Index for Mathematics with approximate Page Numbering

Addition

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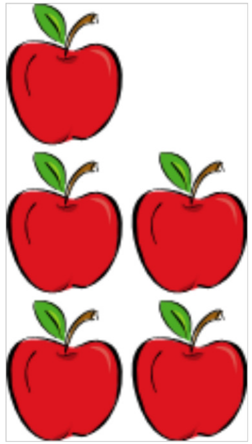


3 + 2 = 5 with apples, a popular choice in textbooks

Addition is the mathematical operation of combining or adding two numbers to obtain an equal simple amount or total. Addition also provides a model for related processes such as joining two collections of objects into one collection. Repeated addition of the number one is the most basic form of counting.

Performing addition is one of the simplest numerical tasks, accessible to infants as young as five months and even some animals.

Notation and terminology

Addition is written using the plus sign "+" between the terms; that is, in infix notation. The result is expressed with an equals sign. For example,

1 + 1 = 2 (verbally, "one plus one equals two")
2 + 2 = 4
5 + 4 + 2 = 11 (see "associativity" below)
3 + 3 + 3 + 3 = 12 (see "multiplication" below)

There are also situations where addition is "understood" even though no symbol appears:

- A column of numbers, with the last number in the column underlined, usually indicates that the numbers in the column are to be added, with the sum written below the underlined number.
- A whole number followed immediately by a fraction indicates the sum of the two, called a *mixed number*. For example,
  3½ = 3 + ½ = 3.5.
  This notation can cause confusion since in most other contexts juxtaposition denotes multiplication instead.

$$\begin{array}{r} 5 \\ \underline{12} \\ 17 \end{array}$$

The numbers or the objects to be added are generally called the "terms", the "addends", or the "summands"; this terminology carries over to the summation of multiple terms. This is to be distinguished from *factors*, which are multiplied. Some authors call the first addend the *augend*. In fact, during the Renaissance, many authors did not consider the first addend an "addend" at all. Today, due to the symmetry of addition, "augend" is rarely used, and both terms are generally called addends.

All of this terminology derives from Latin. " Addition" and " add" are English words derived from the Latin verb *addere*, which is in turn a compound of *ad* "to" and *dare* "to give", from the Indo-European root *do-* "to give"; thus to *add* is to *give to*. Using the gerundive suffix *-nd* results in "addend", "thing to be added".

Likewise from *augere* "to increase", one gets "augend", "thing to be increased".

The resultant	12
To whom it shall be addede	8
The nombre to be addede	4

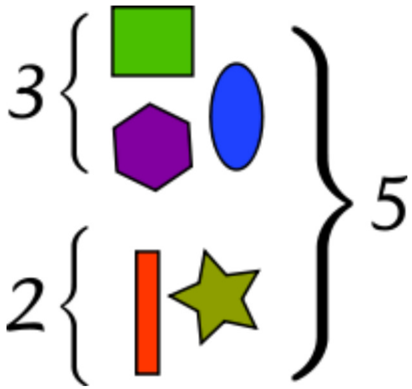
Redrawn illustration from *The Art of Nombryng*, one of the first English arithmetic texts, in the 15th century

"Sum" and "summand" derive from the Latin noun *summa* "the highest, the top" and associated verb *summare*. This is appropriate not only because the sum of two positive numbers is greater than either, but because it was once common to add upward, contrary to the modern practice of adding downward, so that a sum was literally higher than the addends. *Addere* and *summare* date back at least to Boethius, if not to earlier Roman writers such as Vitruvius and Frontinus; Boethius also used several other terms for the addition operation. The later Middle English terms "adden" and "adding" were popularized by Chaucer.

Interpretations

Addition is used to model countless physical processes. Even for the simple case of adding natural numbers, there are many possible interpretations and even more visual representations.

Combining sets



Possibly the most fundamental interpretation of addition lies in combining sets:

- When two or more collections are combined into a single collection, the number of objects in the single collection is the sum of the number of objects in the original collections.

This interpretation is easy to visualize, with little danger of ambiguity. It is also useful in higher mathematics; for the rigorous definition it inspires, see *Natural numbers* below. However, it is not obvious how one should extend this version of addition to include fractional numbers or negative numbers.

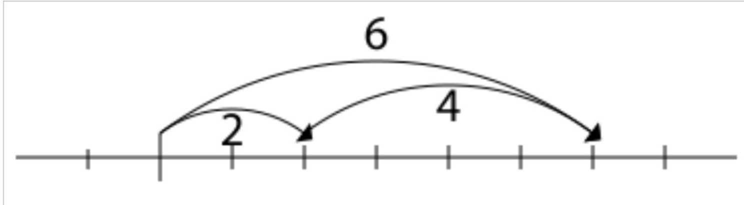
One possible fix is to consider collections of objects that can be easily divided, such as pies or, still better, segmented rods. Rather than just combining collections of segments, rods can be joined end-to-end, which illustrates another conception of addition: adding not the rods but the lengths of the rods.

Extending a length

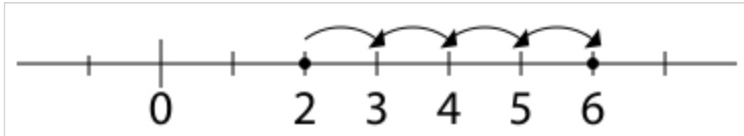
A second interpretation of addition comes from extending an initial length by a given length:

- When an original length is extended by a given amount, the final length is the sum of the original length and the length of the extension.

The sum $a + b$ can be interpreted as a binary operation that combines a and b, in an algebraic sense, or it can be interpreted as the addition of b more units to a. Under the latter interpretation, the parts of a sum $a + b$ play asymmetric roles, and the operation $a + b$ is viewed as applying the unary operation $+b$ to a. Instead of calling both a and b addends, it is more appropriate to call a the **augend** in this case, since a plays a passive role. The unary view is also useful when discussing subtraction, because each unary addition operation has an inverse unary subtraction operation. and *vice versa*.



A number-line visualization of the algebraic addition 2 + 4 = 6. A translation by 2 followed by a translation by 4 is the same as a translation by 6.



A number-line visualization of the unary addition 2 + 4 = 6. A translation by 4 is equivalent to four translations by 1.

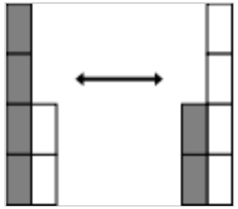
Properties

Commutativity

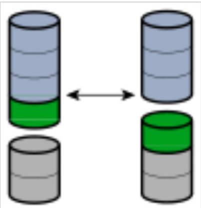
Addition is commutative, meaning that one can reverse the terms in a sum left-to-right, and the result will be the same. Symbolically, if a and b are any two numbers, then

$$a + b = b + a.$$

The fact that addition is commutative is known as the "commutative law of addition". This phrase suggests that there are other commutative laws: for example, there is a commutative law of multiplication. However, many binary operations are not commutative, such as subtraction and division, so it is misleading to speak of an unqualified "commutative law".


4 + 2 = 2 + 4 with blocks

Associativity


2+(1+3) = (2+1)+3 with segmented rods

A somewhat subtler property of addition is associativity, which comes up when one tries to define repeated addition. Should the expression

"$a + b + c$"

be defined to mean $(a + b) + c$ or $a + (b + c)$? That addition is associative tells us that the choice of definition is irrelevant. For any three numbers a, b, and c, it is true that

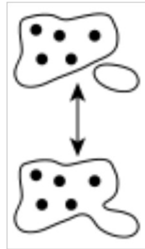
$$(a + b) + c = a + (b + c).$$

For example, (1 + 2) + 3 = 3 + 3 = 6 = 1 + 5 = 1 + (2 + 3). Not all operations are associative, so in expressions with other operations like subtraction, it is important to specify the order of operations.

Zero and one

When adding zero to any number, the quantity does not change; zero is the identity element for addition, also known as the additive identity. In symbols, for any a,

$$a + 0 = 0 + a = a.$$



5 + 0 = 5 with bags of dots

This law was first identified in Brahmagupta's *Brahmasphutasiddhanta* in 628, although he wrote it as three separate laws, depending on whether a is negative, positive, or zero itself, and he used words rather than algebraic symbols. Later Indian mathematicians refined the concept; around the year 830, Mahavira wrote, "zero becomes the same as what is added to it", corresponding to the unary statement $0 + a = a$. In the 12th century, Bhaskara wrote, "In the addition of cipher, or subtraction of it, the quantity, positive or negative, remains the same", corresponding to the unary statement $a + 0 = a$.

