_3 = 143$ N each.

At the two-sided wedge, the cutting edge forces are considerably larger than the driving-in force.

How do the cutting edge forces change when the angle at the point of the wedge becomes smaller?

In fig. 5.25. three wedges are examined. In the selected three cases, the driving-in force is equally large. The cutting edge forces are different, however.

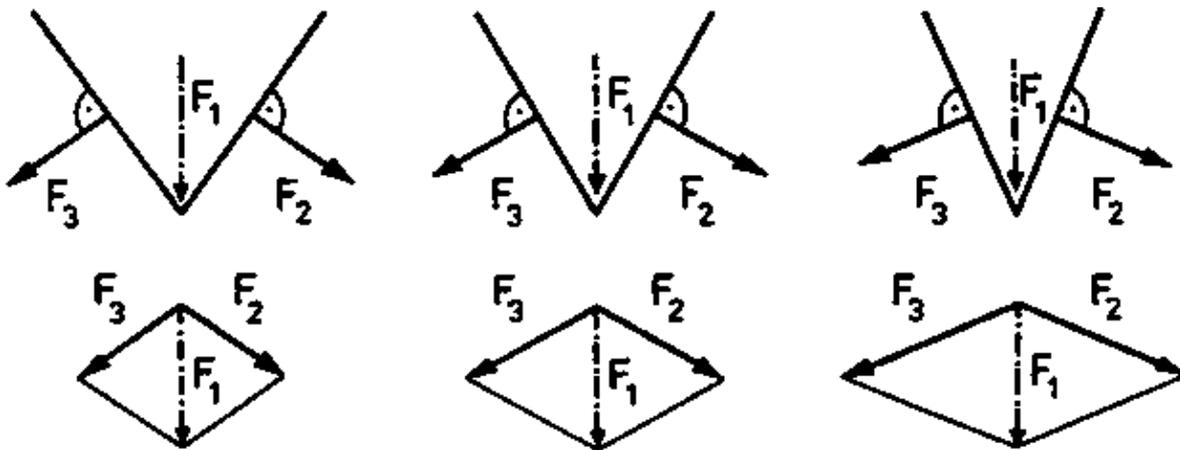


Fig. 5.25. The more pointed the wedge, the larger are the cutting edge forces F_2 and F_3 .

The parallelograms of forces in fig. 5.25. show: When the two-sided wedge becomes more pointed, i.e. when the angle at the point of the wedge becomes smaller, then the cutting edge forces will grow.

The more pointed the wedge, the larger are the cutting edges F_2 and F_3 .

The two-sided wedge is applied in many parting tools like tongs, scissors, chisels and knives. But it also serves for increasing the transmission force at the vee belt.

5.6.5. One-sided wedge

In machine construction, the one-sided wedge is chiefly employed as mounting and adjusting wedge.

The forces at an ordinary key (fig. 5.1.) are to be examined more precisely.

The longitudinal key is a mounting wedge. It has the task to establish a disconnectable joint between a belt pulley and a shaft. The longitudinal key is guided in the shaft in a groove. At driving-in, the key is forced in the interspace between shaft and belt pulley. On the sides of the key, i.e. at the contact surfaces to shaft and belt pulley, very large forces are developed. These forces cause

- that the belt pulley and the shaft are firmly pressed together,
- that due to the developed frictional forces, shaft and belt pulley are rigidly joined together at normal load.

When the key is knocked out again, shaft and belt pulley are loosened from each other. The belt pulley can be pushed off the shaft.

Between longitudinal key and shaft and between longitudinal shaft and belt pulley, surface forces are developed. These surface forces (fig. 5.26.) act vertically on the contact surfaces in both cases.

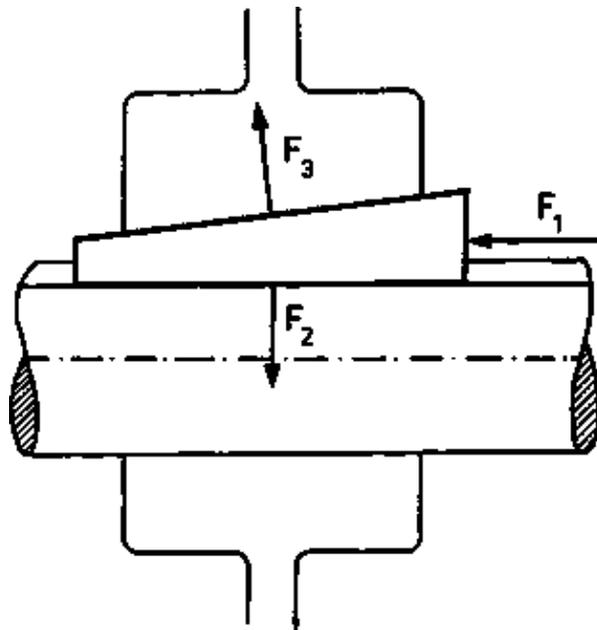


Fig. 5.26. Conversion of force at the one-sided wedge (surface forces)

In order to render the resolution of forces possible more simply and easier to survey, the surface forces are combined to one resultant each (fig. 5.27.).

In fig. 5.27., the following designations were used: F_1 : resultant of driving-in force

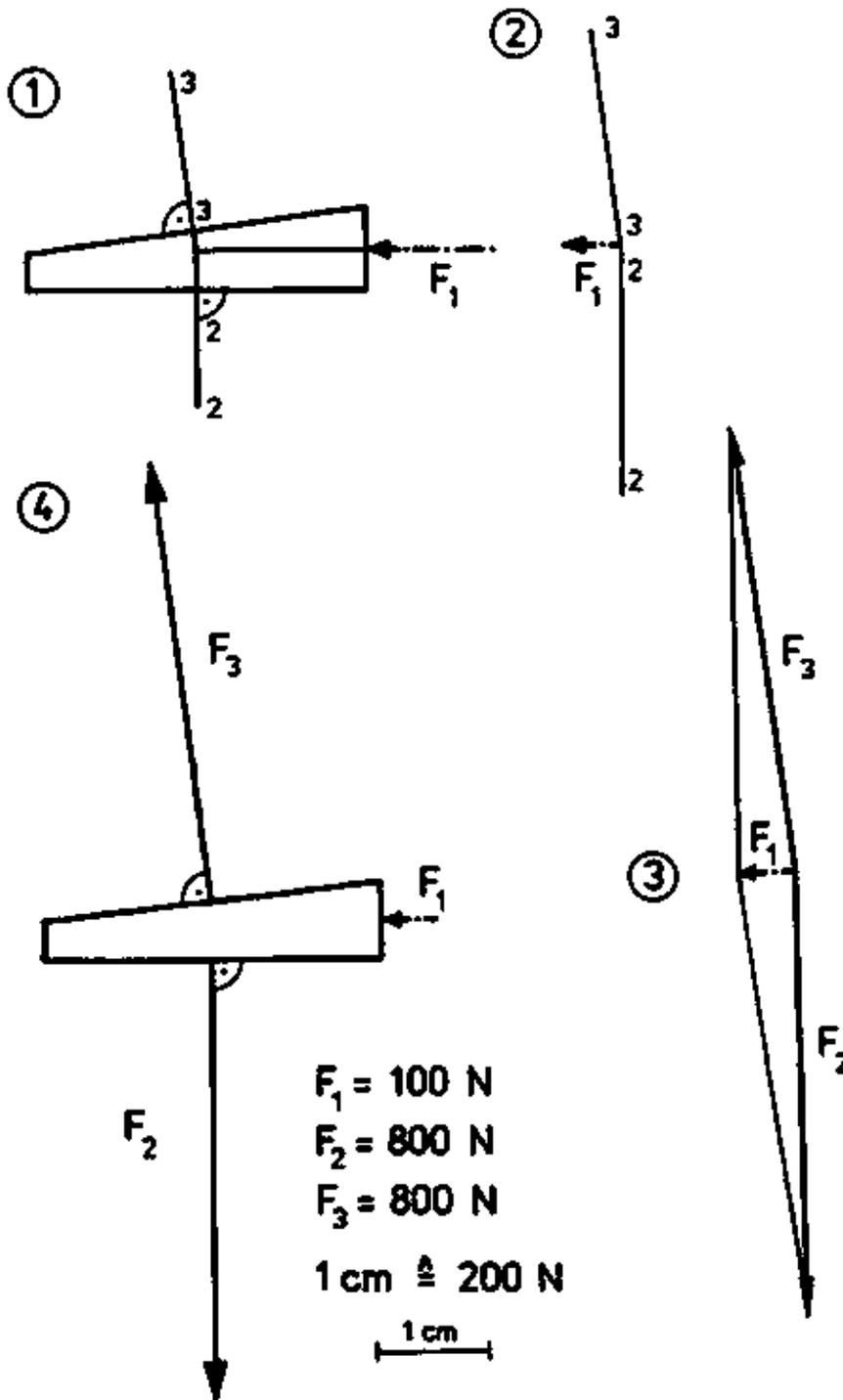


Fig. 5.27. Diagrammatic determination of the side forces F_2 and F_3 at the one-sided wedge

F_2 : resultant of the forces between key and shaft
 F_3 : resultant of the forces between key and belt pulley

The unknown side forces F_2 and F_3 can be ascertained diagrammatically. Fig. 5.27. shows the single operations. The directions of forces 2-2 and 3-3 are fixed in the layout plan (fig. 5.27.(1)). The two straight lines stand vertically on the side surfaces of the wedge. The driving-in force F_1 is consequently to be broken up into two forces, the lines of action of which are known. Beside the layout plan, F_1 is represented in the scale of forces. Through the starting point of F_1 parallel lines are drawn to the straight lines 2-2 and 3-3 (fig. 5.27.(2)). Then the parallelogram of forces is complete (fig. 5.27.(3)) by drawing parallel lines to the straight lines 2-2 and 3-3 through the head of force-arrow F_1 . The sides of the parallelogram are the required forces F_2 and F_3 . These forces can then be measured out from the parallelogram by means of the scale of forces. The directions of F_2 and F_3 are finally transferred to the layout plan (fig. 5.27.(4)).

With flat wedges, the result is that F_2 and F_3 are always approximately equal.

How do the side forces F_2 and F_3 change when the angle of the longitudinal key becomes smaller?

In fig. 5.28., the resolution of forces at three longitudinal keys was examined, proceeding from the driving-in force being equal in each case, but the angle of the key becoming smaller.

When evaluating the three parallelograms of forces, it becomes known that:

The smaller the angle of the wedge, the larger are the side forces.

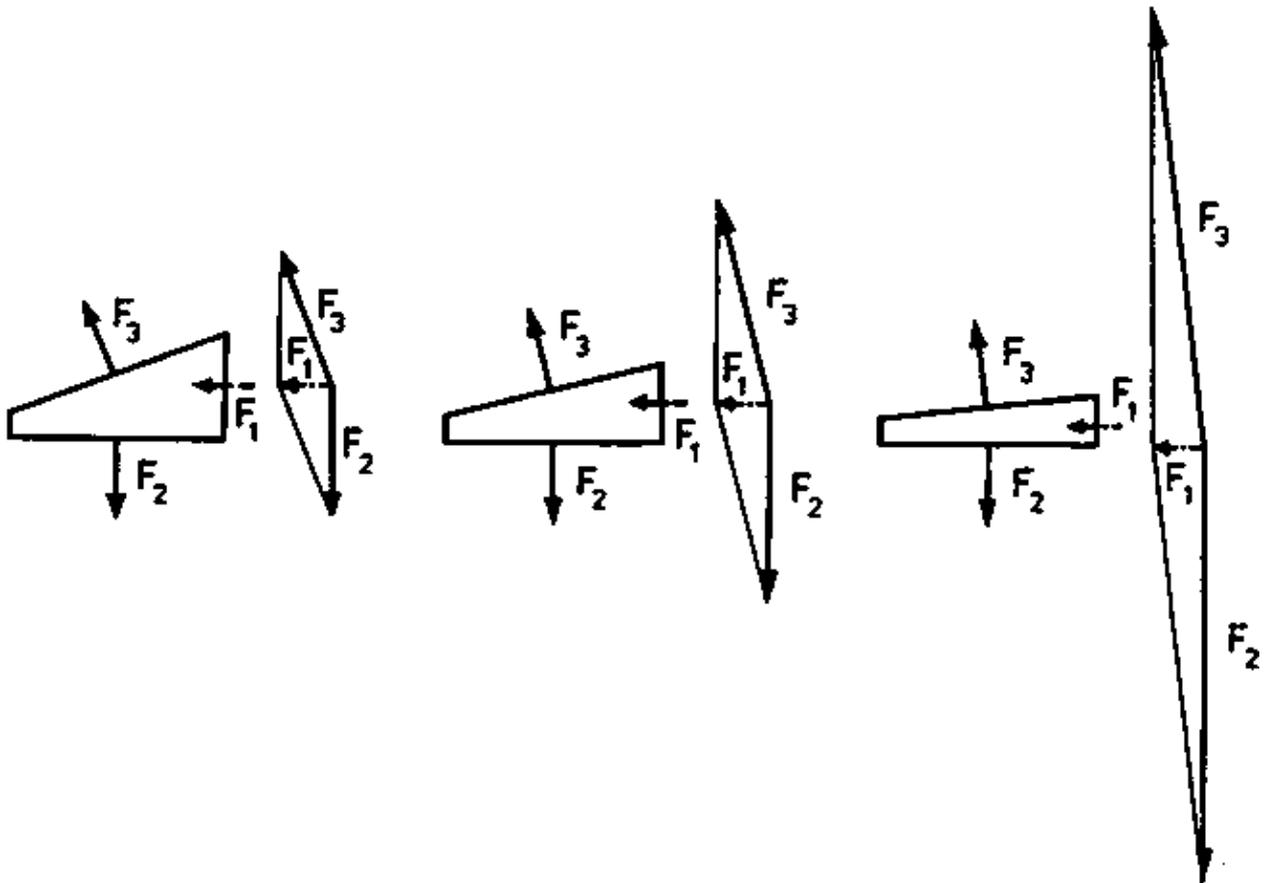


Fig. 5.28. The flatter the wedge, the larger are the side forces F_2 and F_3

In machine construction, longitudinal keys are applied, which are inclined at a ratio of 1:100. The inclination means that the degree of slope only amounts to 1 mm per 100 mm base length!

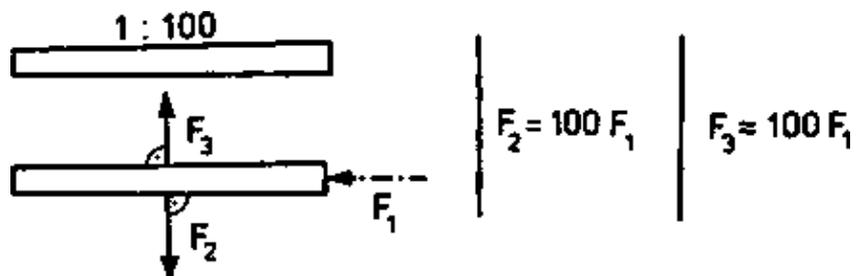


Fig. 5.29. Side forces at the one-sided wedge with taper 1:100

5.6.6. Thread

The thread is used in machine construction in manifold forms:

As thread for transmitting motions it is used, in connection with a screw, for the precise motion of machine parts or for the generation of large forces (screw press).

As fastening screw thread it serves for the secure connection of two or more workpieces with one another.

In the following, the forces are to be examined, which occur in a screwed connection (fig. 5.30.).

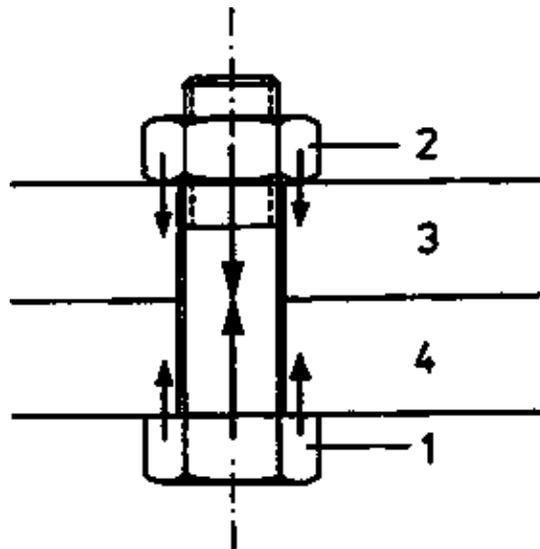


Fig. 5.30. Forces at the screwed connection. 1 screw; 2 nut; 3 and 4; sheet metal parts

Fig. 5.30. shows two sheet metal parts with a borehole. A hexagon-head screw is piloted through this hole. A hexagon nut is tightened with a wrench (fig. 5.31.). The nut rotates about the central axis of the screw and presses at high force on to the upper plate (see drawing). At the same time, the nut tries to draw the screw through the hole. The head of the screw is, however, larger than the hole. Therefore the screw is forced against the bottom plate (see drawing). In the core of the screw, a large tensile force is formed. The two plates are tightly compressed, and large frictional forces are formed between the plates. Thereby the sheet metal parts are firmly connected with each other.

The screwed connection is a fastening joint. The nut can be screwed off again, and plates and screw can be loosened from each other.

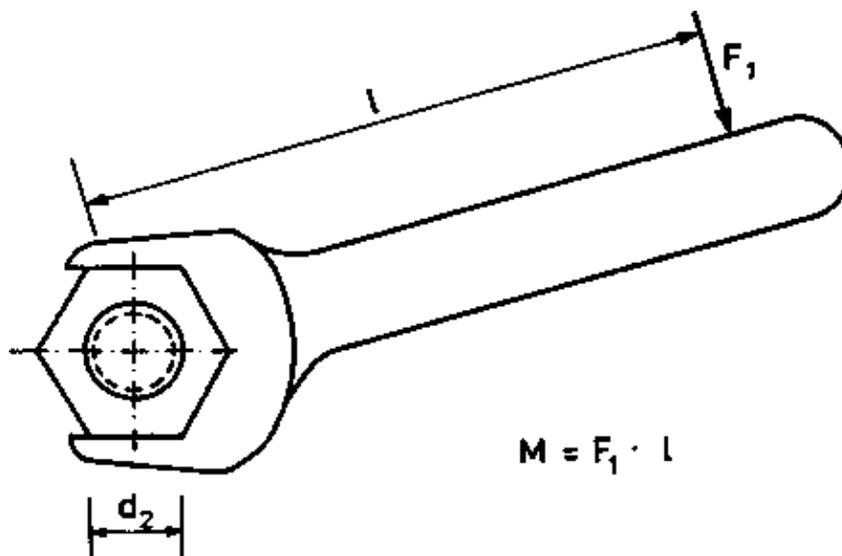


Fig. 5.31. At tightening of the nut, the wrench acts as a lever. F_1 ; manual force.

Effects of the screwed connection:

- nut is pressed against the plate
- screw head is pressed against the plate
- plates are compressed
- in the core of the screw there is a large tensile force

These actions are caused by tightening the nut by means of a wrench.

The manual force acting at the wrench could not produce those actions by itself.

Hence, the manual force has to be converted within the screw.

In the following, some conversions of force are made visible to illustrate the action of the thread and render an estimation of the quantity of the involved forces possible.

The conversion of force is chiefly effected by the thread.

The thread is an inclined plane wound around a cylinder (core of the screw).

Fig. 5.32. shows the wound-off thread.

The nut contains a thread, too. When the nut is turned on the screw, the inclined plane of the nut glides on the inclined plane of the screw downwards or upwards.

The following conversion of force is developed:

1. Action of lever principle

The manual force F_1 (fig. 5.32.) acts at the wrench. The longer lever 1, the larger is the effect.

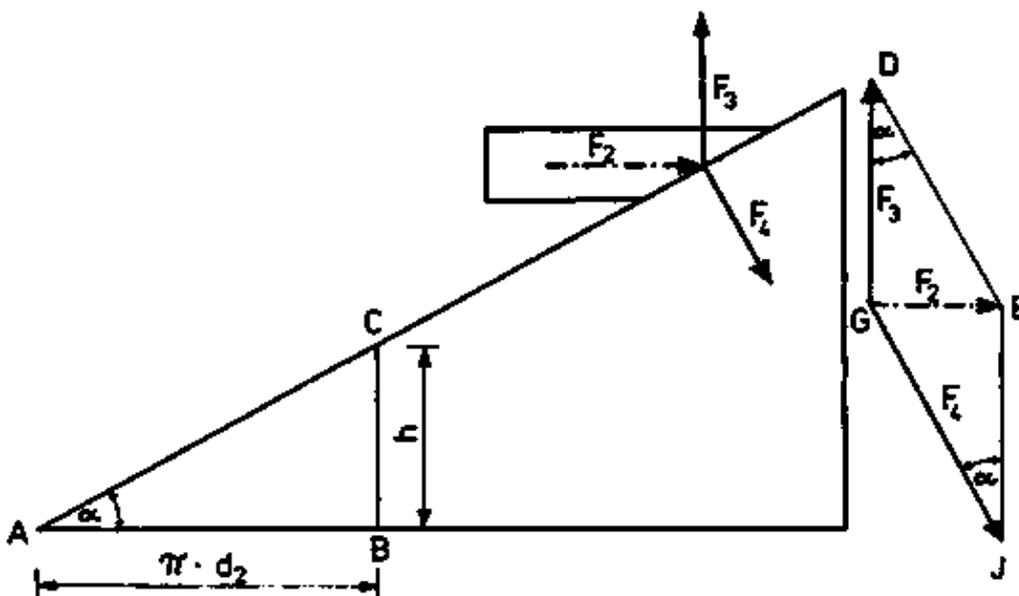
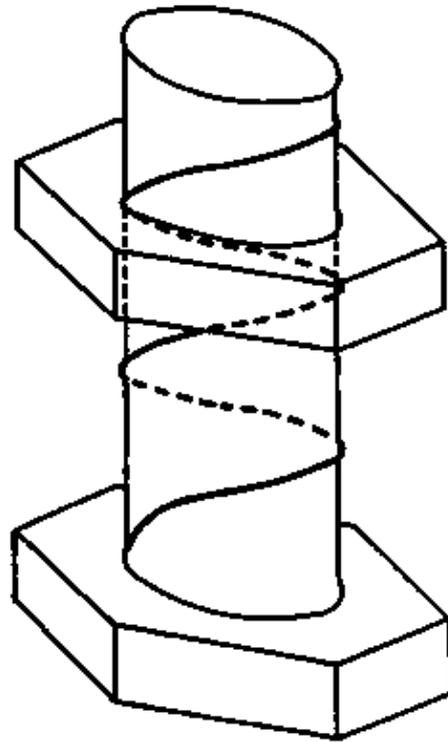


Fig. 5.32. When wound off, a thread yields an inclined plane. The height of the incline (pitch) was highly exaggerated in the representation.

The product from force arm is called torque M .

$$M = F_1 \cdot l$$

When the lever arm is selected too large – i.e. when the torque at nut or screw becomes too large –, then the screw may be destroyed.

The torque turns the nut about the axis of the screw. By that, it produces the force F_2 in the thread. The force F_2 acts in the plane, in which the wrench is turned. Force F_2 acts as surface force distributed over the flanks of the thread. Its resultant acts in the centre of the thread flanks. This diameter is termed d_2 in machine

construction. Hence, the force F acts at a distance of $\frac{d_2}{2}$ from the rotation axis of the screw.

As the two torques are equal, it applies

$$M = F_1 \cdot l$$

$$M = F_2 \cdot \frac{d_2}{2}$$

Hence

$$F_2 = \frac{M l}{\frac{d_2}{2}}$$

with

$$M_1 = M_2$$

and

$$M_1 = F_1 \cdot l$$

becomes

$$F_2 = \frac{2 \cdot l}{d_2} \cdot F_1$$

As lever l of the wrench is much larger than $\frac{d_2}{2}$, the force F_2 becomes considerably larger than manual force F_1 .

2. Conversion of force at the inclined plane The thread of the nut glides on the thread of the screw. When both threads are wound off, then two inclined planes glide one upon the other (fig. 5.32.). In the contact surface between nut and screw, force F_2 acts at an acute angle to the inclined plane. This force is now broken up into two forces with the following directions:

F_3 : The force F_3 is the tensile force in the core of the screw. The force F_3 is also termed initial stressing force. Force F_3 presses the metal sheet parts together.

F_4 : The force F_4 stands vertically on the thread flanks. It transmits the force in the contact surface from the nut to the screw.

Fig. 5.32. shows the resolution of force in the parallelogram of forces.

In order to make the resolution of force visible, the thread was represented highly superelevated in fig. 5.32.

When the thread is wound off, then the path designated p is the distance between two courses of thread.

With metric ISO thread, coarse screw thread, these distances amount to, e.g.:

Metric ISO thread, coarse screw thread

Designation	Diameter of centre of thread flanks D_2	Distance of two courses of thread
M 12	10.863 mm	1.75 mm
M 20	18.376 mm	2.50 mm

From that follows, with the designations of fig. 5.32.

	? d_2	P
M 12	34.127 mm	1.75 mm
M 20	57.730 mm	2.50 mm

At such a small slope, F_3 and F_4 cannot be ascertained diagrammatically any longer. The drawings will become too large and inaccurate. For this reason, the graphic solution has to be recorded by way of calculation.

The triangle ABC represented in fig. 5.35., is similar to triangle DGE for the triangles coincide in three angles.

In similar triangles, the sides are at the same ratios to each other. Hence it applies $\frac{F_3}{F_2} = \frac{d_2 \cdot \pi}{p}$

From that follows
$$F_3 = \frac{\pi \cdot d_2}{p} \cdot F_2$$

F_2 can be calculated as a function of the torque:

$$F_2 = \frac{M}{\frac{d_2}{2}}$$

$$F_3 = \frac{\pi \cdot d_2}{p} \cdot \frac{M \cdot 2}{d_2}$$

$$F_3 = \frac{\pi \cdot M \cdot 2}{p}$$

Or, reduced to manual force F_1 :

$$F_3 = \frac{\pi \cdot d_2}{p} \cdot \frac{F_1 \cdot l}{\frac{d_2}{2}}$$

$$F_3 = \frac{\pi \cdot F_1 \cdot l \cdot 2}{p}$$

At tightening with a torque wrench, the thread M 12 is loaded with the following torques:

M 12	tight	60 N • m
M 12	high-strength	75 N • m

From that follows for the initial stressing force

$$F_3 = \frac{\pi \cdot 75 \text{ N} \cdot \text{m} \cdot 2}{1.75 \text{ mm}} = \frac{\pi \cdot 75 \cdot \text{N} \cdot \text{m} \cdot 2}{0.00175 \text{ m}}$$

$$F_3 = 269.300 \text{ N}$$

This value cannot be attained, however. As a large part of the torque is used at tightening of nut for overcoming the large frictional forces (μF_4), the actual initial stressing force is smaller.

The actual initial stressing force F_v only amounts to

$$F_v = ? \cdot F_3$$

The efficiency of the screw can be estimated at

$$? = 0.2 \dots 0.3$$

In steel construction, the following initial stressing forces in the screw are scheduled arithmetically with high-strength prestressed screws, the nuts of which are tightened with a torque wrench:

Designation of thread	Initial stressing force F_v
-----------------------	-------------------------------

M 8	23 kN
M 10	36 kN
M 12	52 kN
M 20	153 kN
M 24	220 kN

3. Conversion of force at thread flanks

So far, considerations were based on the assumption that the thread flanks run vertically to the screw axis (fig. 5.33.). This is very rarely the case, however. The metric ISO threads used in screws have helical thread flanks.

Consequently, force F_4 is further converted, the forces F_5 and F_6 (fig. 5.33.). The initial stressing forces are not changed by this conversion, however. Only the frictional forces are increased.

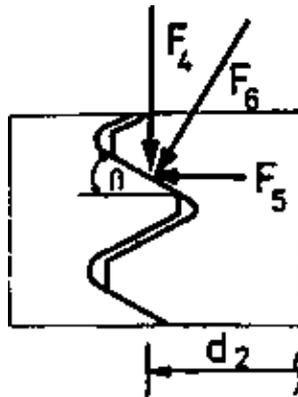


Fig. 5.33. Vertical thread flanks and helical thread flanks

Summary:

1. At the screw connection, several conversions of force will occur:

- One-sided lever at the wrench;
- Resolution of force at the inclined plane;
- Resolution of force at the thread flanks.

2. On tightening the nut, the core of screw is subject to tension. A large tensile force (initial stressing force) is formed in the screw. This initial stressing force is between 23 kN and 220 kN with high-strength prestressed screws. By means of the initial stressing force, the structural parts to be joined are pressed together.

3. In the screw, large frictional forces occur. On tightening and loosening of the nut, these frictional forces have to be overcome. Due to these frictional forces, the screws are selfretaining, i.e. the nut will not get loose on its own.

5.7. Centre of gravity

5.7.1. Application

The drilling machine represented in fig. 5.14. has a large base. This is to attain that the centre of gravity of the machine remains above the base at all positions of the arm. When the centre of gravity is above the base, the machine will stand secure and stable, and will not tilt over.

It is considered as a virtue of motor vehicles if their centre of gravity is as low as possible. Then the motor vehicle has a good roadability, and its inclination to overturning in a turn is reduced.

In punching tools, the punch has to be mounted in the centre of gravity of the line curve of the caster knife.

In the theory of the strength of materials (see para' 6.5.), the position of the centre of gravity of the sectional area is mentioned when dealing with the bending stress of structural parts.

What is the centre of gravity?

Where is the centre of gravity of a body, of a surface and a continuous line?

5.7.2. Centre of gravity of a body and static stability

The centre of gravity of a body is the point of application of the resultant force of all weights of all segments of the body.

It is possible to imagine the entire mass of the body to be united in the centre of gravity. When the entire mass of the body is united in the centre of gravity of the body, it has the same outward effect as if it were distributed in the body.

The centre of gravity of regular bodies follows from perception. The centre of gravity of the sphere is on the centre of the sphere. The centre of gravity of a cuboid is on the intersection point of the planes which bisect the lateral surfaces of the cuboid.

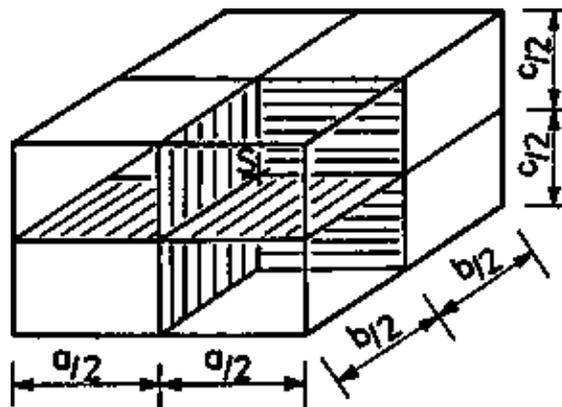


Fig. 5.34. Centre of gravity of a cuboid. S: centre of gravity.

When the body is built up of a uniform material, we can proceed from the fact that the entire mass of the body is united in the centre of gravity. The weight of the cuboid then acts in the centre of gravity. The total weight of the cuboid, which acts in the centre of gravity, has the same outward effect as the total weight of the body if the mass of the cuboid is uniformly distributed in the body.

Use was made of that in figs. 5.18. and 5.21. already. When examining the effects of the inclined plane and of the hoisting sling, only the effect of the weight was examined, which acts in the centre of gravity.

When considering the static stability of machines, containers and other technical equipment, only the centre of gravity is examined.

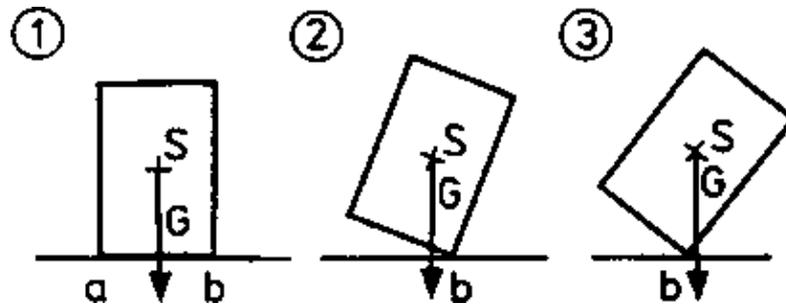


Fig. 5.35. Which body tilts over?

Fig. 5.35. shows a view of a cuboid. The total weight acts in the centre of gravity.

Position 1: The body stands firmly on the base. The line of action of G goes through the base. The body takes a position of rest.

Position 2: The body was tilted off its position of rest by a lateral force. With that, it moved about the tilting edge b . The line of action of G has to passed over the tilting edge yet, however. Under these conditions, the body will move back to its position of rest (position 1) by itself.

Position 3: The body was also tilted about tilting edge b by a lateral force. Now, the line of action of G has passed over tilting edge b , however. Under these conditions, the body will tilt over without the action of additional forces.

When the line of action of the weight acting in the centre of gravity passes over the tilting edge, the body will tilt over.

For this reason, the following measures for increasing the static stability are taken in engineering and production:

1. The tilting edge is arranged as far away from the line of action of G as possible. This can be attained by

- a base as large as possible and
- additional supporting legs standing wide apart.

By additional supporting legs standing wide apart, the static stability is increased (fig. 5.36.).

2. The parts or materials in a machine or technical equipment are, for increasing the static stability, arranged in such a way, that the centre of gravity is as low as possible. This is achieved e.g. by using heavy feet for the machine.

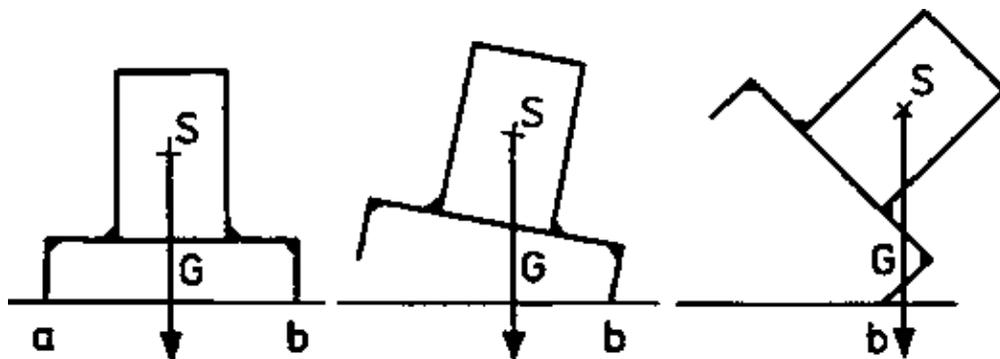


Fig. 5.36. By supporting legs standing wide apart, the static stability is increased.

5.7.3. Centre of gravity of a surface

The centre of gravity of surfaces can be ascertained by way of experiments. To this end, the area to be examined is cut out of uniformly thick material, e.g. of steel sheet of 1 mm thickness.

Then the area is suspended freely rotatable (position 1 in fig. 5.37.). The resulting weight of the sheet metal disk always shows toward the earth's centre, and goes through the centre of gravity. Besides, the disk rotates in such a way that the line of action of G runs through the centre of rotation.

The centre of gravity has to be on the line of action of the weight. This is the straight line a-b.

Then the area is rotated, suspended again, and the straight line c-d is obtained (position 2 in fig. 5.37.).

The centre of gravity lies both on the straight line a-b and on the straight line c-d. This demand is only met by the intersecting point of the two straight lines. The intersecting point of straight line a-b with straight line c-d is the centre of gravity, consequently (position 3 in fig. 5.37.).

Now the material can be changed. Instead of steel sheet, cardboard, plywood or another material is selected: instead of 1 mm thickness, 2.0 mm or 3.0 mm are selected.

In each case, the same centre of gravity will be obtained. Therefore the material and the thickness can be disregarded while talking of the centre of gravity of a surface directly.

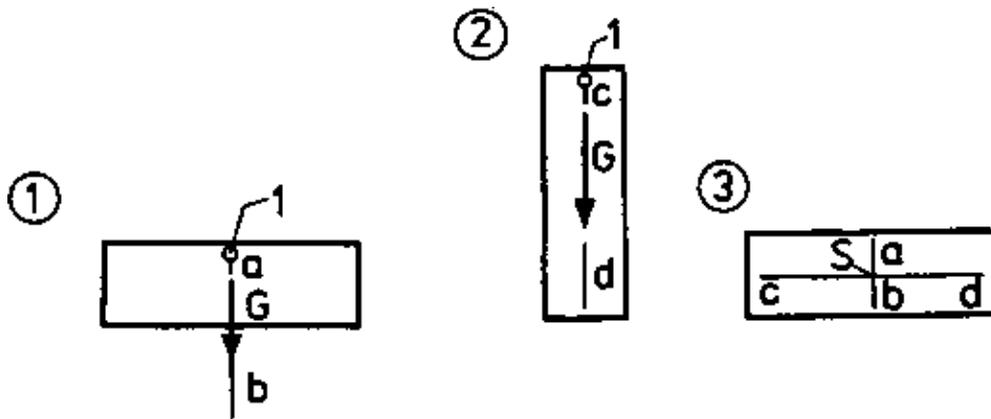


Fig. 5.37. Determination of the centre of gravity of a surface; S centre of gravity. 1 centre of rotation.

The position of the centre of gravity of a circle, rectangle and triangle is represented in fig. 5.38.

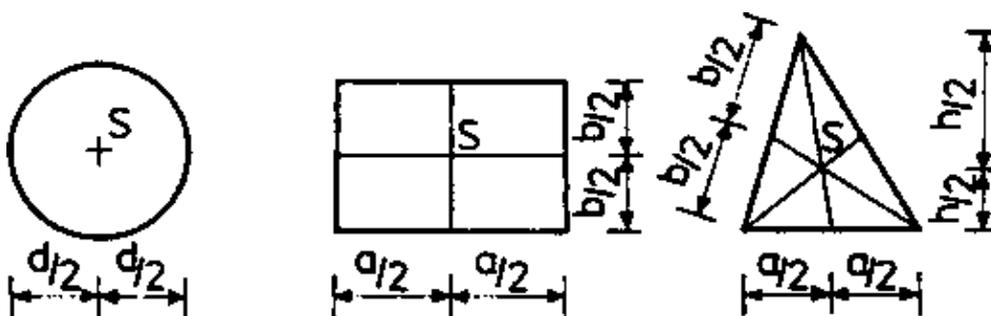


Fig. 5.38. Position of the centre of gravity of a circle, rectangle and triangle.

The centre of gravity of a circle is its central point. The centre of gravity of a rectangle is on the intersecting point of the bisector.

The centre of gravity of a triangle lies on the intersecting point of the bisector, too. The sides of the triangle are halved and connected with the opposite corner of the triangle. The centre of gravity of a triangle always lies at $h/3$.

5.8. Tasks

5.1. Ascertain graphically the resultant force to the layout plans represented in fig. 5.39.

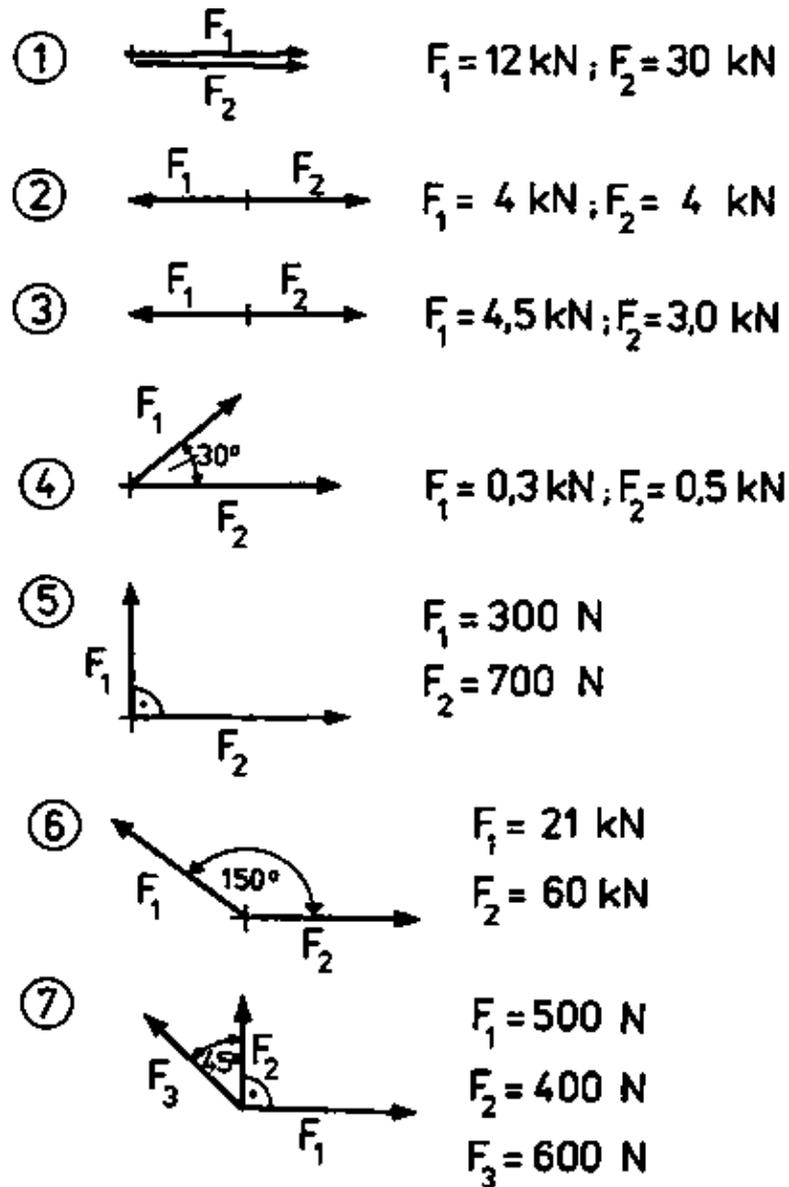
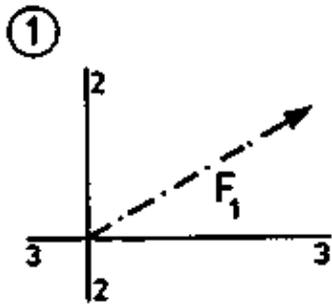


Fig. 5.39. Layout plans to task 5.2.

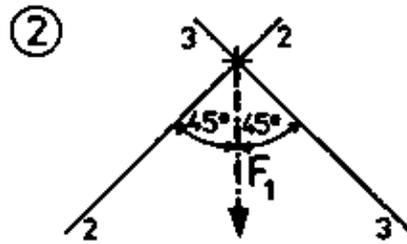
5.2. The following forces act horizontally from left to right.

$$\begin{aligned} F_1 &= 10 \text{ N} \\ F_2 &= 75 \text{ kN} \\ F_3 &= 0.9 \text{ kN} \end{aligned}$$

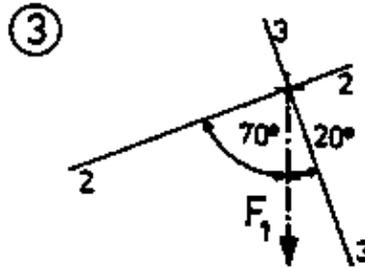
Represent these forces graphically by the aid of selected scales of forces!