In the context of integers, addition of one also plays a special role: for any integer a, the integer $(a + 1)$ is the least integer greater than a, also known as the successor of a. Because of this succession, the value of some $a + b$ can also be seen as the b^{th} successor of a, making addition iterated succession.

Units

In order to numerically add physical quantities with units, they must first be expressed with common unit. For example, if a measure of 5 feet is extended by 2 inches, the sum is 62 inches, since 60 inches is synonymous with 5 feet. On the other hand, it is usually meaningless to try to add 3 meters and 4 square meters, since those units are incomparable; this sort of consideration is fundamental in dimensional analysis.

Performing addition

Innate ability

Studies on mathematical development starting around the 1980s have exploited the phenomenon of habituation: infants look longer at situations that are unexpected. A seminal experiment by Karen Wynn in 1992 involving Mickey Mouse dolls manipulated behind a screen demonstrated that five-month-old infants *expect* 1 + 1 to be 2, and they are comparatively surprised when a physical situation seems to imply that 1 + 1 is either 1 or 3. This finding has since been affirmed by a variety of laboratories using different methodologies. Another 1992 experiment with older toddlers, between 18 to 35 months, exploited their development of motor control by allowing them to retrieve ping-pong balls from a box; the youngest responded well for small numbers, while older subjects were able to compute sums up to 5.

Even some nonhuman animals show a limited ability to add, particularly primates. In a 1995 experiment imitating Wynn's 1992 result (but using eggplants instead of dolls), rhesus macaques and cottontop tamarins performed similarly to human infants. More dramatically, after being taught the meanings of the

Arabic numerals 0 through 4, one chimpanzee was able to compute the sum of two numerals without further training.

Elementary methods

Typically children master the art of counting first, and this skill extends into a form of addition called "counting-on"; asked to find three plus two, children count two past three, saying "four, *five*", and arriving at five. This strategy seems almost universal; children can easily pick it up from peers or teachers, and some even invent it independently. Those who count to add also quickly learn to exploit the commutativity of addition by counting up from the larger number.

Decimal system

The prerequisitive to addition in the decimal system is the internalization of the 100 single-digit "addition facts". One could memorize all the facts by rote, but pattern-based strategies are more enlightening and, for most people, more efficient:

- *One or two more*: Adding 1 or 2 is a basic task, and it can be accomplished through counting on or, ultimately, intuition.
- *Zero*: Since zero is the additive identity, adding zero is trivial. Nonetheless, some children are introduced to addition as a process that always increases the addends; word problems may help rationalize the "exception" of zero.
- *Doubles*: Adding a number to itself is related to counting by two and to multiplication. Doubles facts form a backbone for many related facts, and fortunately, children find them relatively easy to grasp. *near-doubles*...
- *Five and ten*...
- *Making ten*: An advanced strategy uses 10 as an intermediate for sums involving 8 or 9; for example, 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14.

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

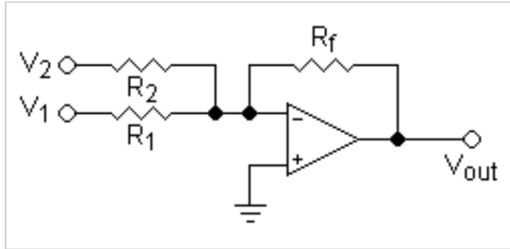
Single-digit addition table with various strategies colored: 0 in blue; 1,2 in light blue; (near) doubles in (light) green; making ten in red; 5,10 in gray.

In traditional mathematics, to add multidigit numbers, one typically aligns the addends vertically and adds the columns, starting from the ones column on the right. If a column exceeds ten, the extra digit is "carried" into the next column. For a more detailed description of this algorithm, see *Elementary arithmetic: Addition*. An alternate strategy starts adding from the most significant digit on the left; this route makes carrying a little clumsier, but it is faster at getting a rough estimate of the sum. There are many different standards-based mathematics methods, but many mathematics curricula such as TERC omit any instruction in traditional methods familiar to parents or mathematics professionals in favour of exploration of new methods.

- Fraction: Addition
- Scientific notation: Operations
- Roman arithmetic: Addition

Computers

Analog computers work directly with physical quantities, so their addition mechanisms depend on the form of the addends. A mechanical adder might represent two addends as the positions of sliding blocks, in which case they can be added with an averaging lever. If the addends are the rotation speeds of two shafts, they can be added with a differential. A hydraulic adder can add the pressures in two chambers by exploiting Newton's second law to balance forces on an assembly of pistons. The most common situation for a general-purpose analog computer is to add two voltages (referenced to ground); this can be accomplished roughly with a resistor network, but a better design exploits an operational amplifier.



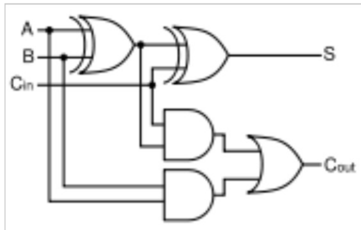
Addition with an op-amp. See Summing amplifier for details.

Addition is also fundamental to the operation of digital computers, where the efficiency of addition, in particular the carry mechanism, is an important limitation to overall performance.


Part of Charles Babbage's Difference Engine including the addition and carry mechanisms

Adding machines, mechanical calculators whose primary function was addition, were the earliest automatic, digital computers. Wilhelm Schickard's 1623 Calculating Clock could add and subtract, but it was severely limited by an awkward carry mechanism. As he wrote to Johannes Kepler describing the novel device, "You would burst out laughing if you were present to see how it carries by itself from one column of tens to the next..." Adding 999,999 and 1 on Schickard's machine would require enough force to propagate the carries that the gears might be damaged, so he limited his machines to six digits, even though Kepler's work required more. By 1642 Blaise Pascal independently developed an adding machine with an ingenious gravity-assisted carry mechanism. Pascal's calculator was limited by its carry mechanism in a different sense: its wheels turned only one way, so it could add but not subtract, except by the method of complements. By 1674 Gottfried Leibniz made the first mechanical multiplier; it was still powered, if not motivated, by addition.

Adders execute integer addition in electronic digital computers, usually using binary arithmetic. The simplest architecture is the ripple carry adder, which follows the standard multi-digit algorithm taught to children. One slight improvement is the *carry skip* design, again following human intuition; one does not perform all the carries in computing 999 + 1, but one bypasses the group of 9s and skips to the answer.



" Full adder" logic circuit that adds two binary digits, *A* and *B*, along with a carry input *Ci*, producing the sum bit, *S*, and a carry output, *Co*.

Since they compute digits one at a time, the above methods are too slow for most modern purposes. In modern digital computers, integer addition is typically the fastest arithmetic instruction, yet it has the largest impact on performance, since it underlies all the floating-point operations as well as such basic tasks as address generation during memory access and fetching instructions during branching. To increase speed, modern designs calculate digits in parallel; these schemes go by such names as carry select, carry lookahead, and the Ling pseudocarry. Almost all modern implementations are, in fact, hybrids of these last three designs.

Unlike addition on paper, addition on a computer often changes the addends. On the ancient abacus and adding board, both addends are destroyed, leaving only the sum. The influence of the abacus on mathematical thinking was strong enough that early Latin texts often claimed that in the process of adding "a number to a number", both numbers vanish. In modern times, the ADD instruction of a microprocessor replaces the augend with the sum but preserves the addend. In a high-level programming language, evaluating $a + b$ does not change either a or b; to change the value of a one uses the addition assignment operator a += b.

Addition of natural and real numbers

In order to prove the usual properties of addition, one must first *define* addition for the context in question. Addition is first defined on the natural numbers. In set theory, addition is then extended to progressively larger sets that include the natural numbers: the integers, the rational numbers, and the real numbers. (In mathematics education, positive fractions are added before negative numbers are even considered; this is also the historical route.)

Natural numbers

There are two popular ways to define the sum of two natural numbers a and b. If one defines natural numbers to be the cardinalities of finite sets, (the cardinality of a set is the number of elements in the set), then it is appropriate to define their sum as follows:

- Let N(S) be the cardinality of a set S. Take two disjoint sets A and B, with N(A) = a and N(B) = b. Then $a + b$ is defined as N($A \cup B$).