$F_1 = 25 \text{ kN}; F_2 = ?; F_3 = ?$



$F_1 = 500 \text{ N}; F_2 = ?; F_3 = ?$



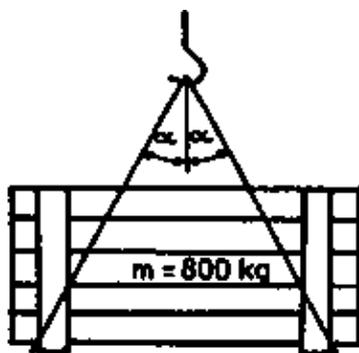
$F_1 = 1,5 \text{ kN}; F_2 = ?; F_3 = ?$

Fig. 5.40. Layout plans to task 5.3.

5.3. Break up the forces F_1 represented in fig. 5.40. into two forces each, in the directions 2–2 and 3–3

5.4. A mass of $m = 800 \text{ kg}$ hangs in a hoisting sling at a crane hook. Calculate the forces in the rope for the cases represented in fig. 5.41.

5.5. A wagon with a mass of $m = 2 \text{ t}$ stands on an inclined plane. Ascertain graphically the slope descending force and normal force for the cases represented in fig. 5.42.

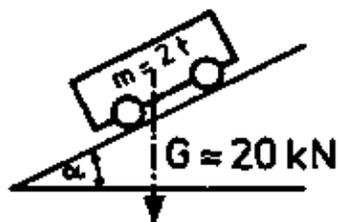


① $\alpha = 30^\circ$

② $\alpha = 40^\circ$

③ $\alpha = 50^\circ$

Fig. 5.41. Layout plan to task 5.4.



① $\alpha = 10^\circ$

② $\alpha = 20^\circ$

③ $\alpha = 30^\circ$

Fig. 5.42. Layout plan to task 5.5.

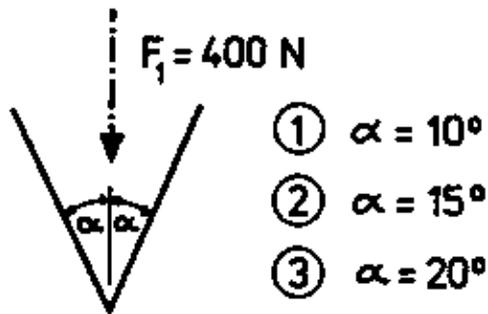


Fig. 5.43. Layout plan to task 5.8.

- 5.6. Name three examples from production or engineering for the application of the inclined plane.
- 5.7. Why has a wagon, which was left on an inclined plane, to be secured against inadvertent motion?
- 5.8. At the two-sided wedge, represented in fig. 5.43., acts a driving-in force $F_1 = 400\text{ N}$. Ascertain graphically the lateral forces.
- 5.9. Name three examples from engineering or production for the application of the two-sided wedge.
- 5.10. How do the frictional forces act at the onesided wedge, which is employed for machine construction for joining two machine elements?
- 5.11. How do the frictional forces act at the thread of a screw?
- 5.12. Name three examples from production where the thread is employed
- a) as fastening screw thread
 - b) as thread for transmitting motions.
- 5.13. Name one example from production or engineering each where the static stability of a body is improved by
- a) a large base,
 - b) wide distances of standing legs,
 - c) a low centre of gravity

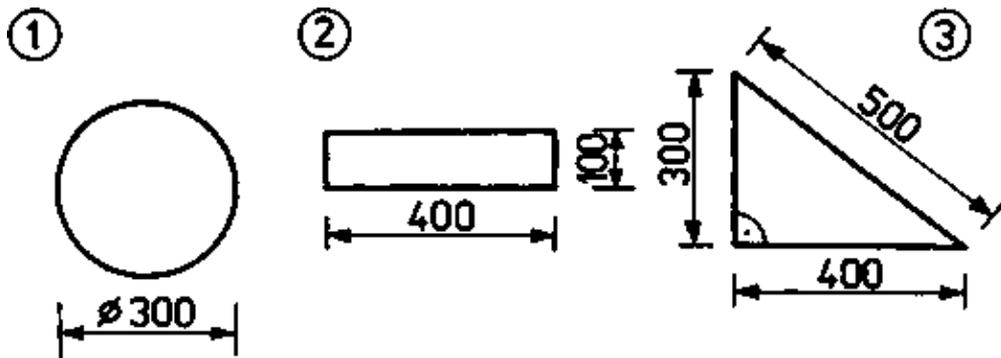


Fig. 5.44. Layout plans to task 5.14.

- 5.14. Determine the position of the centre of gravity of the planes represented in fig. 5.44. Enter the position of the centre of gravity into a diagram true to scale.

6. Strength of materials

6.1. Introduction

6.1.1. Problems of the strength of materials

In engineering and production there act manifold forces:

- All machines, devices and structural parts have a weight, and press with this force on their bearing surface.
- By means of lever, pulley, rope, toothed gear, belt pulley, and others, the forces are converted and transmitted.
- Any change of the path of motion and speed (slowing-down or acceleration) requires the action of a force, see also para' 2.3.2.

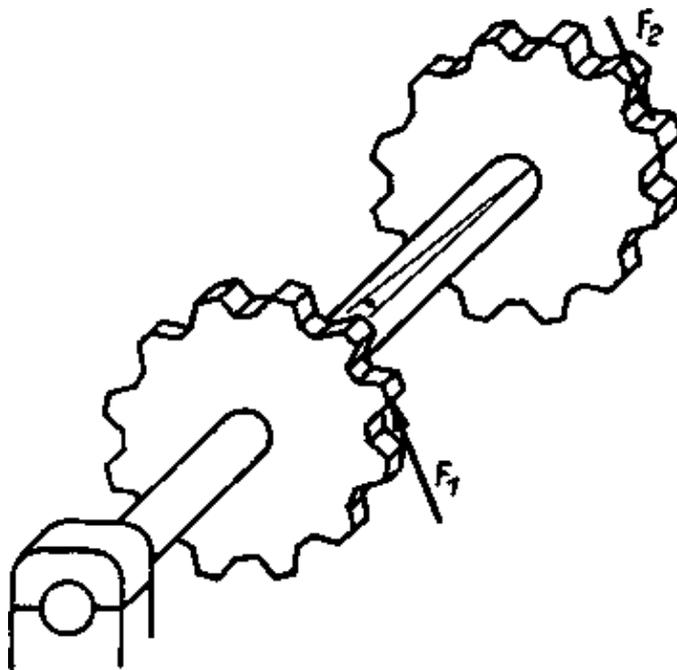


Fig. 6.1. Torsional stress of a shaft. The shape of the gears was highly simplified

By means of these forces, the machine elements and structural members are pulled, upset, bent or twisted.

The material is stressed in quite different ways. The shaft transmits a torque. It generally consists of a solid section (fig. 6.1.).

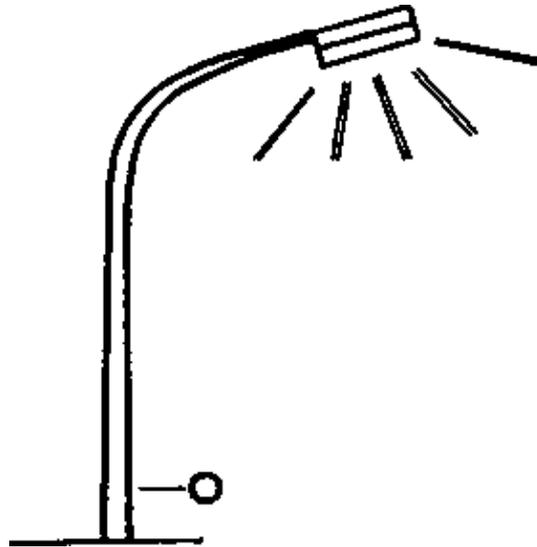


Fig. 6.2. Lighting poles often consist of a thin-walled hollow section only

Lighting poles (fig. 6.2.) are firmly anchored in the ground. At the head they carry a luminous body. The high pole only consists of a circular hollow section with very thin walls. The wall of few millimetres thickness can fully resist the wind load.



Fig. 6.3. Rolled sections of steel

The girder of the crane track in workshops consists, if manufactured from steel, of an I-section. In steel construction, rolled sections (fig. 6.3.) are used at large numbers.



Fig. 6.4. Corrugated board is much used for packaging

With the packaging of industrial goods, foodstuffs etc., corrugated board (fig. 6.4.) figures large. The paper itself has a low strength. If glued together in a wave-like form, however, the corrugated board, coming to exist, obtains such a high strength that it can form large cardboard boxes and carry them. Corrugated iron, used as roofing, is of similar shape.

In nature, forms also occur which are specially adapted to their task. The wheat-ear is carried by a thin stalk. The heavy ear stresses a very slender stalk. When observing this stalk more closely, you will find that it is hollow inside like the lighting pole. The stalk consists only of a tube with very thin walls.

Why do these peculiar forms of sections occur? Why are steel and other metals rolled as I, angle (L) and channel (U) sections?

In view of corrosion, it would be more favourable to select solid sections. Solid sections would have but a small surface, and could be pointed at smaller expense.

To these and other questions, the strength of materials gives an answer.

Strength analysis examines the deformations and stresses, occurring in elastically solid bodies under the action of external forces (loads).

Stresses are surface forces, which are formed inside the body. By strength analysis, the size of these stresses is calculated. It also calculates the deformation, occurring when the body is stressed. From the calculated stresses and deformations it can be derived, what kind of material at what dimensions has to be selected. By means of strength analysis, the engineer calculates machines and devices, structural

components and whole buildings. By these calculations the engineer secures that the machines and devices, components and buildings can absorb the scheduled loads.

The skilled worker works according to finished drawings. The skilled worker is supposed to know selected fundamentals on strength of materials to help them understand the construction of machines and other technical constructions better. Knowledge helps to avoid errors.

6.1.2. Forms of stress

The stress of individual machine members and components is highly different.

With the screw connection it was represented that the shank of the screw is stressed for tension. The plates to be joined are pressed together. The girder of the crane track sags by few millimetres.

All different stresses can be traced back to six basic forms of stress.

These basic forms of stress are:

- Tensile stress
- Compressive stress at the compact body;
- Compressive stress at the selder body (buckling);
- Bending stress;
- Torsional stress and
- Shearing stress.

With concentric tension (fig. 6.5.), the force acts on the centre of the sectional area and acts in the direction of the bar axis. If tension is mentioned in the following, this type of stress is meant. With tension, the body is extended, it becomes somewhat longer.



Fig. 6.5. Tensile stress

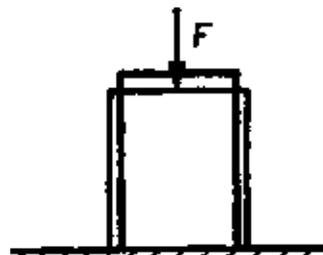


Fig. 6.6. Compressive stress

With concentric pressure, the force compresses the body. It is applied in the centre of gravity of the sectional area and acts in the direction of the bar axis.

If the body has a compact shape, it becomes shorter and wider (fig. 6.6.). If the body is slender, however, like e.g. the drill of a drilling machine, then the body will give way sideways in case of excessive load, fig. 6.7. This deformation is also termed buckling.

With bending, the stress acts at right angles to the bar axis. According to the kind, how the bar is supported, it gets deformed. If it is supported in bearings to the right and left, then it will get deflected as shown in fig. 6.8a, left. Fig. 6.8b, right, shows the deflection with the bar being clamped at one side.



Fig. 6.7. Buckling stress



Fig. 6.8. Bending stress

Of course, the deformations are very small. In machine construction, they amount to fractions of millimetres, and even a large bridge will sag but few centimetres.

The shear stress also acts at right angles to the bar axis (fig. 6.9.). The outer force tries to shift two neighbouring sections of the material against each other. Such a stress occurs in riveted joints, stud bolt joints and also in welded joints.

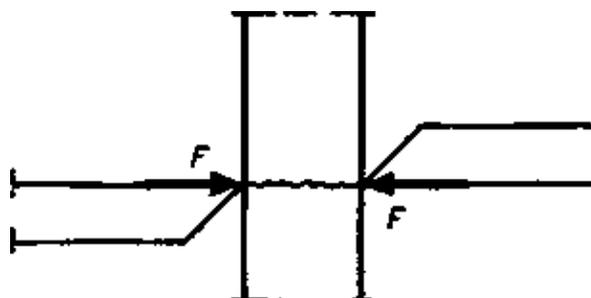


Fig. 6.9. Shearing stress.

And, to conclude with, the torsional stress very frequently occurs in machine construction. Fig. 6.1. shows a shaft with two gears. In the front gear there acts a force F_1 , which produces a torque. The gear is firmly connected with the shaft. Thereby the torque is transmitted to the shaft, and is conducted by the shaft to the back gear. This gear transmits the torque to other gears. In the shaft, the torque tries to twist neighbouring sections against each other. As deformation, a torsion of the shaft will occur. When a straight line is marked on the unloaded shaft, then this straight will run twisted by a small angle after loading.

Apart from these stresses, which are determined by the geometrical shape of the machine, the load in proportion to time figures large.

A static loading is applied and retains its quantity. Typical static loadings are the weights of machines and devices.

Alternating loads frequently change their quantity. Thus single components of an internal combustion engine are stressed several thousand times a minute alternately, first on tension, then on compression.

6.1.3. Tensions

When the loads act on the respective machine parts, components etc., then corresponding bearing forces are formed in the bearing. These forces in the bearing are also termed supporting forces. Loads and supporting forces act on the machine part, e.g. a shaft, from outside. Thereby inside forces come to exist in the interior of the material, which stress the coherence of the single cross sections. These inside forces are the tensions. Tension is the force component, which has to be carried by one unit area.

$$\text{Tension} = \text{force component per unit area}$$

The tension may stand at right angles to the sectional area (fig. 6.10.). A tension in this position is termed normal tension, and provided with the formula sign σ .

If the tension takes its course in the sectional area, however (fig. 6.11.), then it is a tangential stress. The tangential stress receives the formula sign τ .

The unit of tension is Pa; kPa; mPa or also kN/cm and N/mm².¹⁾

¹⁾ See also appendix SI units

Normal tension occur with tensile, compressive and bending stress. Tangential stresses occur with shear stress and torsional stress.

The way how tensions are distributed in the cross section, is described in the following paragraphs.

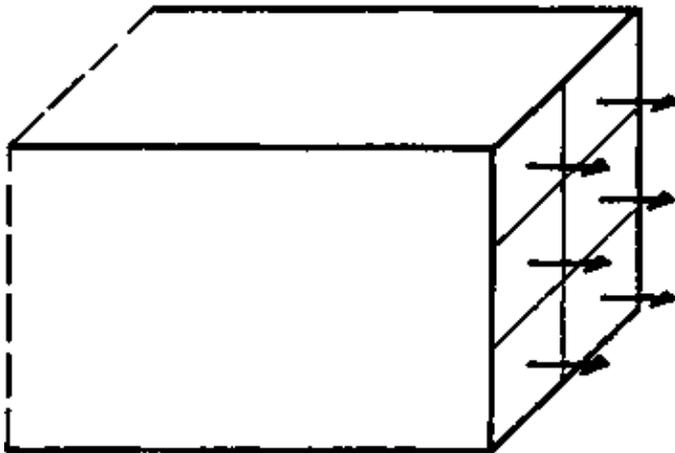
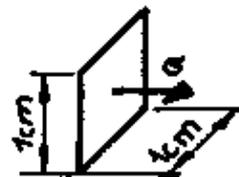


Fig. 6.10. Normal stress ?



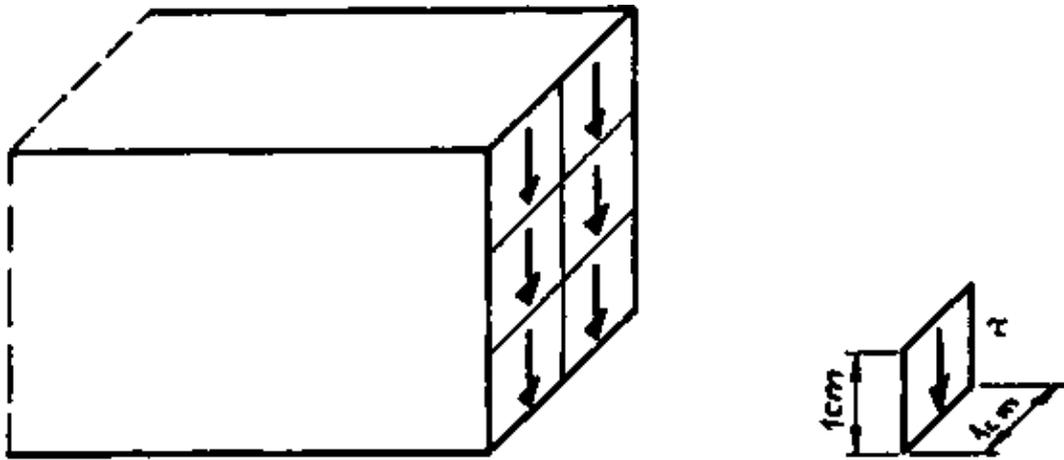


Fig. 6.11. Tangential stress ?

6.2. Change in length

6.2.1. Change in length by external forces

The simplest deformation is the increase in length (extension) with the action of a tensile force.

In order to recognize how a material, e.g. a steel bar, deforms under load, the bar is stressed by a large material testing machine for tension. The machine seizes the bar with special clamping devices, and loads it by uniformly increasing tensile force.

Fig. 6.12. shows a bar of 300 mm length and a square cross section of 20 mm × 20 mm. In the bar axis, the bar is subjected to tension by force F. The force F has to act on both sides of the bar, see also para' 5.4. counterforce.

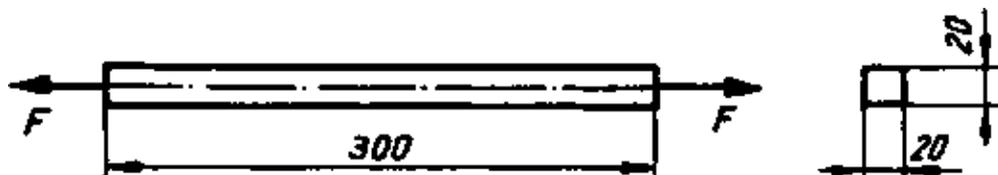


Fig. 6.12. Tensile stress at a steel bar.

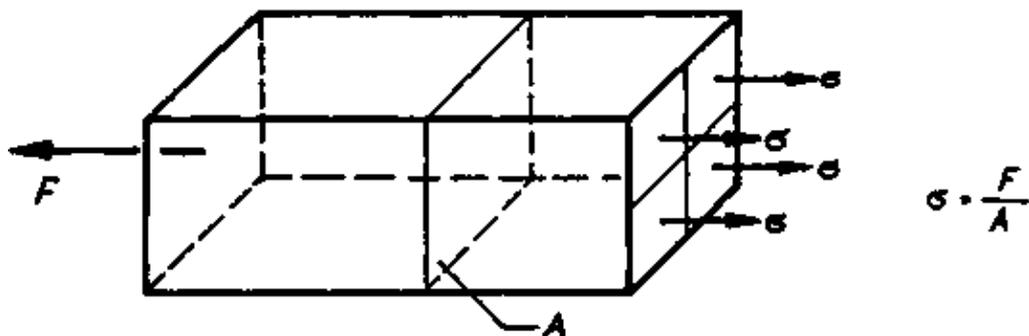


Fig. 6.13. With tensile load, the tensile stress ? comes to exist.

Force F wants to break the bar. This is prevented by the strength of the material. Under load, a uniformly distributed tension is formed in the sectional area. Fig. 6.13. shows the tension in the cross section.

The tension stands at right angles to the sectional area. It is a normal tension ?.

Each square centimetre of the sectional area carries the same force component. The tension in the cross section is alike at all places. Then applies

$$\sigma = \frac{F}{A}$$

σ = tensile stress
 F = external force
 A = sectional area

With this stress, the bar becomes longer. Fig. 6.14. shows that the bar of an original length l_0 was extended to a length $l = l_0 + \Delta l$.

The increase in length is designated with Δl .

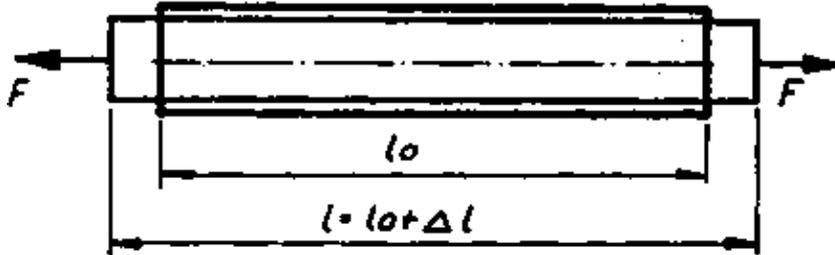


Fig. 6.14. With tensile load the bar becomes longer.

The increase in length depends on the basic length l_0 of the bar. Long bars have a larger increase in length than short bars if an equally large tensile force acts on them. In order to enable a comparison of bars at any length, the proportion of increase in length to basic length is designated as strain ϵ .

$$\text{Strain} = \frac{\text{increase in length}}{\text{basic length}}$$

$$\epsilon = \frac{\Delta l}{l_0} \quad (6.2.)$$

or

$$\epsilon = \frac{\Delta l}{l_0} \cdot 1,000\%$$

The strain has no unit. Sometimes the strain is stated in %.

In the following, a tensile test is evaluated as an example, which was performed at the steel bar represented in fig. 6.12. Table 6.1. shows the ascertained values.

The bar has a basic length of 300 mm.

In unloaded state, the bar is 300 mm long.

The sectional area amounts to

$$\begin{aligned}
 A &= 2.0 \text{ cm} \cdot 2.0 \text{ cm} \\
 A &= 4 \text{ cm}^2
 \end{aligned}$$

In unloaded state, $\Delta l = 0$ and $\epsilon = 0$.