Here, $A \cup B$ is the union of A and B. An alternate version of this definition allows A and B to possibly overlap and then takes their disjoint union, a mechanism which allows any common elements to be separated out and therefore counted twice.

The other popular definition is recursive:

- Let n^+ be the successor of n, that is the number following n in the natural numbers, so $0^+=1$, $1^+=2$. Define $a + 0 = a$. Define the general sum recursively by $a + (b^+) = (a + b)^+$. Hence $1+1=1+0^+=(1+0)^+=1^+=2$.

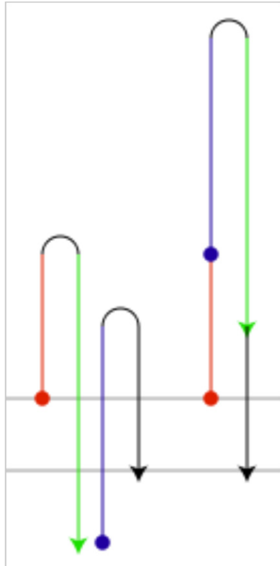
Again, there are minor variations upon this definition in the literature. Taken literally, the above definition is an application of the Recursion Theorem on the poset $\mathbf{N}^2$. On the other hand, some sources prefer to use a restricted Recursion Theorem that applies only to the set of natural numbers. One then considers a to be temporarily "fixed", applies recursion on b to define a function "a + ", and pastes these unary operations for all a together to form the full binary operation.

This recursive formulation of addition was developed by Dedekind as early as 1854, and he would expand upon it in the following decades. He proved the associative and commutative properties, among others, through mathematical induction; for examples of such inductive proofs, see *Addition of natural numbers*.

Integers

The simplest conception of an integer is that it consists of an absolute value (which is a natural number) and a sign (generally either positive or negative). The integer zero is a special third case, being neither positive nor negative. The corresponding definition of addition must proceed by cases:

- For an integer n, let $|n|$ be its absolute value. Let a and b be integers. If either a or b is zero, treat it as an identity. If a and b are both positive, define $a + b = |a| + |b|$. If a and b are both negative, define $a + b = -(|a|+|b|)$. If a and b have different signs, define $a + b$ to be the difference between $|a|$ and $|b|$, with the sign of the term whose absolute value is larger.



Defining (-2) + 1 using only addition of positive numbers: $(2 - 4) + (3 - 2) = 5 - 6$.

Although this definition can be useful for concrete problems, it is far too complicated to produce elegant general proofs; there are too many cases to consider.

A much more convenient conception of the integers is the Grothendieck group construction. The essential observation is that every integer can be expressed (not uniquely) as the difference of two natural numbers, so we may as well *define* an integer as the difference of two natural numbers. Addition is then defined to be compatible with subtraction:

- Given two integers $a - b$ and $c - d$, where a, b, c, and d are natural numbers, define $(a - b) + (c - d) = (a + c) - (b + d)$.

Rational numbers (Fractions)

Addition of rational numbers can be computed using the least common denominator, but a conceptually simpler definition involves only integer addition and multiplication:

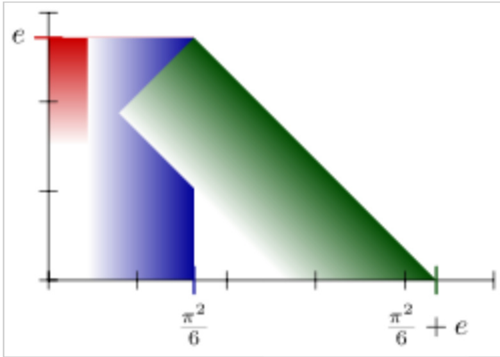
- Define $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$.

The commutativity and associativity of rational addition is an easy consequence of the laws of integer arithmetic. For a more rigorous and general discussion, see *field of fractions*.

Real numbers

A common construction of the set of real numbers is the Dedekind completion of the set of rational numbers. A real number is defined to be a Dedekind cut of rationals: a non-empty set of rationals that is closed downward and has no greatest element. The sum of real numbers *a* and *b* is defined element by element:

- Define $a+b=\{q+r \mid q\in a, r\in b\}.$

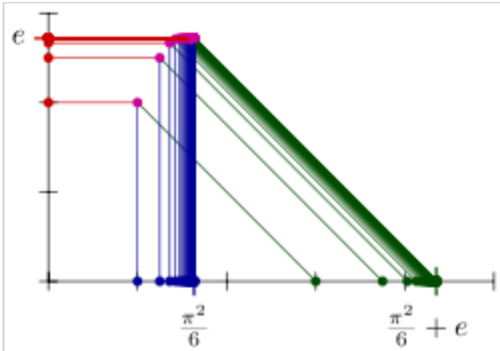


Adding $\pi^2/6$ and *e* using Dedekind cuts of rationals

This definition was first published, in a slightly modified form, by Richard Dedekind in 1872. The commutativity and associativity of real addition are immediate; defining the real number 0 to be the set of negative rationals, it is easily seen to be the additive identity. Probably the trickiest part of this construction pertaining to addition is the definition of additive inverses.

Unfortunately, dealing with multiplication of Dedekind cuts is a case-by-case nightmare similar to the addition of signed integers. Another approach is the metric completion of the rational numbers. A real number is essentially defined to be the a limit of a Cauchy sequence of rationals, lim a_n. Addition is defined term by term:

- Define $\lim_n a_n + \lim_n b_n = \lim_n (a_n + b_n).$



Adding $\pi^2/6$ and *e* using Cauchy sequences of rationals

This definition was first published by Georg Cantor, also in 1872, although his formalism was slightly different. One must prove that this operation is well-defined, dealing with co-Cauchy sequences. Once that task is done, all the properties of real addition follow immediately from the properties of rational numbers. Furthermore, the other arithmetic operations, including multiplication, have straightforward, analogous definitions.

Generalizations

> *There are many things that can be added: numbers, vectors, matrices, spaces, shapes, sets, functions, equations, strings, chains...* — Alexander Bogomolny

There are many binary operations that can be viewed as generalizations of the addition operation on the real numbers. The field of abstract algebra is centrally concerned with such generalized operations, and they also appear in set theory and category theory.

Addition in abstract algebra

In linear algebra, a vector space is an algebraic structure that allows for adding any two vectors and for scaling vectors. A familiar vector space is the set of all ordered pairs of real numbers; the ordered pair (*a*,*b*) is interpreted as a vector from the origin in the Euclidean plane to the point (*a*,*b*) in the plane. The sum of two vectors is obtained by adding their individual coordinates:

$(a,b) + (c,d) = (a+c,b+d)$.

This addition operation is central to classical mechanics, in which vectors are interpreted as forces.

In modular arithmetic, the set of integers modulo 12 has twelve elements; it inherits an addition operation from the integers that is central to musical set theory. The set of integers modulo 2 has just two elements; the addition operation it inherits is known in Boolean logic as the " exclusive or" function. In geometry, the sum of two angle measures is often taken to be their sum as real numbers modulo 2π. This amounts to an addition operation on the circle, which in turn generalizes to addition operations on many-dimensional tori.

The general theory of abstract algebra allows an "addition" operation to be any associative and commutative operation on a set. Basic algebraic structures with such an addition operation include commutative monoids and abelian groups.

Addition in set theory and category theory

A far-reaching generalization of addition of natural numbers is the addition of ordinal numbers and cardinal numbers in set theory. These give two different generalizations of addition of natural numbers to the transfinite. Unlike most addition operations, addition of ordinal numbers is not commutative. Addition of cardinal numbers, however, is a commutative operation closely related to the disjoint union operation.

In category theory, disjoint union is seen as a particular case of the coproduct operation, and general coproducts are perhaps the most abstract of all the generalizations of addition. Some coproducts, such as *Direct sum* and *Wedge sum*, are named to evoke their connection with addition.

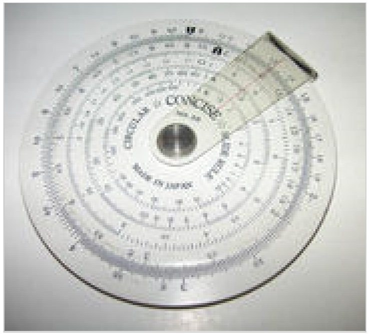
Related operations

Arithmetic

Subtraction can be thought of as a kind of addition—that is, the addition of an additive inverse. Subtraction is itself a sort of inverse to addition, in that adding x and subtracting x are inverse functions.