Now the bar was loaded with $F = 11.2 \text{ kN}$. By the aid of very precise measuring instruments it was ascertained that the bar now has a length of $l = 300.04 \text{ mm}$. Hence, the increase in length amounts to 0.04 mm.

From that follows for the strain according to equation 6.2.

$$\varepsilon = \frac{\Delta l}{l_0}$$

$$\varepsilon = \frac{0.04\text{mm}}{300\text{mm}} = 0.000133$$

Table 6.1. Values to the tensile test

F	0	11.2	22.4	33.6	44.8	56.0	67.5	kN
l	l ₀ = 300.00	300.04	300.08	300.12	300.16	300.20	300.24	mm
Δl	0	0.04	0.08	0.12	0.16	0.20	0.24	mm
$\varepsilon = \frac{\Delta l}{l_0}$	0	0.133	0.267	0.400	0.533	0.667	0.800	10 ⁻³
$\sigma = \frac{F}{A_0}$	0	2.8	5.6	8.4	11.2	14.0	16.8	$\frac{\text{kN}}{\text{cm}^2}$
$\sigma = \frac{F}{A_0}$	0	28	56	84	112	140	168	MPa

The tension, according to equation 6.1., amounts to

$$\sigma = \frac{F}{A_0}$$

$$\sigma = \frac{11.2\text{kN}}{4\text{cm}^2} = 2.8\text{kN/cm}^2$$

$$\sigma = 28\text{MPa}$$

In the same way, the bar was loaded with

F = 22.4 kN; F = 33.6 kN etc. Each time, the length l was measured and tension σ and strain ε were calculated.

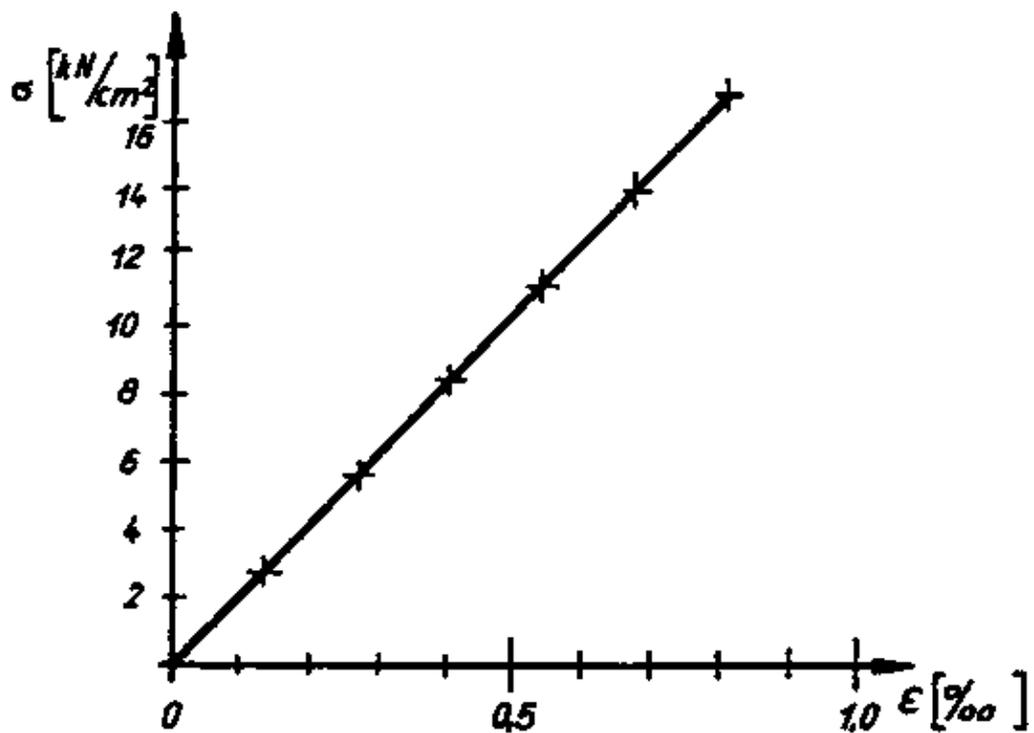


Fig. 6.15. Tension-strain diagram to table 6.1.

Grey cast iron	$E = 7.5 \text{ to } 10.8 \text{ MN/cm}^2$ $= 75,000 \text{ to } 105,000 \text{ MPa}$
Copper	$E = 12.5 \text{ MN/cm}^2$ $= 125,000 \text{ MPa}$
Brass	$E = 8 \text{ to } 10 \text{ MN/cm}^2$ $= 80,000 \text{ to } 100,000 \text{ MPa}$
Nickel	$E = 20 \text{ to } 22 \text{ MN/cm}^2$ $= 200,000 \text{ to } 220,000 \text{ MPa}$
Silver	$E = 7 \text{ to } 8 \text{ MN/cm}^2$ $= 70,000 \text{ to } 80,000 \text{ MPa}$
Steel	$E = 20 \text{ to } 22 \text{ MN/cm}^2$ $= 200,000 \text{ to } 220,000 \text{ MPa}$
Cast steel	$E = 21 \text{ MN/cm}^2$ $= 210,000 \text{ MPa}$
Tungsten	$E = 35 \text{ to } 40 \text{ MN/cm}^2$ $= 350,000 \text{ to } 400,000 \text{ MPa}$
Zinc	$E = 11 \text{ to } 13 \text{ MN/cm}^2$ $= 110,000 \text{ to } 130,000 \text{ MPa}$
Tin	$E = 4 \text{ to } 5 \text{ MN/cm}^2$ $= 40,000 \text{ to } 50,000 \text{ MPa}$

When the elastic modulus is known, the change in length for a given load can be calculated.

Example

A steel bar of 400 mm length with a diameter of 20 mm is subjected to tension by a force $F = 30 \text{ kN}$ (fig. 6.16.).

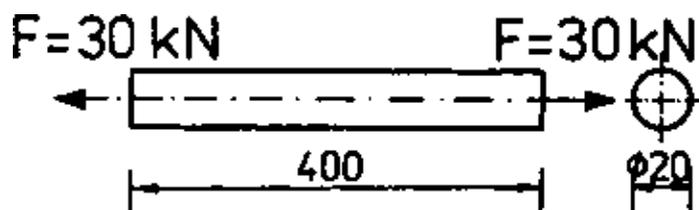


Fig. 6.16. Tensile stress at a bar

How large is the tension, the strain and the increase in length:

Given:

$$l_0 = 400 \text{ mm}$$

$$d = 20 \text{ mm}$$

$$F = 30 \text{ kN}$$

Required:

Tension ?
Strain ?

Increase in length ?l

Solution:

Tension

$$\sigma = \frac{F}{A}$$

$$A = \frac{\pi d^2}{4} = \frac{3.14 \cdot (2 \text{ cm})^2}{4} = 3.14 \text{ cm}^2$$

$$\sigma = \frac{30 \text{ kN}}{3.14 \text{ cm}^2} = 9.55 \text{ kN/cm}^2$$

$$? = 95.5 \text{ MPa}$$

Strain

$$? = ? E$$

$$\varepsilon = \frac{\sigma}{E}$$

$$\varepsilon = \frac{9.55 \text{ kN/cm}^2}{21 \text{ MN/cm}^2} = \frac{9.55 \text{ kN/cm}^2}{21.000 \text{ kN/cm}^2}$$

$$? = 0.000455$$

Increase in length:

$$\varepsilon = \frac{\Delta l}{l_0}$$

$$?l = l_0 ?$$

$$?l = 400 \text{ mm} \cdot 0.000455$$

$$?l = 0.182 \text{ mm}$$

Result:

$$\sigma = 9.55 \frac{\text{kN}}{\text{cm}^2}$$

The increase in length only amounts to ?l = 0.182 mm at a tension

Note:

Even at large tensions, but small changes in length will occur.

6.2.2. Change in length by temperature variation

Solid bodies increase their lengths if heated up. They reduce their lengths if the temperature drops.

Precise measurements proved:

The change in length is dependent

- on the material,
- on the temperature differential and
- on the basic length of the body.

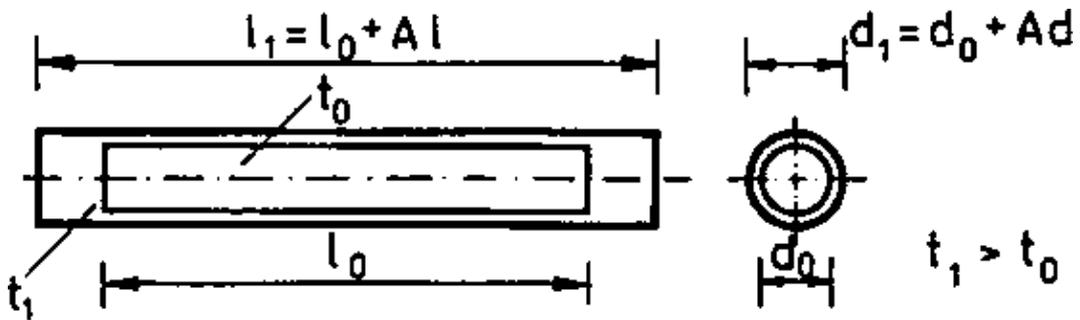


Fig. 6.17. Increase in length by heating up. The change in length was represented highly exaggerated.

The basic length l_0 belongs to the initial temperature t_0 . When the bar is heated up to temperature t_1 , it expands to length l_1 .¹⁾

¹⁾ On the unit of temperature see appendix SI units.

The evaluation showed that the increase in length amounts to

$$\Delta l_t = \alpha_t \cdot \Delta t \cdot l_0$$

$$l_1 = l_0 + \Delta l$$

Δl : increase in length

α : material-dependent factor (see table 6.3.)

Δt : $t_1 - t_0$ = temperature differential

l_0 : initial length

Table 6.3. Coefficient of thermal expansion

Steel	$\alpha = 0.000,010$ to $0.000,012$ $1/K$
Invar steel	$\alpha = 0.000,001$ $1/K$
Aluminium alloys	$\alpha = 0.000,025$ $1/K$
Copper	$\alpha = 0.000,018$ $1/K$
Zinc	$\alpha = 0.000,031$ to $0.000,035$ $1/K$

Example

A steel bar has a temperature of 20 °C and a length of $l_0 = 400.00$ mm.

How large is its length if it is heated up to 70 °C?

Given:

$$l = 400 \text{ mm}$$

$$t_0 = 20 \text{ °C}$$

$$t_1 = 70 \text{ °C}$$

Required:

$$\Delta l$$

Solution:

The temperature differential amounts to

$$\Delta t = t_1 - t_0$$

$$\Delta t = 70 \text{ °C} - 20 \text{ °C} = 50 \text{ K}$$

From that follows the increase in length

$$\Delta t = t - t_0$$

$$\Delta l = 0.000012 \frac{1}{K} \cdot 50K \cdot 400mm = 0.24mm$$

Result:

The increase in length amounts to 0.24 mm.

If machine elements and components are heated up and their expansion is obstructed, large tensions will come to exist in the component, which may even result in the destruction of the machine element or component.

For this reason

- pipings, which have to convey e.g. water of highly differing temperatures, are equipped with expansion bends, which permit a change in length (fig. 6.18.);
- bridges and roof structures have a movable support, which also permits changes in length.

In machine construction, the change in length at variations of temperature is utilized in pressed connections.



Fig. 6.18. Pipe expansion bends prevent damage to pipings, which convey liquids at highly differing temperatures.

Fig. 6.19. shows how an inner ring of an antifriction bearing can be fastened to a shaft without further auxiliary devices. In state 1, both elements have the same temperature $t_0 = 20 \text{ }^\circ\text{C}$. In this state, the ring cannot be pushed on to the shaft as the diameter of the shaft is too large.

In state 2, the shaft is cooled to $t_1 = -190 \text{ }^\circ\text{C}$. The ring retains the temperature $t_0 = 20 \text{ }^\circ\text{C}$. Due to cooling, the shaft reduces its length and its diameter. Now the ring can be slipped on to the shaft.

In state 3, both elements have a temperature $t_0 = 20 \text{ }^\circ\text{C}$ again. The shaft tried to expand again, and pressed against the inner ring of the antifriction bearing. This again was expanded a little, and presses against the shaft. Thus large contact forces come to exist in the area of contact between shaft and inner ring of antifriction bearing. Thereby large frictional forces exist in the area of contact. These frictional forces prevent a shifting of the two elements, and unremovably connect ring and shaft with each other.

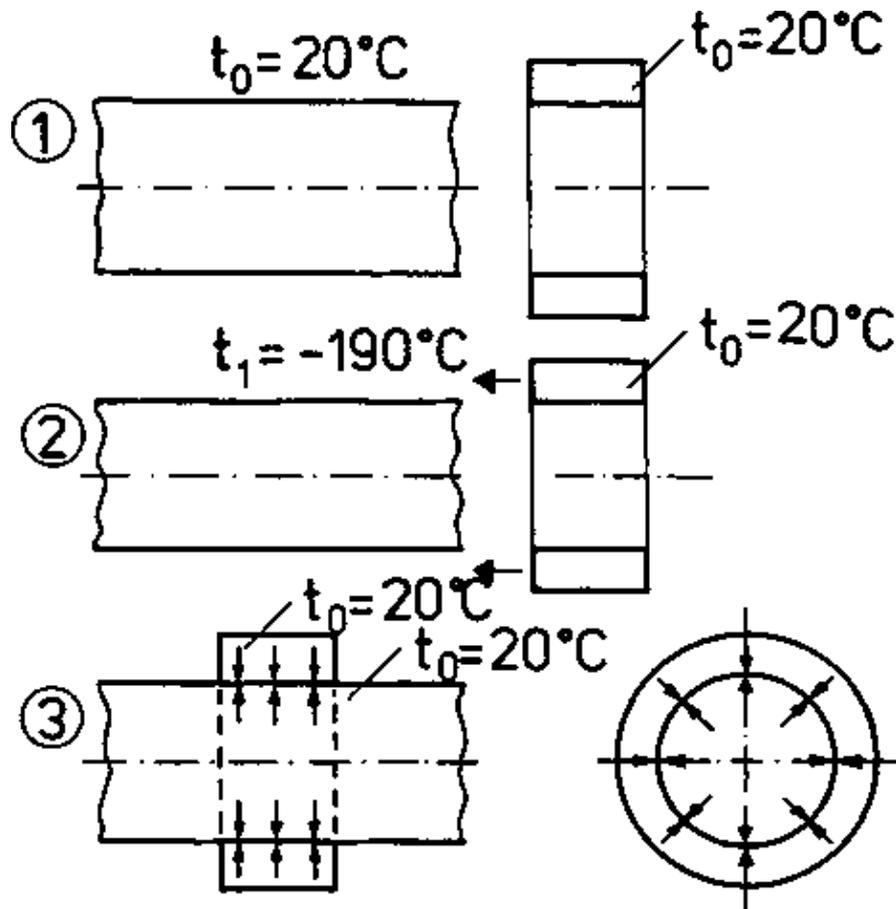


Fig. 6.19. Operations with the manufacture of a pressed connection

Summary

By the action of tensile force or temperature variation, the length of solid bodies is changed. The change in length is dependent on

- the material,
- the size of the tensile force,
- the temperature differential and
- the basic length of the body.

The expansion is the proportion of increase in length and basic length.

6.3. Stress-strain diagram

6.3.1. Tensile test

In materials testing machines it is tested how materials behave if subjected to tension. Thus, e.g., a steel bar is subjected to continuously increasing tension. Its load and increase in length are measured, and represented in a stress-strain diagram.

A steel grade of widespread application is St. 38. This steel combines a sufficient strength with good toughness. Fig. 6.20. shows the stress-strain diagram, which comes to exist when testing St 38.

The force is uniformly distributed over the sectional area. Hence is applicable

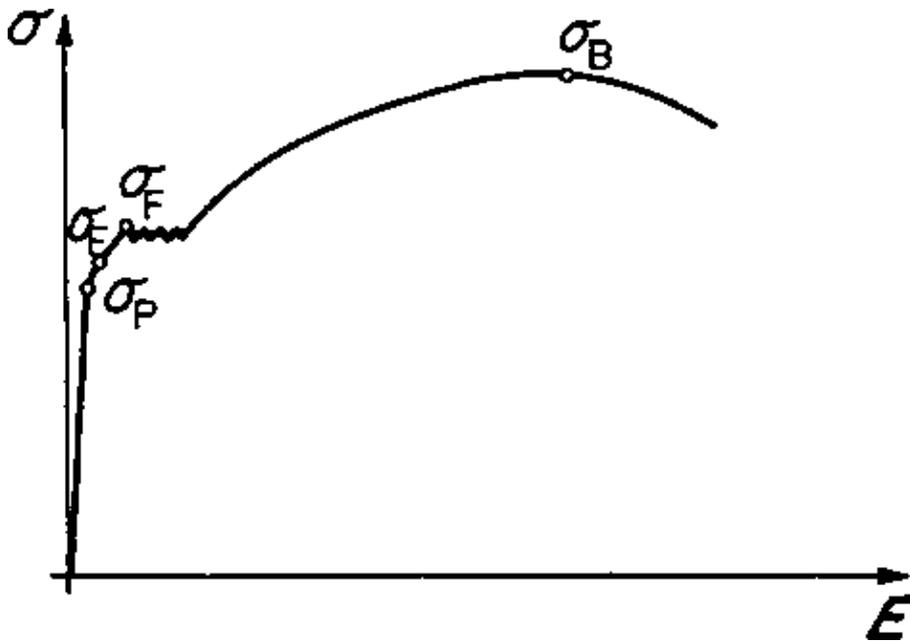


Fig. 6.20. Stress–strain diagram of steel St 38 at a tensile test.

$$\sigma = \frac{F}{A_0}$$

?: stress

F: tensile force in the bar

A₀: sectional area in unloaded state

The increase in length is measured by highly sensitive measuring instruments.

The strain follows from the increase in length.

$$\varepsilon = \frac{\Delta l}{l_0}$$

?: strain

!l: increase in length

l₀: length in unloaded state

In the first phase, but slight increases in length will occur. Although the stress rises, the bar gets only little longer. The increase in length is not visible with the naked eye.

Up to the limit of proportionality ?_p, Hooke's law is applicable. The strain is proportional to the stress (see also para' 6.2.).

After the stress has exceeded the limit of proportionality, the strain increases somewhat more quickly than the stress. In this, and in the following ranges, the values for ? and ? no longer lie on a straight line.

Up to the limit of elasticity ?_E, the bar is fully elastic, practically. When the bar is relieved of load, it assumes its original form and length again. If the stress rises beyond the limit of elasticity, however, then a permanent deformation (increase in length) will remain at the bar.

If the stress is further increased, the steel will start to flow at reaching the yield point ?_F. The bar becomes distinctly longer without the load being increased. Then the steel strain–hardens again. The load can be further increased.. At a further increase of the stress, large increases in length will occur.

The seemingly highest point of stress is reached by the bar at tensile strength ?_B. If this point is exceeded, the steel forms a neck and breaks.

For the frequently used steel grades St 38 and St 50, the following minimum strength values are applicable:

	Yield point σ_F	Tensile strength σ_B
Steel	14 kN/cm ²	38 kN/cm ²
St 38	140 MPa	380 MPa
Steel	30 kN/cm ²	50 kN/cm ²
St 50	300 MPa	500 MPa

6.3.2. Admissible strain and safety

A bar of St 38 will break only if subjected to a stress $\sigma = 38 \text{ kN/cm}^2$, i.e. $\sigma = 380 \text{ MPa}$.

38 kN/cm are the weight of 3.8 m³ of water! A bar with a cross section of 1 cm² is able to carry almost 4 m³ of water.

This high strength cannot be fully utilized, however. The following reasons tell against that:

- Before the bar reaches these high stresses, it gets too strongly deformed. To this high stress belongs a strain of approx. 0.25. A bar of 200 mm length is expanded to a length of 250 mm, i.e. the increase in length amounts to 50 mm !

Deformations of that size cannot be permitted in steel and machine construction, otherwise the machines and structures cannot fulfill their tasks.

- Although the steels are manufactured at high uniformity, a small fault in material cannot be absolutely excluded.

For these and other reasons, only a fraction of the carrying capacity of the material is exhausted. An admissible stress is fixed as a limit.

The stress existing in the component may not exceed the admissible stress.

It must be applicable

existing stress \leq admissible stress

Table 6.4. Admissible stresses at static loading

	Steel St. 38	Steel St. 52
Tension, pressure, bend	16.0 kN/cm ² = 160 MPa	24.0 kN/cm ² = 240 MPa
Shearing stress	9.2 kN/cm ² = 92 MPa	13.9 kN/cm ² = 139 MPa

When the admissible strain utilizes only part of the strength of a material, part of the carrying capacity remains unused for carrying reserve. It serves for safety.

$$\text{Safety} = \frac{\text{ultimate tensile strength}}{\text{admissible stress}}$$

The safety must always be larger than 1.

For example, with a steel St 38, there are the

Admissible stress = 16 kN/cm² = 160 MPa

Ultimate tensile strength = 38 kN/cm² = 380 MPa

From that follows

$$\text{Safety} = \frac{38\text{kN/cm}^2}{16\text{kN/cm}^2} = 2.375$$

The admissible stresses stated in table 6.4. are applicable to static loadings. In some machines, however, strong and frequently changing loads occur. With changing loads, the admissible stress decreases.

Summary

The stress–strain diagram renders information on the behaviour of materials if subjected to tension. The highest point of tensile stress is the tensile strength. On exceeding this point, the material breaks.

For technical reasons and for reasons of safety, only part of the tensile strength is utilized. With various materials, the admissible stress is fixed as a limit which may not be exceeded. At static loading, this admissible stress is larger than with strongly changing load.

6.4. Tensile and compressive stress

6.4.1. Occurrence

Ropes conduct tensile forces. These forces produce a tensile stress in the rope.