Given a set with an addition operation, one cannot always define a corresponding subtraction operation on that set; the set of natural numbers is a simple example. On the other hand, a subtraction operation uniquely determines an addition operation, an additive inverse operation, and an additive identity; for this reason, an additive group can be described as a set that is closed under subtraction.

Multiplication can be thought of as repeated addition. If a single term x appears in a sum n times, then the sum is the product of n and x. If n is not a natural number, the product may still make sense; for example, multiplication by −1 yields the additive inverse of a number.


A circular slide rule

In the real and complex numbers, addition and multiplication can be interchanged by the exponential function:

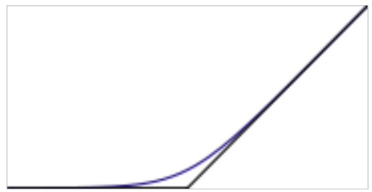
$e^{a + b} = e^a e^b$.

This identity allows multiplication to be carried out by consulting a table of logarithms and computing addition by hand; it also enables multiplication on a slide rule. The formula is still a good first-order approximation in the broad context of Lie groups, where it relates multiplication of infinitesimal group elements with addition of vectors in the associated Lie algebra.

There are even more generalizations of multiplication than addition. In general, multiplication operations always distribute over addition; this requirement is formalized in the definition of a ring. In some contexts, such as the integers, distributivity over addition and the existence of a multiplicative identity is enough to uniquely determine the multiplication operation. The distributive property also provides information about addition; by expanding the product $(1 + 1)(a + b)$ in both ways, one concludes that addition is forced to be commutative. For this reason, ring addition is commutative in general.

Division is an arithmetic operation remotely related to addition. Since $a/b = a(b^{-1})$, division is right distributive over addition: $(a + b) / c = a / c + b / c$. However, division is not left distributive over addition; $1/ (2 + 2)$ is not the same as $1/2 + 1/2$.

Ordering


Log-log plot of $x + 1$ and max $(x, 1)$ from $x = 0.001$ to 1000

The **maximum operation** "max (a, b)" is a binary operation similar to addition. In fact, if two nonnegative numbers a and b are of different orders of magnitude, then their sum is approximately equal to their maximum. This approximation is extremely useful in the applications of mathematics, for example in truncating Taylor series. However, it presents a perpetual difficulty in numerical analysis, essentially since "max" is not invertible. If b is much greater than a, then a straightforward calculation of $(a + b) - b$ can accumulate an unacceptable round-off error, perhaps even returning zero. See also *Loss of significance*.

The approximation becomes exact in a kind of infinite limit; if either a or b is an infinite cardinal number, their cardinal sum is exactly equal to the greater of the two. Accordingly, there is no subtraction operation for infinite cardinals.

Maximization is commutative and associative, like addition. Furthermore, since addition preserves the ordering of real numbers, addition distributes over "max" in the same way that multiplication distributes over addition:

$$a + \max(b, c) = \max(a + b, a + c).$$

For these reasons, in tropical geometry one replaces multiplication with addition and addition with maximization. In this context, addition is called "tropical multiplication", maximization is called "tropical addition", and the tropical "additive identity" is negative infinity. Some authors prefer to replace addition with minimization; then the additive identity is positive infinity.

Tying these observations together, tropical addition is approximately related to regular addition through the logarithm:

$\log(a + b) \approx \max(\log a, \log b)$,

which becomes more accurate as the base of the logarithm increases. The approximation can be made exact by extracting a constant h, named by analogy with Planck's constant from quantum mechanics, and taking the " classical limit" as h tends to zero:

$$\max(a,b) = \lim_{h\to 0} h\log(e^{a/h} + e^{b/h}).$$

In this sense, the maximum operation is a *dequantized* version of addition.

Other ways to add

Incrementation, also known as the successor operation, is the addition of 1 to a number.

Summation describes the addition of arbitrarily many numbers, usually more than just two. It includes the idea of the sum of a single number, which is itself, and the empty sum, which is zero. An infinite summation is a delicate procedure known as a series.

Counting a finite set is equivalent to summing 1 over the set.

Integration is a kind of "summation" over a continuum, or more precisely and generally, over a differentiable manifold. Integration over a zero-dimensional manifold reduces to summation.

Linear combinations combine multiplication and summation; they are sums in which each term has a multiplier, usually a real or complex number. Linear combinations are especially useful in contexts where straightforward addition would violate some normalization rule, such as mixing of strategies in game theory or superposition of states in quantum mechanics.

Convolution is used to add two independent random variables defined by distribution functions. Its usual definition combines integration, subtraction, and multiplication. In general, convolution is useful as a kind of domain-side addition; by contrast, vector addition is a kind of range-side addition.

In literature

- In chapter 9 of Lewis Carroll's *Through the Looking-Glass*, the White Queen asks Alice, "And you do Addition? ... What's one and one and one and one and one and one and one and one and one and one?" Alice admits that she lost count, and the Red Queen declares, "She can't do Addition".
- In George Orwell's *Nineteen Eighty-Four*, the value of 2 + 2 is questioned; the State contends that if it declares 2 + 2 = 5, then it is so. See *Two plus two make five* for the history of this idea.

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Angle

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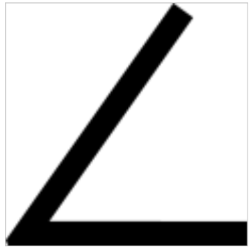


"∠", the angle symbol.

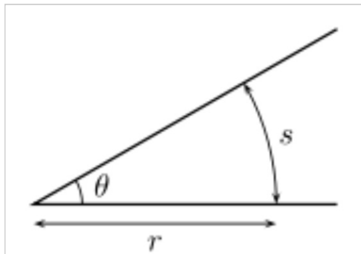
In geometry and trigonometry, an **angle** (in full, **plane angle**) is the figure formed by two rays sharing a common endpoint, called the vertex of the angle. The magnitude of the angle is the "amount of rotation" that separates the two rays, and can be measured by considering the length of circular arc swept out when one ray is rotated about the vertex to coincide with the other (see "Measuring angles", below). Where there is no possibility of confusion, the term "angle" is used interchangeably for both the geometric configuration itself and for its angular magnitude (which is simply a numerical quantity).

The word *angle* comes from the Latin word *angulus*, meaning "a corner". The word *angulus* is a diminutive, of which the primitive form, *angus*, does not occur in Latin. Cognate words are the Latin *angere*, meaning "to compress into a bend" or "to strangle", and the Greek ἀγκύλος *(ankylos)*, meaning "crooked, curved"; both are connected with the PIE root **ank-*, meaning "to bend" or "bow".

History

Euclid defines a plane angle as the inclination to each other, in a plane, of two lines which meet each other, and do not lie straight with respect to each other. According to Proclus an angle must be either a quality or a quantity, or a relationship. The first concept was used by Eudemus, who regarded an angle as a deviation from a straight line; the second by Carpus of Antioch, who regarded it as the interval or space between the intersecting lines; Euclid adopted the third concept, although his definitions of right, acute, and obtuse angles are certainly quantitative.

Measuring angles



The angle θ is the quotient of s and r.

In order to measure an angle θ, a circular arc centered at the vertex of the angle is drawn, e.g. with a pair of compasses. The length of the arc s is then divided by the radius of the circle r, and possibly multiplied by a scaling constant k (which depends on the units of measurement that are chosen):

$$\theta = \frac{s}{r}(k)$$

The value of θ thus defined is independent of the size of the circle: if the length of the radius is changed then the arc length changes in the same proportion, so the ratio s/r is unaltered.

In many geometrical situations, angles that differ by an exact multiple of a full circle are effectively equivalent (it makes no difference how many times a line is rotated through a full circle because it always ends up in the same place). However, this is not always the case. For example, when tracing a curve such as a spiral using polar coordinates, an extra full turn gives rise to a quite different point on the curve.

Units

Angles are considered dimensionless, since they are defined as the ratio of lengths. There are, however, several units used to measure angles, depending on the choice of the constant k in the formula above. Of these units, treated in more detail below, the *degree* and the *radian* are by far the most common.

With the notable exception of the radian, most units of angular measurement are defined such that one full circle (i.e. one revolution) is equal to n units, for some whole number n. For example, in the case of degrees,

A full circle of n units is obtained by setting

in the formula above. (Proof. The formula above can be rewritten as

One full circle, for which

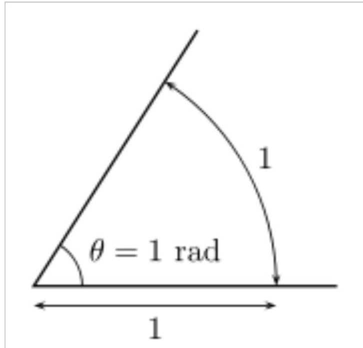
units, corresponds to an arc equal in length to the circle's circumference, which is $2\pi r$, so

. Substituting n for θ and $2\pi r$ for s in the formula, results in

)

- The **degree**, denoted by a small superscript circle (°) is 1/360 of a full circle, so one full circle is 360°. One advantage of this old sexagesimal subunit is that many angles common in simple geometry are measured as a whole number of degrees. (The problem of having *all* "interesting" angles measured as whole numbers is of course insolvable.) Fractions of a degree may be written in normal decimal notation (e.g. 3.5° for three and a half degrees), but the following sexagesimal subunits of the "degree-minute-second" system are also in use, especially for geographical coordinates and in astronomy and ballistics:
  - The **minute of arc** (or **MOA**, **arcminute**, or just **minute**) is 1/60 of a degree. It is denoted by a single prime (′). For example, 3° 30′ is equal to 3 + 30/60 degrees, or 3.5 degrees. A mixed format with decimal fractions is also sometimes used, e.g. 3° 5.72′ = 3 + 5.72/60 degrees. A nautical mile was historically defined as a minute of arc along a great circle of the Earth.
  - The **second of arc** (or **arcsecond**, or just **second**) is 1/60 of a minute of arc and 1/3600 of a degree. It is denoted by a double prime (″). For example, 3° 7′ 30″ is equal to 3 + 7/60 + 30/3600 degrees, or 3.125 degrees.

- The **radian** is the angle subtended by an arc of a circle that has the same length as the circle's radius ($k = 1$ in the formula given earlier). One full circle is 2π radians, and one radian is $180/\pi$ degrees, or about 57.2958 degrees. The radian is abbreviated *rad*, though this symbol is often omitted in mathematical texts, where radians are assumed unless specified otherwise. The radian is used in virtually all mathematical work beyond simple practical geometry, due, for example, to the pleasing and "natural" properties that the trigonometric functions display when their arguments are in radians. The radian is the (derived) unit of angular measurement in the SI system.



$\theta = s/r$ rad = 1 rad.

- The **mil** is *approximately* equal to a milliradian. There are several definitions.

- The **full circle** (or **revolution**, **rotation**, **full turn** or **cycle**) is one complete revolution. The revolution and rotation are abbreviated *rev* and *rot*, respectively, but just *r* in *rpm* (revolutions per minute). 1 full circle = 360° = 2π rad = 400 gon = 4 right angles.

- The **right angle** is 1/4 of a full circle. It is the unit used in Euclid's Elements. 1 right angle = 90° = $\pi/2$ rad = 100 gon.

- The **angle of the equilateral triangle** is 1/6 of a full circle. It was the unit used by the Babylonians, and is especially easy to construct with ruler and compasses. The degree, minute of arc and second of arc are sexagesimal subunits of the Babylonian unit. 1 Babylonian unit = 60° = $\pi/3$ rad ≈ 1.047197551 rad.
- The **grad**, also called **grade**, **gradian**, or **gon** is 1/400 of a full circle, so one full circle is 400 grads and a right angle is 100 grads. It is a decimal subunit of the right angle. A kilometer was historically defined as a centi-gon of arc along a great circle of the Earth, so the kilometer is the decimal analog to the sexagesimal nautical mile. The gon is used mostly in triangulation.

- The **point**, used in navigation, is 1/32 of a full circle. It is a binary subunit of the full circle. Naming all 32 points on a compass rose is called " boxing the compass". 1 point = 1/8 of a right angle = 11.25° = 12.5 gon.

- The astronomical **hour angle** is 1/24 of a full circle. The sexagesimal subunits were called **minute of time** and **second of time** (even though they are units of angle). 1 hour = 15° = $\pi/12$ rad = 1/6 right angle ≈ 16.667 gon.

- The **binary degree**, also known as the **binary radian** (or **brad**), is 1/256 of a full circle. The binary degree is used in computing so that an angle can be efficiently represented in a single byte.

- The **grade of a slope**, or **gradient**, is not truly an angle measure (unless it is explicitly given in degrees, as is occasionally the case). Instead it is equal to the tangent of the angle, or sometimes the sine. Gradients are often expressed as a percentage. For the usual small values encountered (less than 5%), the grade of a slope is approximately the measure of an angle in radians.

Positive and negative angles

A convention universally adopted in mathematical writing is that angles given a sign are **positive angles** if measured counterclockwise, and **negative angles** if measured clockwise, from a given line. If no line is specified, it can be assumed to be the x-axis in the Cartesian plane. In many geometrical situations a negative angle of $-\theta$ is effectively equivalent to a positive angle of "one full rotation less θ". For example, a clockwise rotation of 45° (that is, an angle of −45°) is often effectively equivalent to a counterclockwise rotation of 360° − 45° (that is, an angle of 315°).

In three dimensional geometry, "clockwise" and "counterclockwise" have no absolute meaning, so the direction of positive and negative angles must be defined relative to some reference, which is typically a vector passing through the angle's vertex and perpendicular to the plane in which the rays of the angle lie.

In navigation, bearings are measured from north, increasing clockwise, so a bearing of 45 degrees is north-east. Negative bearings are not used in navigation, so north-west is 315 degrees.

Approximations

- 1° is approximately the width of a little finger at arm's length
- 10° is approximately the width of a closed fist at arm's length.
- 20° is approximately the width of a handspan at arm's length.

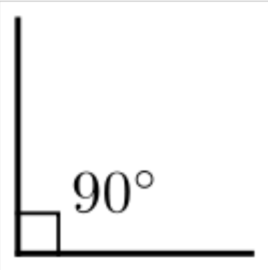
Identifying angles

In mathematical expressions, it is common to use Greek letters (α, β, γ, θ, φ, ...) to serve as variables standing for the size of some angle. (To avoid confusion with its other meaning, the symbol π is not used for this purpose.) Lower case roman letters (a, b, c, ...) are also used. See the figures in this article for examples.

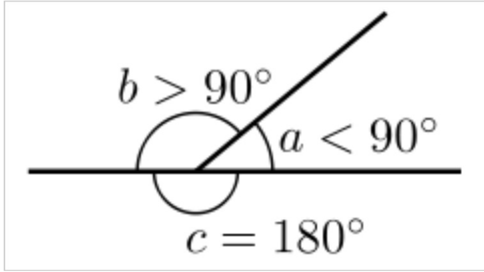
In geometric figures, angles may also be identified by the labels attached to the three points that define them. For example, the angle at vertex A enclosed by the rays AB and AC (i.e. the lines from point A to point B and point A to point C) is denoted $\angle$BAC or BÂC. Sometimes, where there is no risk of confusion, the angle may be referred to simply by its vertex ("angle A").

Potentially, an angle denoted, say, $\angle$BAC might refer to any of four angles: the clockwise angle from B to C, the anticlockwise angle from B to C, the clockwise angle from C to B, or the anticlockwise angle from C to B, where the direction in which the angle is measured determines its sign (see Positive and negative angles). However, in many geometrical situations it is obvious from context that the positive angle less than or equal to 180° degrees is meant, and no ambiguity arises. Otherwise, a convention may be adopted so that $\angle$BAC always refers to the anticlockwise (positive) angle from B to C, and $\angle$CAB to the anticlockwise (positive) angle from C to B.

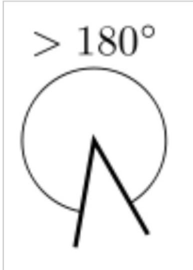
Types of angles



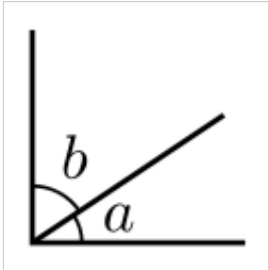
Right angle.



Acute (*a*), obtuse (*b*), and straight (*c*) angles. Here, *a* and *b* are supplementary angles.



Reflex angle.