Fig. 5.18. shows a hoisting sling, and in fig. 5.19. the forces were ascertained, which are generated in the rope by a load. These forces cause the tensile stress.

In the same way, belts in the belt drive are subjected to tension. Fig. 3.19. showed how a force was transmitted by the belt from one belt pulley to another. The belt is subjected to tension, too.

Chains have various shapes. Common to all chains is that they can conduct tensile forces only. The single links of the chain are subjected to tension.

In para' 5.6.6. it was ascertained that at tightening the nut of a screw connection, large tensile forces are formed in the core. In the core of the screw there acts a tensile stress.

Compressive stresses occur in all bearings.

A very frequent form of compressive stress is the surface pressure between two bodies. Thus a large surface pressure is formed in the contact area of wedge and belt pulley, and of wedge and shaft of a keyed joint (figs. 5.27 and 5.29.).

In the base of machines and devices, the surface pressure is caused by the weight of the machines and devices.

Tensile stress	Surface pressure	Compressive stress
ropes;	keyed joint;	machine foundations
belts;	bases of machines and devices	
screws in the core	loading of ceilings with stacked goods	

6.4.2. Stress distribution

Compressive and tensile stresses are normal stresses. They stand vertically on the sectional area. The force is evenly distributed over the cross section. The stress is equal at all points of the cross section. Therefore the following formula is applicable:

$$\sigma = \frac{F}{A}$$

?: compressive or tensile stress

F: force

A: sectional area

With the calculation, the smallest useful cross section in each case is to be examined. The largest stress will occur where the sectional area is smallest.

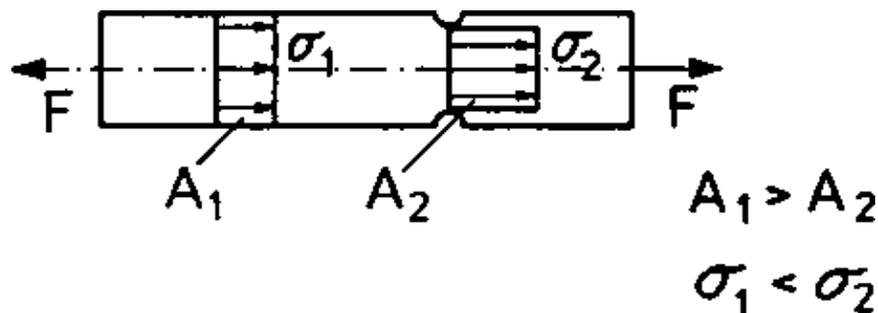


Fig. 6.21. At the weakened cross section A_2 , the largest tensile stress will occur.

In fig. 6.21. it is shown that the cross section A_1 is larger than the cross section A_2 . The force F must be carried by both cross sections.

It applies

$$\sigma_1 = \frac{F}{A_1} \quad \text{and} \quad \sigma_2 = \frac{F}{A_2}$$

If $A_1 > A_2$ it follows $\sigma_1 < \sigma_2$.

The largest stress occurs in the endangered cross section. With surface pressure, a uniform distribution of the force over the area is assumed.

The surface pressure is designated by the formula sign. p .

$$p = \frac{F}{A} \quad (6.5.)$$

p : surface pressure

F : compressive force

A : base

Note:

With tensile and compressive stress, the form of the cross section has no influence on the size of the stresses.

6.5. Bending stress

6.5.1. Occurrence

All machine elements and structural parts, which absorb forces across the bar axis and are supported on bearings, are subjected to bending. The bending stress occurs very frequently.

Fig. 6.22. shows an axle with wheels. The axle is loaded by the vehicle by the forces F . The wheels carry the axle. The axle is subjected to bending. In fig. 6.22. below, the course of deflection is shown. The bearings were drawn symbolically.

Shafts are subjected to bending, too. Fig. 6.23. contains a shaft, which is loaded by two belt pulleys, and rests on two bearings. In the lower part of fig. 6.23., the simplified representation and the form of the bending line are stated.

If an axle is of relatively short construction and firmly clamped at one side only, it is termed bolt.

Fig. 6.24. shows a bolt, which is loaded by a toothed gear. In the lower part of fig. 6.24. it was represented how the bolt gets deformed if subjected to bending.

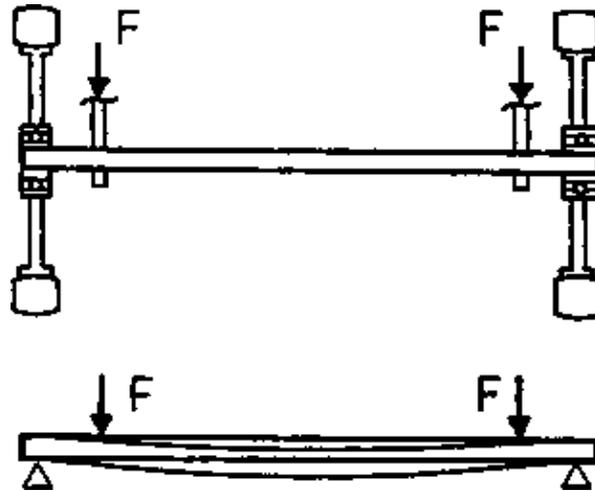


Fig. 6.22. Axle with wheels. Construction of wheels strongly simplified.

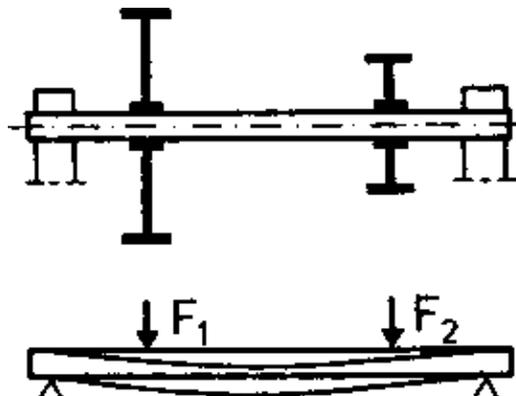


Fig. 6.23 Shaft with two belt pulleys.

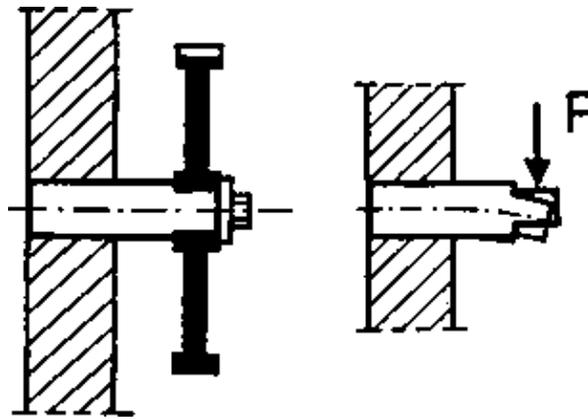


Fig. 6.24. Toothed gear supported by a bolt.

The steel girders, which carry the crane in workshop halls, and the beams of steel or reinforced concrete, which carry the ceilings and roofs, are subjected to bending, too.

6.5.2. Stress distribution

The stress distribution with bending is represented in fig. 6.25. This girder is generally termed girder on two columns. Axles, shafts, ceiling beams can be girders on two columns. The girder is stressed at right angles to its axis. It sags. By that, the bottom side is strained and the top side compressed. In para' 6.2., the correlation of change in length and strain was specified already.

For girders according to fig. 6.25, applies:

Bottom side	Strain	Increase in length	Tensile stress
Top side	Compression	Decrease in length	Compressive stress

With bending, normal stresses occur.

The stresses stand on the cross section vertically.

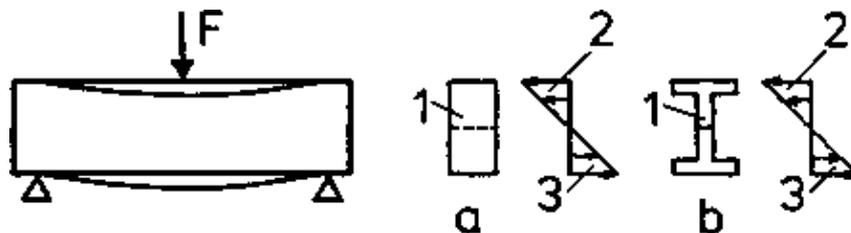


Fig. 6.25. Stress distribution with bending.

1 Neutral fibre; 2 Compressive stresses at top side; 3 Compressive stresses at bottom side; a Rectangular section; b I-section.

If a girder is loaded and supported (carried) in the way as represented in fig. 6.25., there occur compressive stresses at the top side of the bar, and tensile stresses at the bottom side.

Both stresses act in the direction of the bar axis. They act in opposite directions, however. Under these conditions, a layer of fibres must lie between the ranges with tensile stresses and those with compressive stresses,

- which is neither strained nor compressed,
- which will not undergo a change in length and
- in which no stresses occur.

This layer of fibres is called neutral layer of fibres.

Exact calculations prove that the neutral layer of fibres always extend through the centre of gravity of the sectional area.

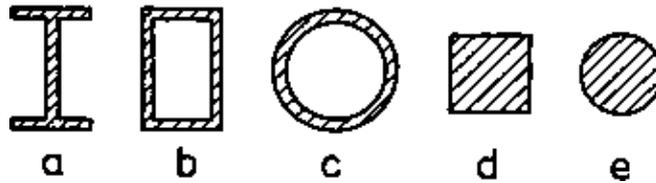


Fig. 6.26. Sections of high load-carrying capacity (a, b, c) and low load-carrying capacity (d, e) at bending stress.

The stress distribution is represented in fig. 6.25, a and b. At the top side there is the largest compressive stress. Then the compressive stress decreases continuously, and reaches the value of zero in the centre of gravity of the sectional area. This layer of fibres is the neutral layer of fibres, which retains its original length.

Below the neutral layer of fibres there begins the tensile stress. The tensile stress reaches its largest value at the bottom side of the bar.

This stress distribution is equal with all cross sections. The stress distribution does not depend on the shape of the cross section.

With a solid section (fig. 6.26, d and e), the material is but little utilized in its load-carrying capacity near the neutral layer of fibres, as but small stresses occur here.

For engineering and production, special sections were developed, which reach a higher load-carrying capacity with bending stress at lower material expenditure. Among these are the cross sections a, b and c in fig. 6.26. These cross sections are marked by the fact that in them, larger portions of the sectional area are arranged on the edge of the cross section, i.e. in the range of high stresses.

With the I-section, e.g., the material is utilized more intensively. In the range of small stresses, i.e. near the neutral fibre, but a narrow web exists. On the top and bottom edge, the largest stresses will occur. The I-section has large surface areas there. By these large surface areas on the edge of the cross section, the I-section can absorb a larger bending stress.

Exact calculations prove:

I-sections and hollow sections can absorb a larger bending stress at equal material expenditure.

Summary

With the bending stress, the top side of girders is subjected to compression, and the bottom side to tension. With bending stress it is therefore most expedient to employ girders with I or hollow sections.

6.6. Buckling stress

6.6.1. Occurrence

All slender machine elements and components are, at higher load, not only compressed, but they buckle sideways, too. This applies both to the drill of a drilling machine and to large steel columns in steel construction, for lighting poles and, in nature, for the stems of cereal plants. The piston rod is subjected to buckling, too (fig. 8.4.).

6.6.2. Direction of buckling and response to buckling

The slender rod can be supported differently on its two ends. Fig. 6.27. shows four possibilities.

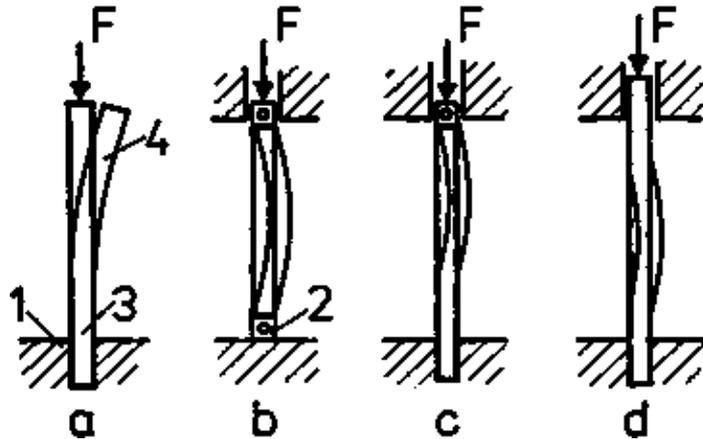


Fig. 6.27. The response to buckling depends on the support of the rod.

1 clamped bearing; 2 hinged joint; 3 stressed rod; 4 buckling line.

In case a, the rod is firmly clamped at one end. The compressive force acts in the direction of the rod axis. At buckling, the rod moves sideways.

In case b, the rod is held by hinged joints at both ends. The buckling line runs from joint to joint.

In case c, the rod is clamped at one end, and held by a joint at the other end.

In case d, the rod is firmly clamped at both sides.

Whereas the rod can buckle completely sideways at application of load in case a, it is held fast at both ends in case b. Thereby the load carrying ability of the rod is increased.

If one side is replaced by a clamped joint, the response to buckling is further reduces, and the rod can carry a larger force F .

To conclude with, the response to buckling is highly limited in case d. This rod can carry the largest force F .

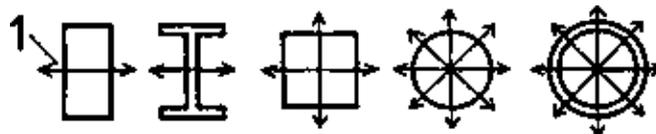


Fig. 6.28. Slender rods buckle into the direction of the smallest width of cross section.

If the rods represented in fig. 6.28. are of equal cross section, their load carrying ability rises from a to d.

The direction of buckling at yielding by buckling depends on the shape of cross section. Buckling always takes place in the direction of smallest width of cross section.

Fig. 6.28. shows the directions of buckling for some selected sections. A circular ring section (tube) can absorb the highest load. Therefore this section is used so frequently in engineering and also in nature.

6.7. Shearing stress

6.7.1. Occurrence

With shearing stress, external forces try to displace single cross sections towards each other. This stress chiefly occurs with fastening devices.

Riveted joints serve for connecting single structural elements.

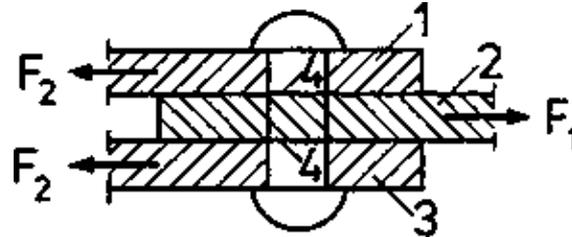


Fig. 6.29. Riveted joints are subjected to shearing off. 1 2 and 3 Metal sheetings, which transmit tensile forces; 4 shear planes.

In the riveted joint, represented in fig. 6.29., the metal sheets 1, 2 and 3 push against the rivet with different directions. The forces acting in the metal sheets try to displace the rivet sections against each other. Thereby stresses are formed in the shear planes, which run in direction of the shear plane.

The shearing stress produces tangential stresses.

Pin joints are stressed similarly.

Riveted joints are applied only in special cases yet. Instead, the single structural members are joined by welds. In welding seams, shearing stresses may occur, too.

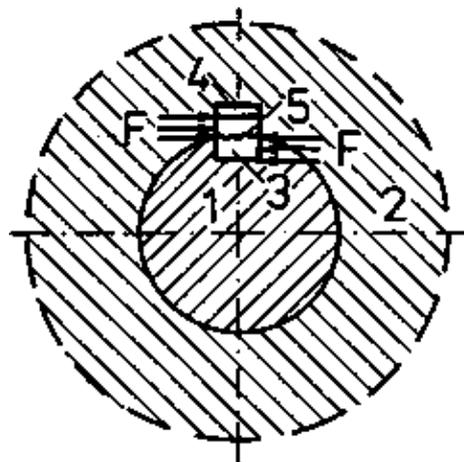


Fig. 6.30. Feather joints are subjected to shearing off.

1 shaft; 2 toothed gear(section);3 spring; 4 air gap; 5 shear plane.

If toothed gears have to run very true, keyed joints cannot be employed as the large contact forces (see para' 5.6.5. and fig. 5.27.) will deform the gear unfavourably. Under these conditions, the key is replaced by a feather. A feather has no attraction, hence it does not produce any contact forces. The feather joints are subjected to shearing off. Fig. 6.30. shows a feather joint as fastening device between a shaft and a toothed gear. The gear was outlined only. The feather is guided in a slot, which permits an air gap between key and feather. Thereby contact forces are excluded. In the feather, a shear plane is formed.

6.7.2. Stress distribution

Shearing stresses produce tangential stresses. The stresses run in direction of the shear plane, see also fig. 6.11.

It is assumed that the force is uniformly distributed over the shear plane.

$$\tau = \frac{F}{A} \quad (6.6.)$$

τ : shearing stress

F: force that stresses the shear plane

A: shear plane

When calculating the shearing stresses, the number of shear planes is to be considered.

As the force is uniformly distributed over the shear plane, the shearing stress is equal at all points of the shear plane.

Note:

The shape of the shear plane has no influence on stressability.

The load carrying ability of shear plane is only determined by its area and material.

6.8. Torsional stress

6.8.1. Occurrence

Torsional stress chiefly occurs at shafts. The task of shafts is the transmission of torques.

In fig. 6.1. a shaft was represented, which carries two gears. Force F_1 acts on the front gear, and this force produces the torque M. This torque is delivered to further gears by the second gear. The torque M has to be transmitted in the shaft from cross section to cross section. This torque displaces adjoining cross sections against one another.

6.8.2. Stress distribution

The largest stresses will occur where the cross sections are displaced largest against one another.

Shafts have circular cross sections. If the cross sections is displaced by angle α , the covered distance δl becomes larger toward the edge (fig. 6.31.). This also applies if angle α is very small. In the centre, $\delta l = 0$.

As can be seen from fig. 6.31., the motion is at the edge of the cross section. The largest stress occurs where there is the largest motion. Hence it is largest at the edge of the cross section, and decreases toward the centre. In the centre, $\delta l = 0$ (fig. 6.32.).

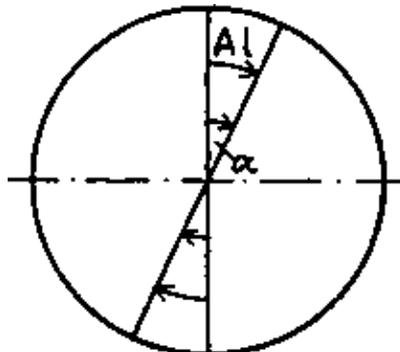


Fig. 6.31. With the torsion of two cross sections against each other, the displacement δl increases toward the edge.

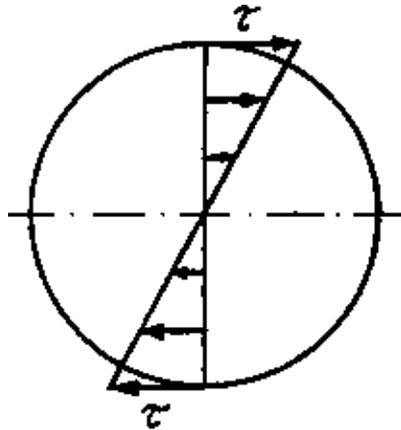


Fig. 6.32. With torsional stress, the stress τ increases toward the edge.

At torsional stress, the following stress distribution will occur:

- tangential stresses will occur;
- at the edge of the cross section, the stresses are largest;
- the stresses become smaller toward the centre;
- in the centre of the cross section, the stresses are equal to zero.

As the stresses near the centre are smaller, the strength of the material is less utilized there. A better utilization of the material is achieved by hollow shafts (fig. 6.33.). Hollow shafts are difficult to construct, however. The distribution of stresses represented in fig. 6.32. only applies to cross sections with an all-round or circular profile. With other cross sections, more complicated distributions of stress will occur.

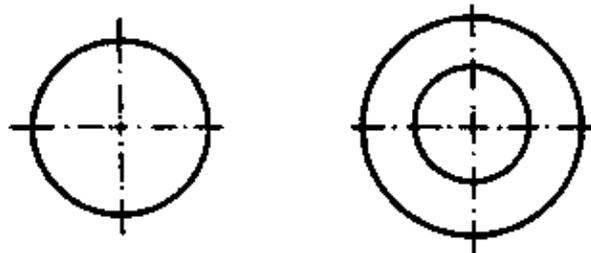


Fig. 6.33. Hollow sections can transmit larger torques at equal employment of material.

6.9. Tasks and questions

- 6.1. How is the stress distributed over the cross sectional area in case of tensile stress?
- 6.2. Name three examples from production and engineering where machine members are subjected to tension.
- 6.3. Describe the behaviour of a steel tube at
 - a) temperature rise
 - b) temperature drop.
- 6.4. Why are high-grade measuring devices made from a steel, whose coefficients of thermal expansion are almost equal to zero?

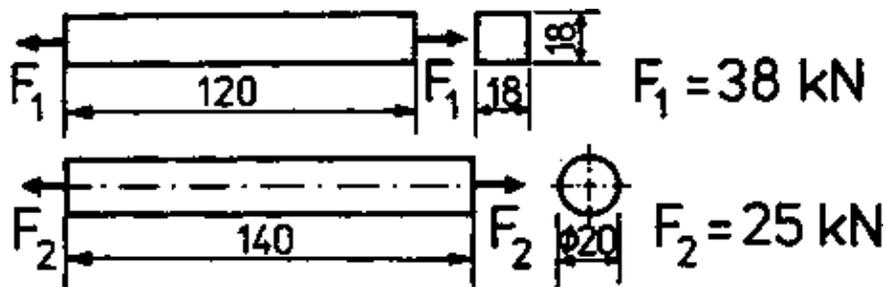


Fig. 6.34. To task 6.5.

6.5. Two steel rods are stressed as represented in fig. 6.34. Calculate the stress σ , the strain ϵ and the increase in length Δl . With the solution of the tasks, $E = 21 \text{ MN/cm}^2$ is to be used.

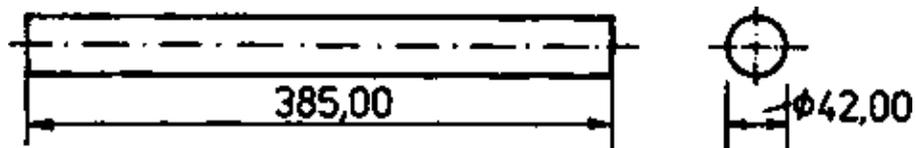


Fig. 6.35. To task 6.6.

6.6. A steel rod has, at 15°C , the dimensions represented in fig. 6.35. How large is diameter d if the rod is cooled down to -190°C ? With the solution of the task, $\alpha = 0.000010 \text{ } ^\circ\text{C}^{-1}$ is to be used.

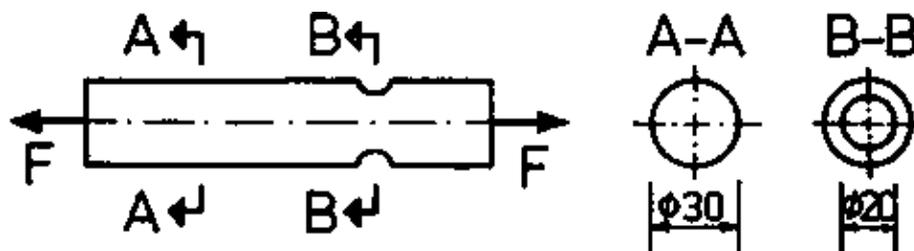


Fig. 6.36. To task 6.7.

6.7. A rod has the shape represented in fig. 6.36. It is subjected to tension by a force $F = 24 \text{ kN}$. Calculate the tensile stresses in the cross sections A and B,

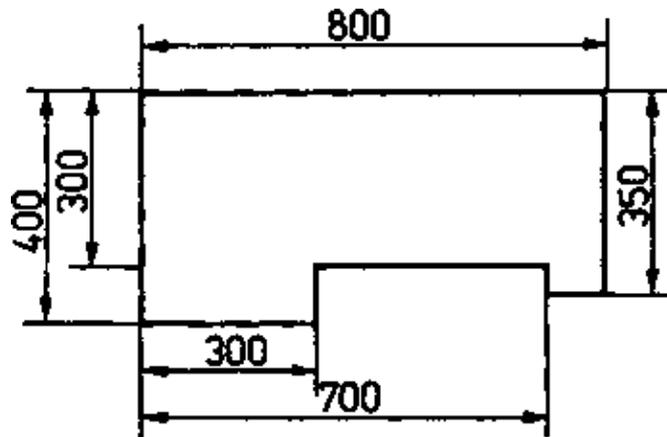


Fig. 6.37. To task 6.8.

6.8. A machine has a weight of 6.2 tons. Its base has the dimensions represented in fig. 6.37. Calculate the unit pressure below the machine.

6.9. The admissible load of ceiling of a storeroom is stated at 5 kN/m^2 . What weight may be piled on 1 m^2 ceiling area?

6.10 Name three examples from production and engineering where machine members are subjected to bending.

6.11. Describe the deflection of the bolt according to fig. 6.24. Draw the stress distribution in the cross section.

State the position of the neutral fibre.

Bear in mind that in this case the strain occurs on the top surface.

6.12. Where is the neutral layer of fibres with a machine member subjected to bending. How large is the change in length of the neutral layer of fibres?

6.13. Name three examples from production and engineering where machine members are subjected to buckling.

6.14. Ascertain for the examples named in task 6.13.

- a) the cross section of the rod and
- b) the support of the rod.

Name the probable direction of buckling.

6.15. Name three examples from production or engineering where machine members are subjected to shearing off.

6.16. Describe the different stress of a keyed and a feather joint.

6.17. Name three machines from production and engineering containing shafts for the transmission of torques.

6.18. What advantages and disadvantages has a shaft with a cross section in the shape of a circular ring?

7. Mechanics of liquids (hydraulics)

7.1. Introduction

Drinking-water is collected in large reservoirs, and flows through pipings to the place of application.

By means of water, hydraulic oil or glycerine, large forces are produced in machine tools, presses and gearings in a simple manner, and transmitted through pipings.

Thereby the properties of liquids are utilized. The mechanics of liquids describes the natural laws, which determine the behaviour of liquids. The mechanics of liquids is also termed hydraulics.

The skilled worker has to know selected natural laws, which are applicable with the employment of liquids, in order to understand the operating mode of hydraulic equipment.

7.2. Volume of liquids

7.2.1. Influence of temperature and pressure

Liquids have no stable shape.

They assume any shape offered by a vessel or piping. Liquids have a definite volume, however. This volume depends on the temperature of the liquid. When heated, liquids will expand, and the volume is increased. When cooled down, liquids will reduce their volumes.

Exception:

Water has its highest density at 4 °C.

It increases its volume both when heated and when cooled down below 4 °C.

1,000 cm³ hydraulic oil increase their volume by 0.75 cm³ per 1 K heating up (mean value).

1,000 cm³ water increase their volume by 0.38 cm per 1 K heating up (mean value for temperature between 4 °C and 100 °C).

Liquids expand when heated up.

To a smaller degree, the volume can be influenced by pressure exerted on the liquid.

Hydraulic oil:	pressure	volume
	10 N/cm ² = 0.1 MPa	1,000 cm ³
	1,000 N/cm ² = 10 MPa	992.1 cm ³

This high rise in pressure reduces the volume by less than 1 percent

Liquids can hardly be compressed.

7.2.2. Connected vessels

Liquids fill all vessels as far as their volume suffices. Fig. 7.1. shows the vessels, which are connected among another. The liquid will flow through the connecting paths till the liquid level is equally high in all vessels.

In connected vessels, the liquid level is equally high.



Fig. 7.1. In connected vessels, the liquid level is equally high.

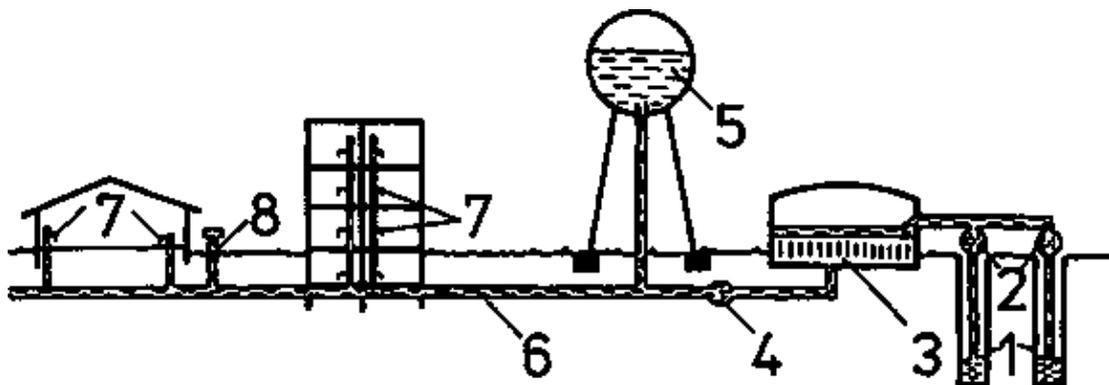


Fig. 7.2. Water supply with elevated tank.

1 deep well; 2 pumps; 3 gravel filter for cleaning of water; 4 pump; 5 elevated tank; 6 pressure pipeline; 7 taps; 8 hydrant for fire brigade.

This natural law is applied with water supply, fig. 7.2.

The water is delivered from deep wells by means of pumps. Then it trickles through filtering layers of gravel,

and is thereby cleaned. Then the water is pumped to the consumers through a pipe system. Part of the water is pumped into an elevated tank, and forms a permanent reserve there for short-time high consumption. Besides, a constant pressure is produced in the pipe system by the water level in the elevated tank. The water is conducted to the consumer through pipings. All taps have to be lower than the water level in the elevated tank. Elevated tank and pipe system form a system of connected vessels.

7.3. Pressure in liquids

7.3.1. Gravitational pressure

The force acting on a reference surface vertically is termed pressure.

Units of pressure:

N/mm²; N/cm²; kN/cm² and Pa; kPa; MPa ¹⁾

¹⁾ see supplement Si units

The pressure is calculated by

$$p = \frac{F}{A} \quad (7.1.)$$

p: pressure

F: force per reference surface

A: reference surface

The pressure in a liquid can be generated by its own weight and by external forces.

The pressure as a result of its own weight is termed gravitational pressure.

Gravitational pressure is a function of

- the height of the head of the liquid pressing on the examined reference surface,
- the density of the liquid.

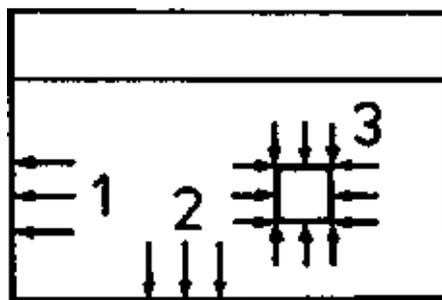


Fig. 7.3. Pressure in liquids.

1 wall pressure; 2 ground pressure; 3 pressure within the liquid.

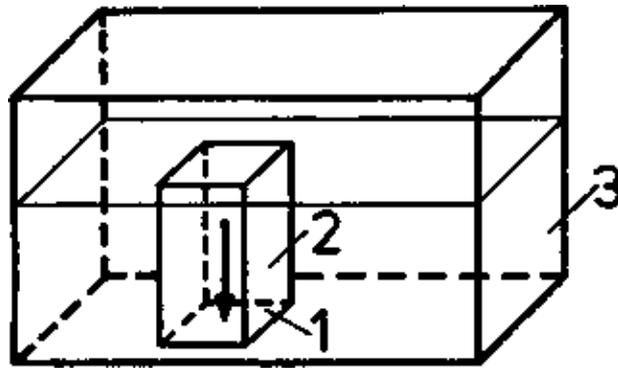


Fig. 7.4. Pressure of a head of liquid.

1 reference surface; 2 head of liquid; 3 liquid in the vessel.

The weight of the head of liquid loading the reference surface is, according to equation. 2.9.

$$G = \rho \cdot g \cdot V$$

$$G = \rho \cdot g \cdot A \cdot h$$

The pressure amounts to

$$p = \frac{G}{A}$$

$$p = \frac{\rho \cdot g \cdot A \cdot h}{A}$$

The size of the reference surface does not influence the size of pressure as A is cancelled in the equation.

$$p = \rho \cdot g \cdot h \quad (7.2.)$$

p: pressure in liquid

ρ : density

g: gravitational acceleration

h: height of liquid head

The shape of the vessel does not influence the ground pressure either. Fig. 7.5. shows three vessels of different shape. It can be proved in the experiment that the ground pressure is of equal size in all three vessels.



Fig. 7.5. The ground pressure is equally large in all three vessels.

7.3.2. Pressure spread in liquids

Pressure spread in liquids can be recognized well at an experiment. Fig. 7.6. shows a ball-shaped vessel filled with a liquid. If a pressure is exerted on the liquid by means of a piston, the same quantity of liquid flows out from each orifice. The orifices are of equal size. Hence, the pressure must be equally large at all orifices.

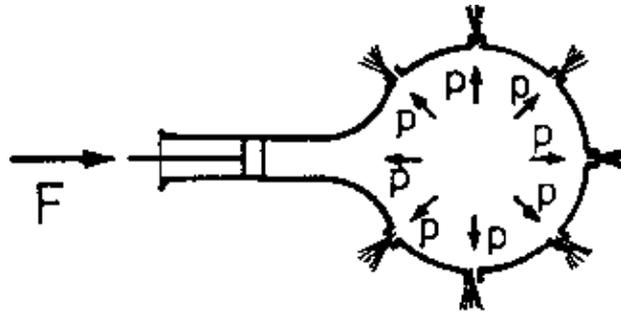


Fig. 7.6. In liquids, the pressure spreads evenly into all direction.

Principle of hydrostatic pressure:

In liquids, the pressure acts evenly into all directions.

Example

A water tank is filled with water 3,400 mm high (fig. 7.7.). How large is the pressure at points A, B, C, D, E, H and I?

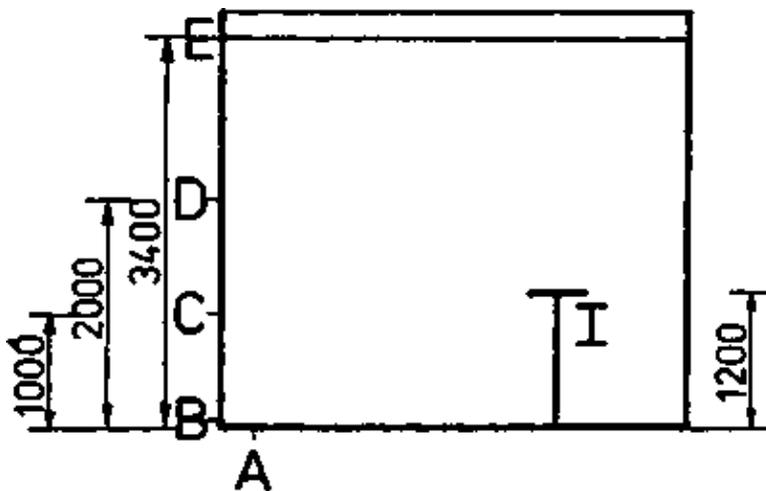


Fig. 7.7. Pressure in a water tank.

Given:

Density of water $\rho = 1 \frac{\text{kg}}{\text{dm}^3}$

Heights of liquid heads acc fig. 7.7.:

$h_A = h_B = 3,400 \text{ mm}$; $h_C = 2,400 \text{ mm}$

$h_D = 1,400 \text{ mm}$; $h_E = 0$; $h_H = h_I = 2,200 \text{ mm}$

Required:

Water pressure at points A to I

p_A ; p_B ; p_C ; p_D ; p_E ; p_H and p_I

Solution:

According to equation (7.2.), the pressure in a liquid is $p = \rho gh$

According to table 2.1., for water applies

$$\rho g = 9.81 \frac{\text{N}}{\text{dm}^3} = \frac{9.81 \text{N}}{1,000 \text{cm}^3}$$

Point A:

$$P_A = \frac{9.81 \text{N}}{1,000 \text{cm}^3} \cdot 340 \text{cm} = 3.34 \text{N/cm}^2$$

$$P_A = \underline{33.4 \text{ kPa}}$$

Point B:

As the pressure in a liquid acts evenly into all directions, the pressure in point B is equal to the pressure in point A.

$$p_B = p_A \quad 3.34 \text{ N/cm}^2 = \underline{33.4 \text{ kPa}}$$

Point C:

$$P_C = \frac{9.81 \text{N}}{1,000 \text{cm}^3} \cdot 240 \text{cm} = 2.35 \text{N/cm}^2$$

$$P_C = \underline{23.5 \text{ kPa}}$$

Point D:

$$P_D = \frac{9.81 \text{N}}{1,000 \text{cm}^3} \cdot 140 \text{cm} = 1.37 \text{N/cm}^2$$

$$P_D = \underline{13.7 \text{ kPa}}$$

Point E:

$$P_E = 0, \text{ as } h_E = 0$$

Point H:

$$P_H = \frac{9.81 \text{N}}{1,000 \text{cm}^3} \cdot 220 \text{cm} = 2.16 \text{N/cm}^2$$

$$P_H = \underline{21.6 \text{ kPa}}$$

Point I:

As the pressure in a liquid acts evenly into all directions, it acts in point I in the same size as at point H.

$$P_I = P_H = 2.16 \text{ N/cm}^2 = \underline{21.6 \text{ kPa}}$$

7.4. Hydraulic systems and subassemblies

7.4.1. Subassemblies for the increase of forces

The principle of pressure spread into all directions in liquids can be utilized for the increase of forces (fig. 7.8.)

Forces. A small force F_1 acts on a piston with a small base area A_1 . This piston transmits the force to a liquid, and generates in the liquid the pressure p .

$$p = \frac{F_1}{A_1}$$

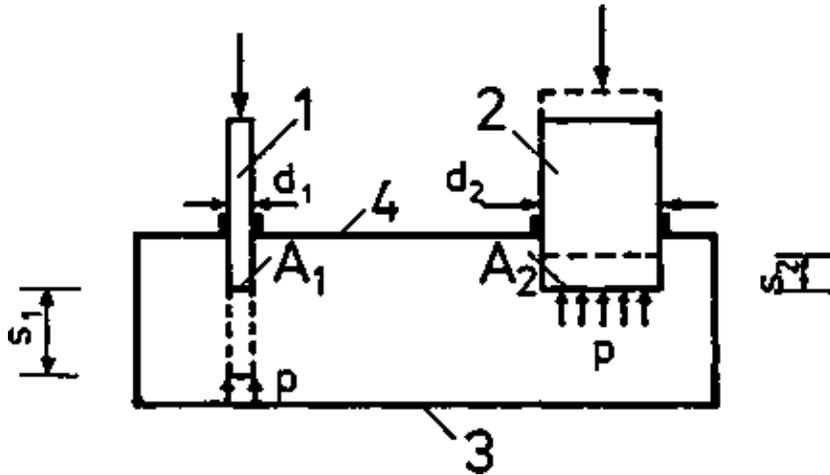


Fig. 7.8. Force-converting hydraulic subassembly.

1 piston 1; 2 piston 2; 3 solid, closed vessel; 4 hydraulic oil.

With this pressure, the liquid acts on piston 2 with a larger base area A_2 . At piston 2, it also applies

$$p = \frac{F_2}{A_2}$$

Pressure p is equal, hence the two equations can be combined.

$$\frac{F_1}{F_2} = \frac{A_1}{A_2} \quad (7.3.)$$

The forces, which act on the piston, are at the same ratio to each other as their areas.

As $A_2 > A_1$ it also applies that $F_2 > F_1$.

Paths. For the following consideration, it is assumed that liquids do not change their volumes at pressure rise. If piston 1 (fig. 7.8.) is pushed forward by the distance s_1 , the volume of displaced liquid amounts to

$$V_1 = A_1 \cdot s_1.$$

Piston 2 moves such that room is created for the displaced liquid.

$$V_2 = A_2 \cdot s_2.$$

As the volume of liquid does not change, it must apply that

$$V_1 = V_2.$$

$$A_1 s_1 = A_2 s_2$$

$$s_2 = \frac{A_1}{A_2} s_1 \quad (7.4.)$$

The paths are at inverse ratio as the areas of piston.

The piston with the small base area covers a longer distance, and piston 2 with the larger base area moves along a short path only.

The Golden Rule of mechanics applies here, too:

What is saved in force, must be added in path.

Work. At piston 1, the work $W_1 = F_1 S_1$ is performed, at piston 2, $W_2 = F_2 S_2$, accordingly.

With $\frac{F_1}{F_2} = \frac{A_1}{A_2}$; $F_2 = \frac{A_2}{A_1} \cdot F_1$

and $\frac{s_1}{s_2} = \frac{A_2}{A_1}$; $s_2 = \frac{A_1}{A_2} \cdot s_1$

follows $W_2 = \frac{A_2}{A_1} \cdot F_1 \cdot \frac{A_1}{A_2} \cdot s_1$

or $W_2 = F_1 \cdot s_1$

hence $W_2 = W_1$.

The conversion of the equation shows, that forces are converted though, but no work is saved.

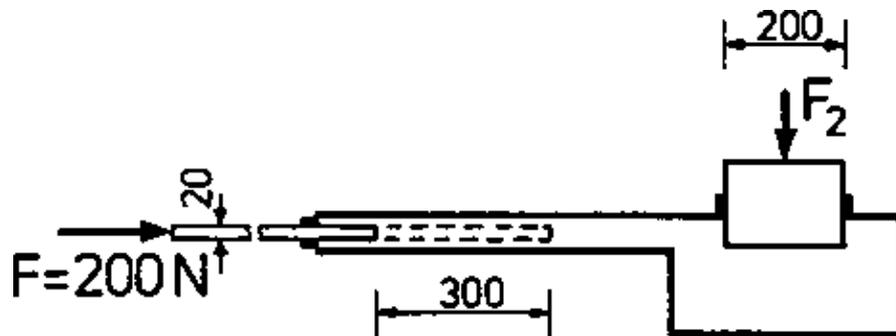


Fig. 7.9. Dimensions to example

Example

A piston (fig. 7.9.) with $d_1 = 20$ mm is moved by a force of 200 N. The piston covers a distance of 300 mm.

What force acts at piston 2?

What path is covered by piston 2?

Given:

- Piston diameter $d_1 = 20$ mm
- $d_2 = 200$ mm
- Force at piston 1 $F_1 = 200$ N
- Travel of piston 1 $s_1 = 300$ mm

Required:

- Force at piston 2 F_2
- Travel of piston 2 s_2

Solution:

According to equation (7.3.) applies

$$\frac{F_1}{F_2} = \frac{A_1}{A_2}$$

$$F_2 = \frac{A_2}{A_1} F_1$$

$$A = \frac{\pi d^2}{4} \rightarrow A_2 = \frac{\pi \cdot (20 \text{ cm})^2}{4} = 314 \text{ cm}^2$$

$$\rightarrow A_1 = \frac{\pi \cdot (2 \text{ cm})^2}{4} = 3.14 \text{ cm}^2$$

$$F_2 = \frac{314 \text{ cm}^2}{3.14 \text{ cm}^2} \cdot 200 \text{ N} = 20,000 \text{ N} = 20 \text{ kN}$$

According to equation (7.4.) applies

$$\frac{s_1}{s_2} = \frac{A_2}{A_1}$$

$$s_2 = \frac{A_1}{A_2} s_1$$

$$s_2 = \frac{3.14 \text{ cm}^2}{314 \text{ cm}^2} \cdot 300 \text{ mm} = 3 \text{ mm}$$

Result:

Piston 2 develops a force of 20 kN.