The complementary angles *a* and *b* (*b* is the complement of *a*, and *a* is the complement of *b*).

- An angle of 90° (π/2 radians, or one-quarter of the full circle) is called a **right angle**.
  Two lines that form a right angle are said to be **perpendicular** or **orthogonal**.
- Angles smaller than a right angle (less than 90°) are called **acute angles** ("acute" meaning "sharp").
- Angles larger than a right angle and smaller than two right angles (between 90° and 180°) are called **obtuse angles** ("obtuse" meaning "blunt").
- Angles equal to two right angles (180°) are called **straight angles**.
- Angles larger than two right angles but less than a full circle (between 180° and 360°) are called **reflex angles**.
- Angles that have the same measure are said to be **congruent**.
- Two angles opposite each other, formed by two intersecting straight lines that form an "X" like shape, are called **vertical angles** or **opposite angles**. These angles are congruent.
- Angles that share a common vertex and edge but do not share any interior points are called **adjacent angles**.
- Two angles that sum to one right angle (90°) are called **complementary angles**.
  The difference between an angle and a right angle is termed the **complement** of the angle.
- Two angles that sum to a straight angle (180°) are called **supplementary angles**.
  The difference between an angle and a straight angle is termed the **supplement** of the angle.
- Two angles that sum to one full circle (360°) are called **explementary angles** or **conjugate angles**.
- An angle that is part of a simple polygon is called an **interior angle** if it lies in the inside of that the simple polygon. Note that in a simple polygon that is concave, at least one interior angle exceeds 180°.
  In Euclidean geometry, the measures of the interior angles of a triangle add up to π radians, or 180°; the measures of the interior angles of a simple quadrilateral add up to 2π radians, or 360°. In general, the measures of the interior angles of a simple polygon with n sides add up to $[(n - 2) \times \pi]$ radians, or $[(n - 2) \times 180]$°.
- The angle supplementary to the interior angle is called the **exterior angle**. It measures the amount of "turn" one has to make at this vertex to trace out the polygon. If the corresponding interior angle exceeds 180°, the exterior angle should be considered negative. Even in a non-simple polygon it may be

possible to define the exterior angle, but one will have to pick an orientation of the plane (or surface) to decide the sign of the exterior angle measure. In Euclidean geometry, the sum of the exterior angles of a simple polygon will be 360°, one full turn.

- Some authors use the name **exterior angle** of a simple polygon to simply mean the explementary (*not* supplementary!) of the interior angle . This conflicts with the above usage.
- The angle between two planes (such as two adjacent faces of a polyhedron) is called a **dihedral angle**. It may be defined as the acute angle between two lines normal to the planes.
- The angle between a plane and an intersecting straight line is equal to ninety degrees minus the angle between the intersecting line and the line that goes through the point of intersection and is normal to the plane.
- If a straight transversal line intersects two parallel lines, corresponding (alternate) angles at the two points of intersection are congruent; adjacent angles are supplementary (that is, their measures add to π radians, or 180°).

A formal definition

Using trigonometric functions

A Euclidean angle is completely determined by the corresponding right triangle. In particular, if θ is a Euclidean angle, it is true that

$$\cos\theta = \frac{x}{\sqrt{x^2 + y^2}}$$

and

$$\sin\theta = \frac{y}{\sqrt{x^2 + y^2}}$$

for two numbers x and y. So an angle in the Euclidean plane can be legitimately given by two numbers x and y.

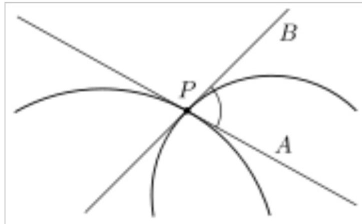
To the ratio $\frac{y}{x}$ there correspond two angles in the geometric range $0 < \theta < 2\pi$, since

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{\sqrt{x^2+y^2}}}{\frac{x}{\sqrt{x^2+y^2}}} = \frac{y}{x} = \frac{-y}{-x} = \frac{\sin(\theta+\pi)}{\cos(\theta+\pi)}.$$

Using rotations

Suppose we have two unit vectors $\vec{u}$ and $\vec{v}$ in the euclidean plane $\mathbb{R}^2$. Then there exists one positive isometry (a rotation), and one only, from $\mathbb{R}^2$ to $\mathbb{R}^2$ that maps *u* onto *v*. Let *r* be such a rotation. Then the relation $\vec{a}\mathcal{R}\vec{b}$ defined by $\vec{b} = r(\vec{a})$ is an equivalence relation and we call **angle of the rotation *r*** the equivalence class $\mathbb{T}/\mathcal{R}$, where $\mathbb{T}$ denotes the unit circle of $\mathbb{R}^2$. The angle between two vectors will simply be the angle of the rotation that maps one onto the other. We have no numerical way of determining an angle yet. To do this, we choose the vector (1,0), then for any point M on $\mathbb{T}$ at distance θ from (1,0) (on the circle), let $\vec{u} = \overrightarrow{OM}$. If we call r_θ the rotation that transforms (1,0) into $\vec{u}$, then $[r_\theta] \mapsto \theta$ is a bijection, which means we can identify any angle with a number between 0 and 2π.

Angles between curves



The angle between the two curves is defined as the angle between the tangents *A* and *B* at *P*

The angle between a line and a curve (mixed angle) or between two intersecting curves (curvilinear angle) is defined to be the angle between the tangents at the point of intersection. Various names (now rarely, if ever, used) have been given to particular cases:—*amphicyrtic* (Gr. *ἀμφί*, on both sides, *κυρτόσ*, convex) or *cissoidal* (Gr. *κισσόσ*, ivy), biconvex; *xystroidal* or *sistroidal* (Gr. *ξυστρίσ*, a tool for scraping), concavo-convex; *amphicoelic* (Gr. *κοίλη*, a hollow) or *angulus lunularis*, biconcave.

The dot product and generalisation

In the Euclidean plane, the angle θ between two vectors **u** and **v** is related to their dot product and their lengths by the formula

$$\mathbf{u}\cdot\mathbf{v} = \cos(\theta)\ \|\mathbf{u}\|\ \|\mathbf{v}\|.$$

This allows one to define angles in any real inner product space, replacing the Euclidean dot product · by the Hilbert space inner product <·,·>.

Angles in Riemannian geometry

In Riemannian geometry, the metric tensor is used to define the angle between two tangents. Where *U* and *V* are tangent vectors and g_{ij} are the components of the metric tensor *G*,

$$\cos\theta = \frac{g_{ij}U^iV^j}{\sqrt{|g_{ij}U^iU^j|\,|g_{ij}V^iV^j|}}.$$

Angles in geography and astronomy

In geography we specify the location of any point on the Earth using a **Geographic coordinate system**. This system specifies the latitude and longitude of any location, in terms of angles subtended at the centre of the Earth, using the equator and (usually) the Greenwich meridian as references.

In astronomy, we similarly specify a given point on the celestial sphere using any of several **Astronomical coordinate systems**, where the references vary according to the particular system.

Astronomers can also measure the **angular separation** of two stars by imagining two lines through the centre of the Earth, each intersecting one of the stars. The angle between those lines can be measured, and is the angular separation between the two stars.

Astronomers also measure the **apparent size** of objects. For example, the full moon has an angular measurement of approximately 0.5°, when viewed from Earth. One could say, "The Moon subtends an angle of half a degree." The small-angle formula can be used to convert such an angular measurement into a distance/size ratio.

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Arabic numerals

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Arabic numerals are the traditional name for the digits invented by Indian mathematicians in around AD 500 and the system by which a sequence of digits (e.g. "436") represents a number. The reason that they are known as Arabic rather than Indian numerals owes to how Arabic speakers conveyed the system from India to Europe during the Middle Ages, leading Europeans to attribute the numerals to the Arabic language. They are the most common symbolic representation of numbers around the world and are considered an important milestone in the development of mathematics.

One may distinguish between the decimal system involved, also known as the Hindu-Arabic numeral system, and the precise glyphs used. The glyphs most commonly used in conjunction with the Latin alphabet since Early Modern times are 0 1 2 3 4 5 6 7 8 9.

They were transmitted first to West Asia, where they find mention in the 9th century, and eventually to Europe in the 10th century. Since knowledge of the numerals reached Europe through the work of Arab mathematicians and astronomers, the numerals came to be called "Arabic numerals."