It only covers a path of 3 mm, however.

Check-up:

For checking purposes, the work performed at piston 1 and at piston 2 is compared.

Work at piston 1:

$$W = F \cdot s$$

$$W_1 = 200 \text{ N} \cdot 30 \text{ cm}$$

$$W_1 = 6,000 \text{ N} \cdot \text{cm}$$

Work at piston 2:

$$W_2 = 20,000 \text{ N} \cdot 0.3 \text{ cm}$$

$$W_2 = 6,000 \text{ N} \cdot \text{cm}$$

$$\underline{W_1 = W_2}$$

At both pistons, the same work is performed. With this calculation, the losses due to friction, leakage etc have been neglected.

7.4.2. Hydraulic press

The hydraulic press increases an initial force, e.g. a hand force, by hydraulic conversion of force (figs. 7.10a. to 7.10b.). The same principle is applied with hydraulic lifting platforms (fig. 2.1.).

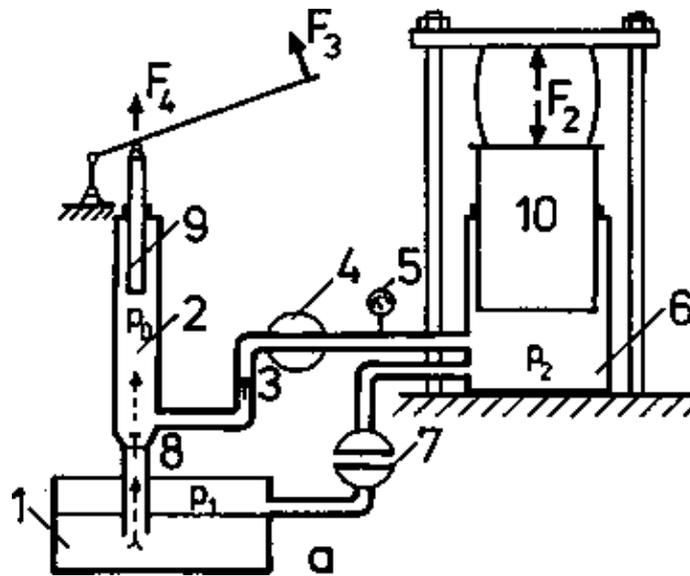


Fig. 7.10a. Hydraulic press at suction.

- 1 open vessel with water or hydraulic oil;
- 2 closed vessel for suction or pressing;
- 3 valve 4 rotary slide valve;
- 5 pressure gauge 6 closed vessel for pressing;
- 7 rotary slide valve 8 valve;
- 9 pump piston 10 power piston.

Drawing-in. Air-pressure p_1 acts on the liquid in the open vessel 1 (fig. 7.10a), see also para' 8.2.2.). When pump piston 9 is moved upwards, an underpressure p_0 comes to exist in vessel 2. As $p_0 < p_1$, the air presses the liquid from vessel 1 into vessel 2. To permit this, valve 8 opens. Valve 3 is closed by the pressure in vessel 6, and thus prevents the liquid from flowing back into this vessel.

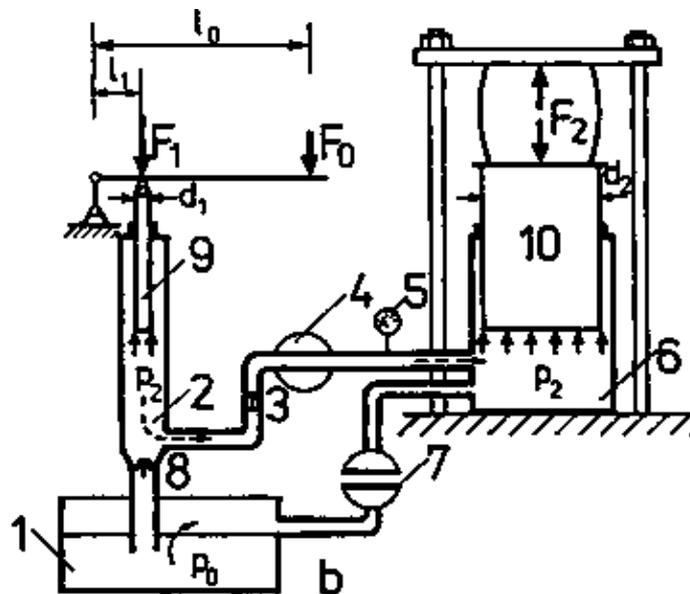


Fig. 7.10b. Hydraulic press at pressing, see also fig. 7.10.

Pressing (fig. 7.10b). The pump piston is pressed downwards by the lever, (fig. 7.10b). The liquid presses against valve 8, and closes it. In vessel 2, the pressure rises till the pressure p_2 of vessel 6 is reached and exceeded. Then valve 3 opens. and the liquid flows from vessel 2 into vessel 6. As the power piston cannot evade without hindrance (counterforce F_2), pressure p_2 increases, and power piston 10 moves upwards by a little distance. After pump piston 9 has reached its lowest point, drawing-in takes place again, and the cycle of operations is repeated. The procedure is repeated till the desired force is attained, or the power piston has covered the desired path (lifting platform).

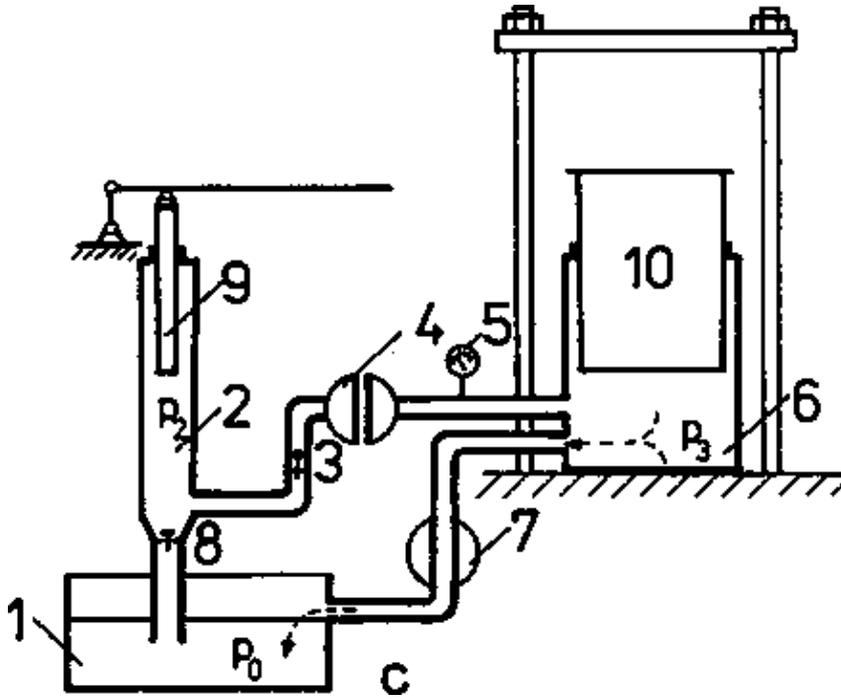


Fig. 7.10c. Hydraulic press at descent of power piston, see also fig. 7.10.

Descent of power piston. After the work is performed, the power piston has to be moved to its starting position again. To this end, rotary slide valve 4 is closed, and rotary slide valve 7 is opened. Under the weight of the piston, the liquid flows back to vessel 1 (fig. 7.10c).

Larger hydraulic presses are not operated by hand pumps, but by motor pumps.

Hydraulic presses operate at working pressures up to $10 \text{ kN/cm}^2 = 100 \text{ MPa}$, and generate forces up to 750 MN.

7.4.3. Subassembly for pressure rise

By applying the principle of even spreading of pressure in liquids into all directions, forces can be increased, and pressures can be raised, too.

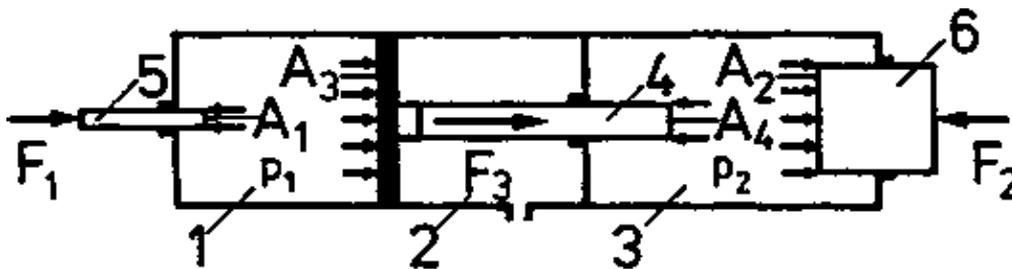


Fig. 7.11. Hydraulic subassembly for pressure rise.

- 1 and 2 closed vessel with hydraulic oil;
- 3 open cylinder with air 4 transmission piston;
- 5 pump piston 6 power piston.

In the subassembly represented in fig. 7.11., force F_1 generates pressure p_1 . This pressure is transmitted to the large area A_1 of the transmission piston, and generates force F_3 . The power piston transmits force F_3 on to the liquid in vessel 2 with the small area A_2 , and generates pressure p_2 there. The liquid presses against the power piston with p_2 . The air in cylinder 3 can escape without hindrance. Hence it does not obstruct the motion of the transmission piston.

The following pressure rise will occur:

$$F_3 = p_1 A_3$$

$$F_3 = p_2 A_4$$

$$F_3 = p_1 A_3 = p_2 A_4$$

$$p_2 = \frac{A_3}{A_4} p_1$$

Since $A_3 >$, $p_2 > p_1$.

Summary

Properties of liquids

- Liquids have no stable shape
- Liquids will expand when heated
- Liquids can hardly be compressed
- In connected vessels, the liquid level is equally high
- In liquids, the pressure is transmitted evenly into all directions.

In hydraulic systems

- the forces, which act on the pistons, are at the same ratio to each other as the piston areas
- the distances covered by the pistons are at inverse ratio to the areas of the pistons.

7.5. Speed of pressure spread and flow velocity

7.5.1. Speed of pressure spread

A pressure, which is, e.g., exerted by the pump piston on the liquid in a closed vessel, is propagated to all sides almost at sound velocity. To that, the liquid need not flow itself, but can propagate a pressure rise in state of rest, too.

In water or oil, the pressure is propagated at a speed of approx. 1,000 m/s.

This speed is so large that the pressure rise takes place in the whole liquid immediately, practically.

7.5.2. Flow velocity

In fig. 7.12. a pipe is represented flown through by a liquid. The pipe has different diameters in the individual sections.

If a volume of liquid V is fed to the pipe through the left opening, the same volume of liquid must flow off at the right opening of the pipe, for liquids cannot be compressed, practically.

Consequently, the same volume of liquid has to flow in the time t both through the pipe section with the large diameter d_1 and through the pipe section with the small diameter d_2 .

Note:

At a narrowing of the pipe, the flow velocity will rise. At a widening of the pipe, the flow velocity will decrease.

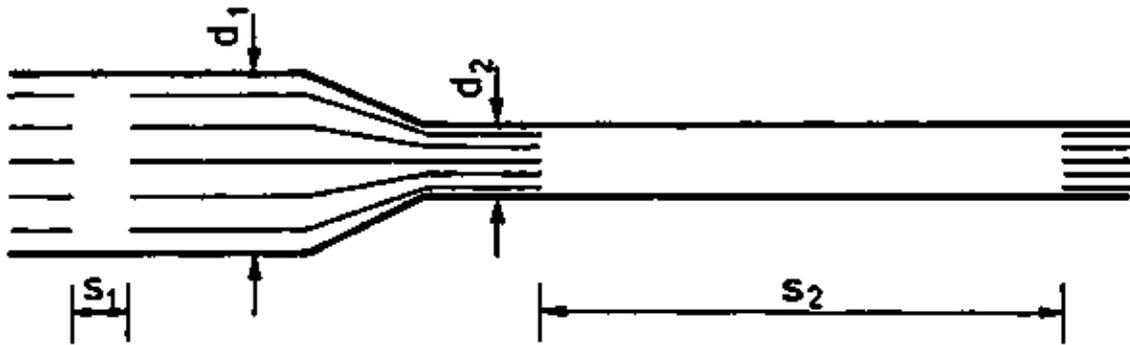


Fig. 7.12. The rate of flow is a function of the pipe diameter. s_1 and s_2 : distances covered by a liquid particle in time t

The volume of liquid flowing through a pipe in time t is termed stream of liquid Q .

$$Q = \frac{V}{t} \quad (7.5.)$$

Q : stream of liquid

V : volume

t : time

The volume of liquid can be calculated. It is the cylinder of liquid moving past a point of the pipe in time t . If s is the distance covered by a liquid particle in time t , then is

$$V = A \cdot s \quad (7.6.)$$

V : volume of liquid

A : sectional area

s : path of a liquid particle (see fig. 1.12.)

7.5.3. Kinds of flow

Two kinds of flow can be distinguished:

- laminar flow and
- turbulent flow.

The path of a liquid particle is termed line of flow.

With laminar flow, the lines of flow run orderly side by side. With narrowings of the pipe, the distance of the lines of flow to each other is reduced (fig. 7.13.).



Fig. 7.13. Laminar flow.

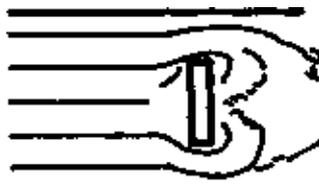


Fig. 7.14. Turbulent flow.

With turbulent flow (fig. 7.14.), the lines of flow are turbulent. They quickly change their direction, and do not run parallel and orderly any longer.

At low speeds, a laminar flow is formed in the pipes. When the speed exceeds the range $v = 2.5$ to 5.0 m/s, a turbulent flow will occur. In hydraulic systems, the laminar flow is preferred.

The kind of flow is influenced by

- the speed of the liquid,
- the roughness of pipe walls,
- number and formation of pipe bends
- branchings, valves and slides,
- viscosity of the liquid.

7.6. Tasks and questions

7.1. The water level in an elevated tank (fig. 7.2.) is 10.5 m above the highest tap of a workshop hall. How large is the water pressure at the tap?

7.2. The following questions refer to the hydraulic press, which is represented in figs. 7.10 a–e.

7.2.1. Explain the task of valve 3.

What will happen if valve 3 does not close tight?

7.2.2. Explain the task of valve 8.

What will happen if valve 8 does not close tight?

7.2.3. The power piston has a diameter of $d_2 = 200$ mm. The pressure in vessel 6 amounts to

- a) $1.75 \text{ kN/cm}^2 = 17.5 \text{ MPa}$
- b) $2.0 \text{ kN/cm}^2 = 20.0 \text{ MPa}$ and
- c) $3.5 \text{ kN/cm}^2 = 35 \text{ MPa}$

Calculate the piston force F_2 .

7.2.4. A force $F_2 = 775$ kN acts on the power piston. The diameter of the power piston amounts to $d_2 = 160$ mm. The power piston is to be lifted by 70 mm.

Calculate the necessary working pressure in vessel 6.

How large is the volume of oil to be pumped in.

7.3. Compare the

- paths of pump piston (5),
- paths of transmission piston (4) and
- paths of power piston (6)

with one another, represented in fig. 7.11. (subassembly for pressure rise. Apply the golden rule of mechanics:

What is saved in force, has to be added in distance.

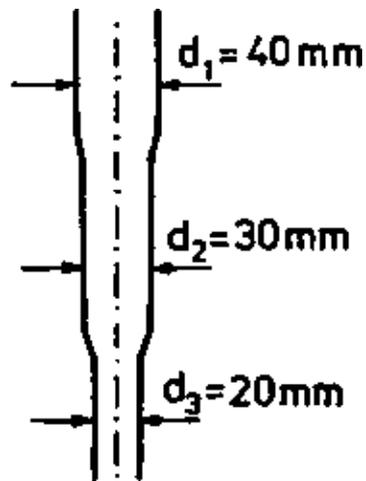


Fig. 7.15. To task 4.

7.4. A flow of liquid of $600 \text{ cm}^3/\text{s}$ flows through the pipe represented in fig. 7.15. Calculate the flow velocity in the three pipe cross sections.

8. Mechanics of gases (pneumatics)

8.1. Introduction

The properties of gases are utilized in various ways in production and engineering.

By means of compressed air, hammers, grinding machines and hand drills can be driven. Compressed air is used for transporting solid matter through pipings (pneumatic delivery). For derusting by means of a sand-blast and with pain spraying, compressed air is employed. Pneumatic brakes are applied in lorries, buses and in rail vehicles. Power cylinders operated by compressed air are widely applied. They are employed for automating production, to turn over workpieces, align and clamp them etc. To conclude with, compressed air is applied, in measurement technique. With non-contacting measurement, the dimensions of workpieces can be determined, without having to touch the workpieces themselves. Hence, measurement is possible without having to interrupt operations, e.g. at a lathe. Pneumatic systems and subassemblies make use of natural laws, which determined the behaviour of gases, and of air in particular. The mechanics of gases describes the natural laws acting in gases. The theory of the behaviour and of the motion of gases is also termed pneumatics. A skilled worker has to know selected fields of pneumatics to render an understanding of the performance of machines, systems and subassemblies operated by compressed air possible.

8.2. Pressure in gases

8.2.1. Units of pressure

The state of a gas is specified by its pressure, its temperature and its volume.

The force by which a gas acts on a reference area, is termed pressure.

$$p = \frac{F}{A} \quad (8.1.)$$

p: pressure

F: acting force

A: reference area

The unit of pressure is $\frac{N}{m^2}$. In remembrance of the great scientist Blaise Pascal (1623 to 1662), this unit bears the independent name of Pascal (Pa).

$$1\text{Pa} = \frac{1\text{N}}{m^2}$$

1 Pa marks a very low pressure. For this reason, megapascal (MPa) is chiefly used.

In engineering, the unit of 1 bar is still frequently used.

$$1\text{ bar} = 100,000\text{ Pa} = 10^5\text{ Pa}$$

The following table gives a comparative survey of the units of pressure:

Table 8.1. Units of pressure

	1 Pa = 1 N/m ²	1 MPa	1 bar
1 Pa = 1 N/m ²	1	10 ⁻⁶ MPa	10 ⁻⁵ bar
1 MPa	10 ⁶ Pa	1	10 bar
1 bar	10 ⁵ Pa	0.1 MPa	1

8.2.2. Air pressure

The earth is surrounded by an atmosphere. At the altitude of sea-level, 1 m³ of air has a mass of 1.2 kg. With rising altitude, the density of air decreases considerably.

The weight of air produces the air pressure (gravitational pressure). At the altitude of sea-level, the normal air pressure is 1.013 bar. The air pressure changes with the weather conditions and with the altitude above sea-level. On a mountain, the air pressure is lower.

For technical purposes, the statement is sufficient:

The normal air pressure is approx. 1 bar = 0.1 MPa.

In engineering, both overpressure and underpressure are utilized.

Fig. 8.1. represents the pressure ratios to each other.

The normal air pressure has the value of 1 bar = 0.1 MPa. The range marked with 2 is termed underpressure, the one marked with 3, overpressure.

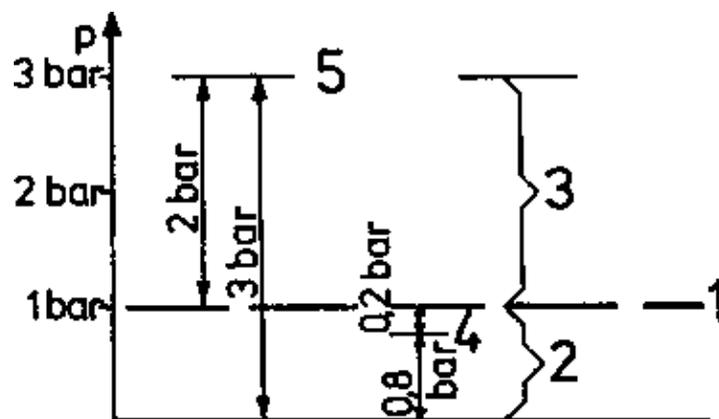


Fig. 8.1. Statements on pressure. 1 normal air pressure; 2 underpressure; 3 overpressure; 4 0.2 bar underpressure; 5 2 bar overpressure

Hence, an underpressure of 0.2 bar = 0.02 MPa (4) corresponds to an absolute pressure of 0.8 bar = 0.08 MPa.

A pressure of a value smaller than zero is not possible.

Hence, an overpressure of 2 bar = 0.2 MPa (5) corresponds to an absolute pressure of 3 bar a 0.3 MPa.

With each statement on pressure it should be stated whether it is

- absolute pressure,
- overpressure or
- underpressure.

As the usual manometers (pressure gauges) only measure the pressure difference to the air pressure, however, the statement overpressure is frequently omitted in technical pneumatics.

The technical statement:

The pneumatic power cylinder operates at $p = 6 \text{ bar} = 0.6 \text{ MPa}$ means:

The pneumatic power cylinder operates at an overpressure of 6 bar = 0.6 MPa.

8.2.3. Pressure spread

The pressure in a gas can be generated by its own gravitational pressure, or be transmitted to the gas by external forces. The pressure of a gas, enclosed in a vessel, may also be raised by a temperature rise.

The pressure acts in a gas uniformly into all directions.

8.3. Volume of gases

8.3.1. Influence of pressure

Gases have no stable shape. They assume any shape offered by a vessel or piping.

In contrast to liquids, gases have no stable volume either. They fill any vessel into which they can ingress.

	stable shape	stable volume
Solids	yes	yes
Liquids	no	yes
Gases	no	no

The state of a gas is specified by statements

- on pressure,
- on temperature and
- on volume.

All three quantities can change with a gas. The same masses of a gas may, consequently, have different volumes.

As early as the 17th century, the scientists Robert Boyle and Edme Mariotte found that – at constant temperature – the product from pressure and volume will always remain constant.

Boyle's and Mariotte's law:

$$p \cdot V = \text{constant (if } t = \text{constant)}$$

$$\text{Hence } p_1 V_1 = p_2 V_2 = p_3 V_3.$$

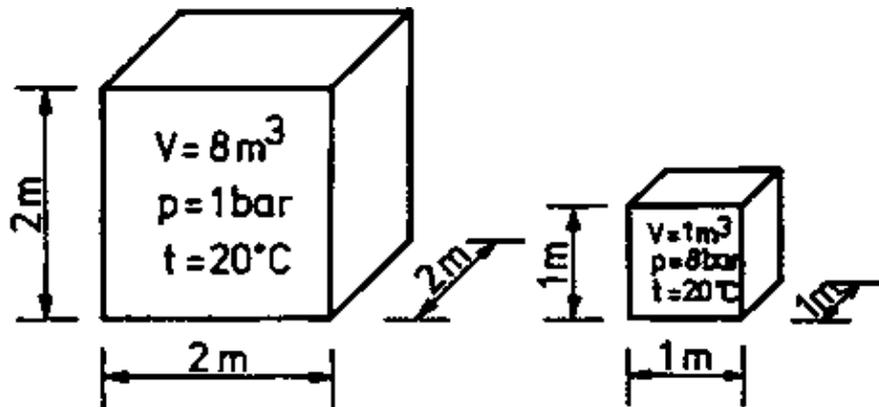


Fig. 8.2. Gases can be highly compressed.

When the pressure rises, the volume decreases (fig. 8.2.)

Gases can be highly compressed.

Example

For autogenous welding, oxygen is used, too.

Oxygen is a gas having a mass of 1.429 kg per 1 m³, at a pressure $p = 1 \text{ bar} = 0.1 \text{ MPa}$. As large quantities of gas are required for welding, the gas is highly compressed for transport. It is transported in thick-walled steel cylinders having capacities from 10 dm³ to 40 dm³.

Oxygen in the steel cylinder

Filled gas cylinder

$$p = 150 \text{ bar} = 15 \text{ MPa}$$

$$V = 40 \text{ dm}^3$$

Gas volume under normal pressure

$$p = 1 \text{ bar} = 0.1 \text{ MPa}$$

$$V = 40 \cdot 150 \text{ dm}^3 = 6,000 \text{ dm}^3 = 6 \text{ m}^3.$$

The gas contained in a steel cylinder of 40 m³ capacity occupies, at normal pressure, a volume of 6 m³.

8.3.2. Influence of temperature

Gases expand highly when heated. Measurements yield

$$V_1 = V_0 + V_0 \cdot \gamma \cdot t_1 \quad (8.2.)$$

V_1 : volume at temperature t_1

V_0 : volume at 0 °C if $p = \text{constant}$

t_1 : temperature of the gas

$$\gamma = \frac{1}{273\text{K}} \quad \text{: temperature coefficient of expansion}$$

When heated, all gases expand equally large.

When the pressure is kept constant, their volume increases as shown in fig. 8.3.

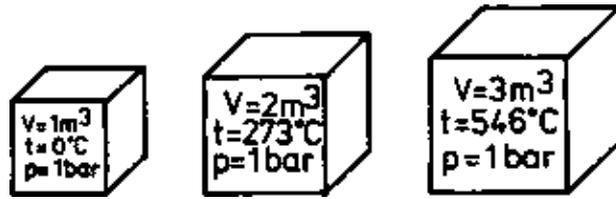


Fig. 8.3. When heated, all gases expand equally large.

8.4. Pneumatic systems

8.4.1. Pneumatic power cylinders

With the automation of production, pneumatic power cylinders are used for various purposes. Fig. 8.4. shows the construction and performance.

In constructional engineering, operations are at an overpressure from 2 bar = 0.2 MPa to 10 bar = 1.0 MPa. Hence, the pressure of the compressed air is 2 bar = 0.2 MPa to 10 bar = 1.0 MPa above normal air pressure. A higher pressure is not used. At flowing into outside air. the compressed air cools down. At exceedingly high pressure values, the compressed air may cool down so much that the outlets will freeze.

The compressed air can get to the piston from two sides. In working position 1, fig. 8.4., the compressed air is conveyed to the piston from the left. In the left space there is overpressure. The right space is opened to the outside air. There is normal air-pressure in the right space.

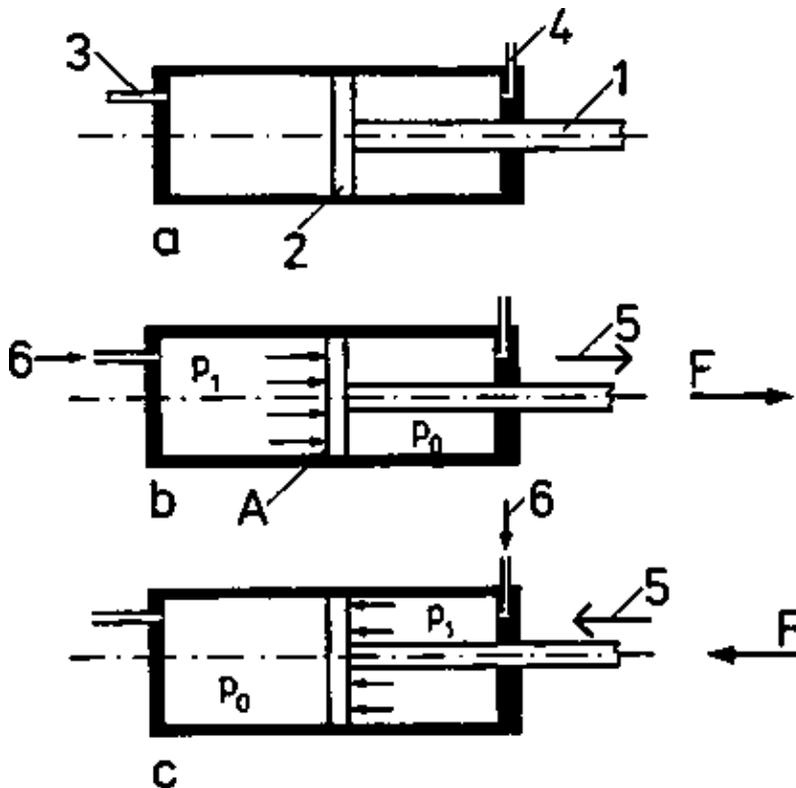


Fig. 8.4. Pneumatic power cylinder.

- 1 piston rod; 2 piston; 3 and 4 pipe connections;
- 5 direction of motion;
- 6 feed pipe of compressed air.

- a: neutral position;
- b: working position 1;
- c: working position 2;

Under these conditions, the compressed air pushes the piston to the right.

Thereby the force

$$F = p \cdot A$$

F: force at the piston
 p: overpressure
 A: piston area

is transmitted to the piston.

The overpressure is calculated according to

$$p = p_1 - p_0$$

In working position 2 (fig. 8.4c), the right space is filled with compressed air, and the left space is opened to the outside air. The compressed air, previously contained in the left space, escapes into the outside air.

Generally applies:

By means of pneumatic power pistons, forces from

$$f = 20 \text{ N to } F = 30.000 \text{ N} = 30 \text{ kN}$$

can be transmitted.

The flow velocity of compressed air amounts up to 10 m/s. As the compressed air flows in very quickly, the piston can be moved fast, too. The piston speed amounts to

$$10 \text{ mm/s to } 1.500 \text{ mm/s.}$$

Therefore the piston can cover large distances, too. The piston can get over distances up to a length of 2,000 mm.

Example

At a pneumatic power cylinder acts an overpressure of 6 bar = 0.6 MPa. The piston diameter amounts to 200 mm. What force acts at the piston?

Given:

Overpressure $p = 6 \text{ bar} = 0.6 \text{ MPa}$

Piston diameter $d = 200 \text{ mm}$

Required:

Force at piston F

Solution:

According to equation (8.1.) applies

$$p = \frac{F}{A}$$

$$F = A \cdot p$$

$$F = 6\text{bar} \cdot \frac{\pi \cdot 20^2}{4} \cdot \text{cm}^2$$

$$1\text{bar} = 0.1\text{MPa} = 10^5 \frac{\text{N}}{\text{m}^2}$$

$$F = \frac{\pi \cdot 20^2}{4} \cdot \text{cm}^2 \cdot 6 \cdot 10^5 \frac{\text{N}}{\text{m}^2}$$

$$1 \text{ m}^2 = 100^2 \text{ cm}^2$$

$$F = \frac{\pi \cdot 20^2}{4} \cdot \text{cm}^2 \cdot 6 \cdot 10^5 \frac{\text{N}}{100^2 \text{ cm}^2}$$

$$F = 18850 \text{ N}$$

Result:

At the piston there acts a force of 18.85 kN.

	Force	Working speed
hydraulic power cylinders	large	small
pneumatic power cylinders	small	large

Pneumatic power cylinders are employed because of their properties:

- for chucking workpieces,
- for transporting single workpieces,
- for positioning workpieces (turning round, rotating and other things).

Even complicated motions can be effected by pneumatic power cylinders.

Fig. 8.5. shows a pneumatic power cylinder, which takes from a stacking magazine one workpiece each, and pushes it to a track with conveying rollers. The rollers are inclined so that the workpiece will roll on of its own, due to the slope descending force (see para' 5.6.3.).

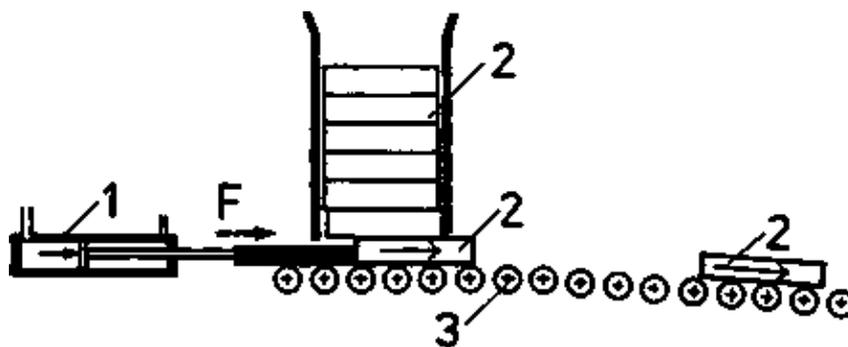


Fig. 8.5. A pneumatic power cylinder transfers single workpieces from a stacking magazine to conveying rollers.

- 1 pneumatic power cylinder;
- 2 workpieces;
- 3 conveying rollers.

8.4.2. Pneumatic conveying plant

Compressed air can carry along small solid particles and transport them in this way.

The domestic vacuum cleaner operates with underpressure. The motor sucks air, with that, dust particles are carried along, and separated from the air again in a filter.

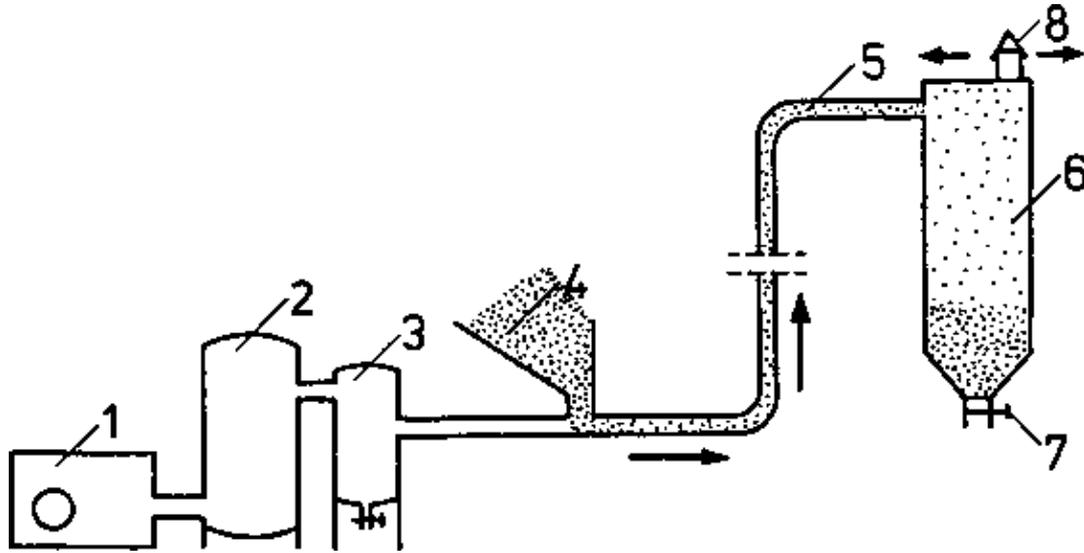


Fig. 8.6. Pneumatic conveyance of chips or similar materials by means of compressed air.

- 1 compressor; 2 stock bin; 3 water separator;
- 4 supply of conveying material;
- 5 delivery line; 6 silo;
- 7 discharge of conveying material;
- 8 filter and outflowing air.

In large conveying systems, operation is with overpressure. Fig. 8.6. shows a pneumatic system for conveying small solid matter like chips, powders etc. A compressor generates the compressed air, which is stored in a pressure vessel, and freed from surplus water in the water separator. The compressed air flows past the conveying material, carries the small solid particles of the conveying material along, and whirls them through the piping. The conveying distance may amount up to 1,000 m.

In a silo, the air flow gets calmed. The small solid particles are deposited, and the compressed air escapes to the outside air through a filter.

By specifically shaped pipes it is possible, to transport even larger solid bodies in transport vessels by means of compressed air.

Summary

In pneumatic systems, the physical properties of gases are utilized for performing work.

Pneumatic power cylinders attain high working speeds by a small force

Pneumatic conveying systems transport solid and fluid matter by using over- or underpressure, respectively.

8.5. Tasks and questions

8.1. Name three examples for the application of compressed air in production.

8.2. Why can hydraulic power cylinders generate larger forces than pneumatic power cylinders?

8.3. Complete the following table:

6	2.2				bar
		330,000	10,000		Pa
			249	1.2	0.5 MPa

8.4. 0.2 m^3 compressed air are under an overpressure of 6 bar = 0.6 MPa. How large is its volume when it escapes to the outside air ($1 \text{ bar} = 0.1 \text{ MPa}$). It is assumed that the temperature does not change.

8.5. What advantages has the application of compressed air? Note origin and remaining of air.

Appendix

Système International d'Unités (selection)

Prefixes

Prefix	Sign of abbr.	Meaning
Giga	G	$1.000.000.000 = 10^9$ units
Mega	M	$1.000.000 = 10^6$ units
Kilo	k	$1.000 = 10^3$ units
Deci	d	$0.1 = 10^{-1}$ units
Centi	c	$0.01 = 10^{-2}$ units
Milli	m	$0.001 = 10^{-3}$ units
Micro	?	$0.000.001 = 10^{-6}$ units
Nano	?	$0.000.000.001 = 10^{-9}$ units

The prefixes deci and centi are to be avoided if possible. They should only be used in connection with metre.

Correct: cm, to be avoided: cN

Basic units

Length	metre	m
Mass	kilogramme	kg
Time	second	s
Electrical current intensity	ampere	A
Temperature	Kelvin	K
Luminous intensity	candela	cd

All other SI units are derived from these basic units.

Selected SI units and conversions

Length l

Conversions

$1 \text{ m} = 10 \text{ dm} = 100 \text{ cm} = 1.000 \text{ mm}$

$1 \text{ mm} = 0.1 \text{ cm} = 0.01 \text{ dm} = 0.001 \text{ m}$

Area A

Conversions

$$1 \text{ m}^2 = 100 \text{ dm}^2 = 10,000 \text{ cm}^2 = 1,000,000 \text{ mm}^2$$

$$1 \text{ mm}^2 = 0.01 \text{ cm}^2 = 0.000,1 \text{ dm}^2 = 0.000,001 \text{ m}^2$$

Volume V

Conversions

$$1 \text{ m}^3 = 1,000 \text{ dm}^3 = 1,000,000 \text{ cm}^3 = 1,000,000,000 \text{ mm}^3$$

$$1 \text{ mm}^3 = 0.001 \text{ cm}^3 = 0.000,001 \text{ dm}^3 = 0.000,000.001 \text{ m}^3$$

Force F

$$1\text{N} = \frac{1\text{kg} \cdot \text{m}}{\text{s}^2}$$

A newton is the force, which accelerates a mass of 1 kg by 1 m/s² in the line along which the force acts.

Conversions

$$1 \text{ MN} = 1,000 \text{ kN} = 1,000,000 \text{ N}$$

$$1 \text{ N} = 0.001 \text{ kN} = 0.000,001 \text{ MN}$$

Work, energy W

$$1\text{J} = 1\text{N} \cdot \text{m} = 1\text{W} \cdot \text{s} = \frac{1\text{Kg} \cdot \text{m}^2}{\text{s}^2}$$

A joule is the work, which is performed if the point of application of force 1 N is displaced by 1 m in the direction of the force.

Energy W

$$1\text{W} = \frac{1\text{J}}{\text{s}} = \frac{1\text{N} \cdot \text{m}}{\text{s}} = \frac{1\text{kg} \cdot \text{m}^2}{\text{s}^3}$$

A watt is the energy of a uniformly proceeding process, in the course of which the work of 1 J is performed in the time of 1 s.

Mechanical stress

$$1 \text{ N/m}^2 = 1 \text{ Pa}$$

Conversions

$$1 \text{ MPa} = 1.000 \text{ kPa} = 1,000,000 \text{ Pa}$$

$$1 \text{ N/m}^2 = 0.000,1 \text{ N/cm}^2 = 0.000.001 \text{ N/mm}^2$$

$$1 \text{ N/mm}^2 = 100 \text{ N/cm}^2 = 10,000 \text{ N/dm}^2 = 1,000,000 \text{ N/m}^2$$

$$1 \text{ N/mm}^2 = 0.1 \text{ kN/cm}^2 = 10 \text{ kN/dm}^2 = 1 \text{ MN/m}^2$$

$$1 \text{ MPa} = 1 \text{ N/mm}^2 = 0.1 \text{ kN/cm}^2 = 100 \text{ N/cm}^2$$

Pressure in gases P

$$1 \text{ N/m}^2 = 1 \text{ Pa}$$

$$1 \text{ bar} = 100,000 \text{ N/m}^2$$

$$1 \text{ bar} = 100 \text{ kN/m}^2 = 0.1 \text{ MN/m}^2$$

$$1 \text{ MN/m}^2 = 1.000 \text{ kN/m}^2 = 10 \text{ bar}$$

Time s

Conversions

$$1 \text{ d} = 24 \text{ h} = 1,440 \text{ min} = 86,400 \text{ s}$$

For day (d), minute (min) and hour (h), prefixes may not be used

Temperature K

	Thermodynamic temperature	Celsius temperature
Freezing point of water	273.15 K	0 °C
Boiling point	373.15 K	100 °C

Temperature difference are stated in K only.

Example: $\Delta t = 40 \text{ °C} - 30 \text{ °C} = 10 \text{ K}$ (10 K read 10 Kelvin)

Survey of important equations

Speed	$v = \frac{s}{t}$
Speed on circular path	$v = r \omega$
Acceleration	$a = \frac{v_1 - v_0}{t}$
Final speed at uniformly accelerated motion	$v_1 = v_0 + at$
Mean speed	$v_m = \frac{v_0 + v_1}{2}$
Covered distance at accelerated motion	$s = v_0 t + \frac{a}{2} t^2$
Law of acceleration	$F = ma$
Weight	$G = mg$
Gravitational acceleration	$g = 9.81 \text{ m/s}^2$
Density	$\rho = \frac{m}{V}$
Lever principle	$F_1 l_1 = F_2 l_2$
Fixed pulley	$F = G/2$
Factorial pulley block	$F = \frac{G}{n}$
Winch	$F = \frac{r}{R} G$
Differential pulley block	$F = \frac{G}{2} \cdot \frac{R-r}{R}$

Belt drive	
Torque	$M = F \frac{d}{2}$
Transmission ratio	$i = \frac{n_1}{n_2} = \frac{d_2}{d_1}$
Ratio of torques	$\frac{M_1}{M_2} = \frac{d_1}{d_2}$
Frictional force	$F_R = F_A?$
Work	$W = Fs$
Work of friction	$W_R = F_A?s$
Lifting work	$W = G \cdot g$ $W = m \cdot g \cdot h$
Work of acceleration	$W = mas$
Potential energy	$W_{pot} = G \cdot h$ $W_{pot} = m \cdot g \cdot h$
Kinetic energy	$W_{kin} = \frac{mv^2}{2}$
Power	$P = \frac{W}{t}$
Tensile and compressive stress	$\sigma = \frac{F}{A}$
Strain	$\varepsilon = \frac{\Delta l}{l_0}$
Hooke's law	$? = ? \cdot E$
Change in length at temperature	$?l = ??tl_0$
Shearing stress	$\pi = \frac{F}{A}$
Pressure in liquids	$p = \frac{F}{A}$
Hydraulic press	$\frac{F_1}{F_2} = \frac{A_1}{A_2}$ $\frac{s_1}{s_2} = \frac{A_2}{A_1}$
Flow velocity	$v = \frac{s}{t}$
Stream of liquid	$Q = \frac{As}{t}$ $Q = Av$

Pressure in gases	$p = \frac{F}{A}$
Boyle's and Mariotte's law	$pV = \text{constant}$
Volume at temperature change	$V_1 = V_0 + V_0 \gamma t_1$ $\gamma = \frac{1}{273K}$