One also distinguishes an eastern arabic form of these numerals and a western arabic form, closer to the modern western, worldwide form.

History

Origins

The symbols for 1 to 9 in the Hindu-Arabic numeral system evolved from the Brahmi numerals. Buddhist inscriptions from around 300 BC use the symbols which became 1, 4 and 6. One century later, their use of the symbols which became 2, 7 and 9 was recorded.

The first universally accepted inscription containing the use of the 0 glyph is first recorded in the 9th century, in an inscription at Gwalior dated to 870. However, by this time, the use of the glyph had already reached Persia, and is mentioned in Al-Khwarizmi's descriptions of Indian numerals. Indian documents on copper plates, with the same symbol for zero in them, dated back as far as the 6th century AD, abound.

0123456789

Numerals sans-serif

Numeral systems by culture

Hindu-Arabic numerals

Indian	Indian family
Eastern Arabic	Brahmi
Khmer	Thai

East Asian numerals

Chinese	Japanese
Suzhou	Korean
Counting rods	

Alphabetic numerals

Abjad	Hebrew
Armenian	Greek (Ionian)
Cyrillic	Āryabhaṭa
Ge'ez	

Other systems

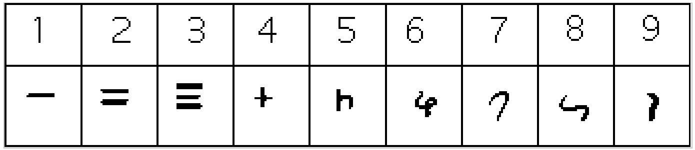
Attic	Mayan
Babylonian	Roman
Egyptian	Urnfield
Etruscan	

List of numeral system topics

Positional systems by base

Decimal (10)

2, 4, 8, 16, 32, 64



Brahmi numerals in India in the 1st century AD

The numeral system came to be known to both the Persian mathematician Al-Khwarizmi, whose book *On the Calculation with Hindu Numerals* written about 825, and the Arab mathematician Al-Kindi, who wrote four volumes, "On the Use of the Indian Numerals" (*Ketab fi Isti'mal al-'Adad al-Hindi*) about 830, are principally responsible for the diffusion of the Indian system of numeration in the Middle East and the West. In the 10th century, Middle-Eastern mathematicians extended the decimal numeral system to include fractions, as recorded in a treatise by Syrian mathematician Abu'l-Hasan al-Uqlidisi in 952–53.

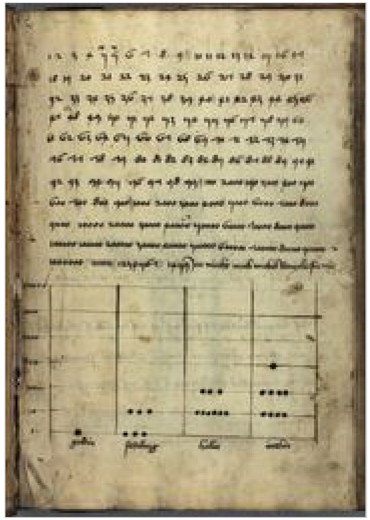
1, 3, 9, 12, 20, 24, 30, 36, 60, more…

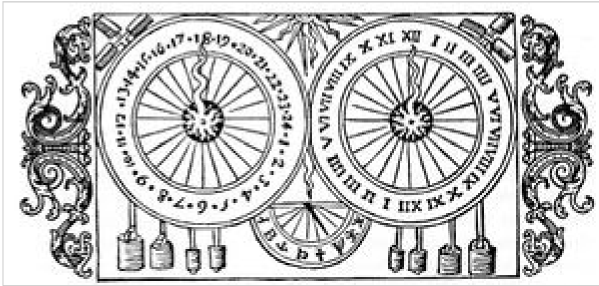
Modern-day Arab telephone keypad with two forms of Hindu-Arabic numerals, Arabic and European

In the Arab world—until modern times—the Arabic numeral system was used only by mathematicians. Muslim scientists used the Babylonian numeral system, and merchants used the Abjad numerals. It was not until Fibonacci that the Arabic numeral system was used by a large population.

A distinctive West Arabic variant of the symbols begins to emerge around the 10th century in the Maghreb and Al-Andalus, called *ghubar* ("sand-table" or "dust-table") numerals.

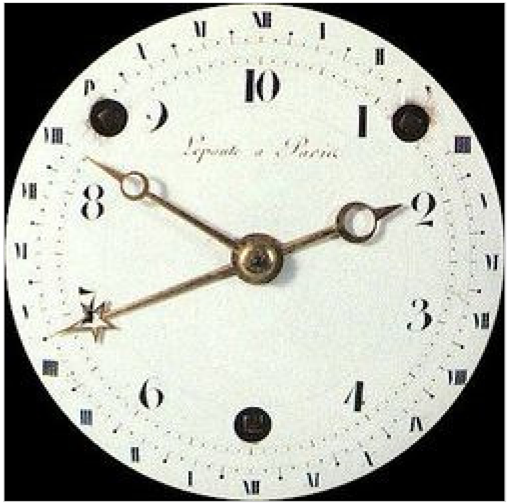
The first mentions of the numerals in the West are found in the *Codex Vigilanus* of 976 . From the 980s, Gerbert of Aurillac (later, Pope Silvester II) began to spread knowledge of the numerals in Europe. Gerbert studied in Barcelona in his youth, and he is known to have requested mathematical treatises concerning the astrolabe from Lupitus of Barcelona after he had returned to France.

Adoption in Europe

A German manuscript page teaching use of Arabic numerals (Talhoffer Thott, 1459). At this time, knowledge of the numerals was still widely seen as esoteric, and Talhoffer teaches them together with the Hebrew alphabet and astrology.



Woodcut showing the 16th century astronomical clock of Uppsala cathedral, with two clockfaces, one with Arabic and one with Roman numerals.

In 825 Al-Khwārizmī, the Persian scientist, wrote a treatise, *On the Calculation with Hindu Numerals*, which was translated into Latin in the 12th century as *Algoritmi de numero Indorum*, where *Algoritmi*, the translator's rendition of the author's name, gave rise to the word *algorithm* (Latin *algorithmus*, "calculation method").



Late 18th century French revolutionary "decimal" clockface.

Fibonacci, an Italian mathematician who had studied in Bejaia (Bougie), Algeria, promoted the Arabic numeral system in Europe with his book *Liber Abaci*, which was written in 1202, still describing the numerals as Indian rather than Arabic.

> "When my father, who had been appointed by his country as public notary in the customs at Bugia acting for the Pisan merchants going there, was in charge, he summoned me to him while I was still a child, and having an eye to usefulness and future convenience, desired me to stay there and receive instruction in the school of accounting. There, when I had been introduced to the art of the Indians' nine symbols through remarkable teaching, knowledge of the art very soon pleased me above all else and I came to understand it.."

The numerals are arranged with their lowest value digit to the right, with higher value positions added to the left. This arrangement was adopted identically into the numerals as used in Europe. The Latin alphabet running from left to right, unlike the Arabic alphabet, this resulted in an inverse arrangement of the place-values relative to the direction of reading.

The European acceptance of the numerals was accelerated by the invention of the printing press, and they became commonly known during the 15th century. Early uses in England include a 1445 inscription on the tower of Heathfield Church, Sussex, a 1448 inscription on a wooden lych-gate of Bray Church, Berkshire, and a 1487 inscription on the belfry door at Piddletrenthide church, Dorset and in Scotland a 1470 inscription on the tomb of the first Earl of Huntly in Elgin, (Elgin, Moray) Cathedral. (See G.F. Hill, *The Development of Arabic Numerals in Europe* for more examples.) By the mid-16th century, they were in common use in most of Europe. Roman numerals remained in use mostly for the notation of Anno Domini years, and for numbers on clockfaces. Sometimes, Roman numerals are still used for enumeration of lists (as an alternative to alphabetical enumeration), and numbering pages in prefatory material in books.

Evolution of symbols

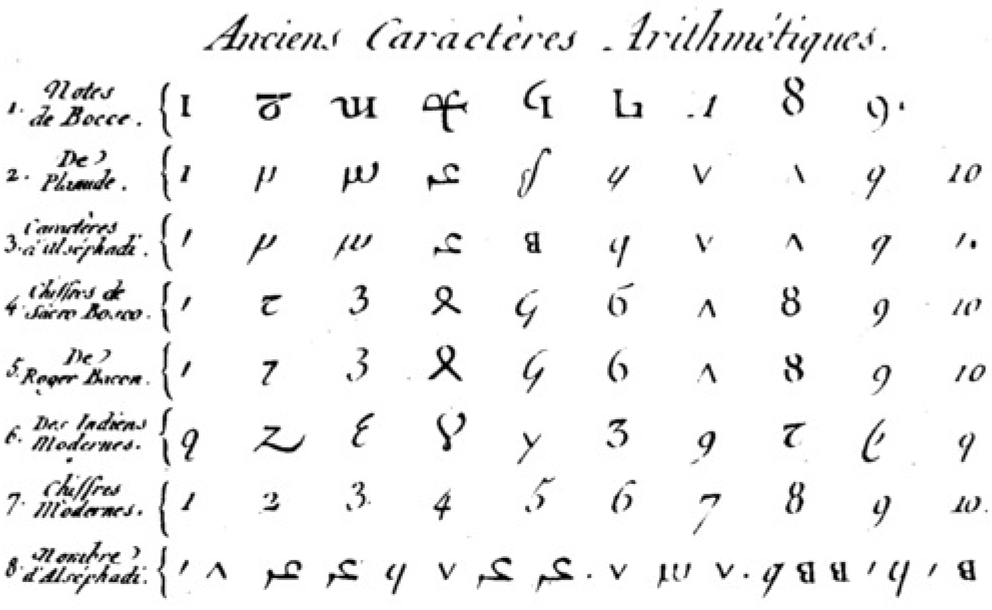
The numeral system employed, known as algorism, is positional decimal notation. Various symbol sets are used to represent numbers in the Arabic numeral system, all of which evolved from the Brahmi numerals. The symbols used to represent the system have split into various typographical variants since the Middle Ages:

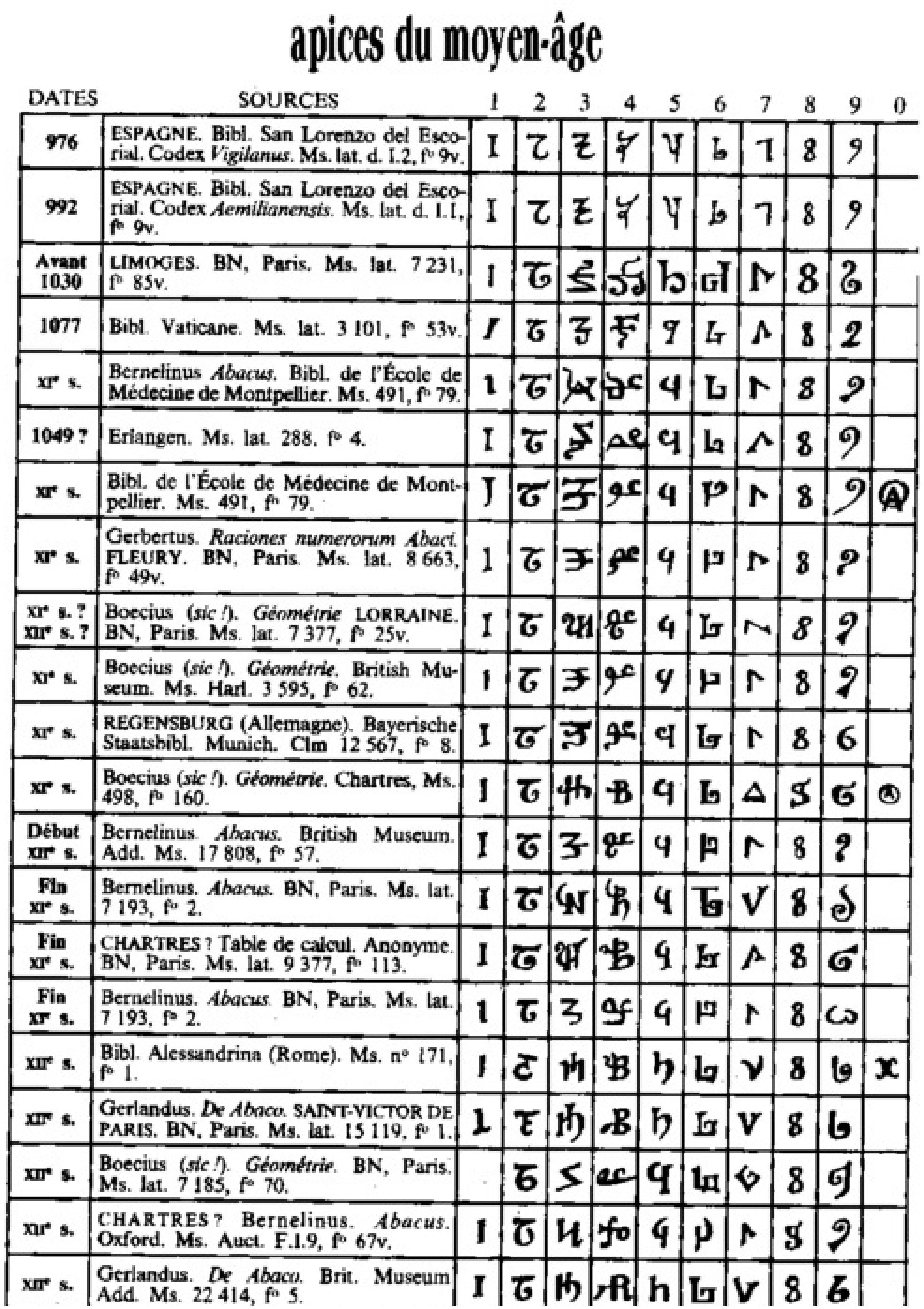
- The widespread Western Arabic numerals used with the Latin alphabet, in the table below labelled *European*, descended from the West Arabic numerals developed in al-Andalus and the Maghreb. (There are two typographic styles for rendering European numerals, known as lining figures and text figures).
- The Arabic-Indic or Eastern Arabic numerals used with the Arabic alphabet developed primarily in what is now Iraq. A variant of the Eastern Arabic numerals used in the Persian and Urdu languages is shown as East Arabic-Indic.

- The Devanagari numerals used with Devanagari and related variants are grouped as Indian numerals.

European	0	1	2	3	4	5	6	7	8	9
Arabic-Indic	٠	١	٢	٣	٤	٥	٦	٧	٨	٩
Eastern Arabic-Indic (Persian and Urdu)	۰	۱	۲	۳	۴	۵	۶	۷	۸	۹
Devanagari (Hindi)	०	१	२	३	४	५	६	७	८	९
Tamil		௧	௨	௩	௪	௫	௬	௭	௮	௯

The evolution of the numerals in early Europe is shown on a table created by the French scholar J.E. Montucla in his *Histoire de la Mathematique*, which was published in 1757:




apices du moyen-âge
DATES
SOURCES
1 2 3 4 5 6 7 8 9 0
976 ESPAGNE. Bibl. San Lorenzo del Escorial. Codex Vigilanus. Ms. lat. d. I.2, f° 9v.
992 ESPAGNE. Bibl. San Lorenzo del Escorial. Codex Aemilianensis. Ms. lat. d. I.1, f° 9v.
Avant 1030 LIMOGES. BN, Paris. Ms. lat. 7 231, f° 85v.
1077 Bibl. Vaticane. Ms. lat. 3 101, f° 53v.
XIe s. Bernelinus Abacus. Bibl. de l'École de Médecine de Montpellier. Ms. 491, f° 79.
1049 ? Erlangen. Ms. lat. 288, f° 4.
XIe s. Bibl. de l'École de Médecine de Montpellier. Ms. 491, f° 79.
XIe s. Gerbertus. Raciones numerorum Abaci. FLEURY. BN, Paris. Ms. lat. 8 663, f° 49v.
XIe s. ? XIIe s. ? Boecius (sic !). Géométrie LORRAINE. BN, Paris. Ms. lat. 7 377, f° 25v.
XIe s. Boecius (sic !). Géométrie. British Museum. Ms. Harl. 3 595, f° 62.
XIe s. REGENSBURG (Allemagne). Bayerische Staatsbibl. Munich. Clm 12 567, f° 8.
XIe s. Boecius (sic !). Géométrie. Chartres, Ms. 498, f° 160.
Début XIIe s. Bernelinus. Abacus. British Museum. Add. Ms. 17 808, f° 57.
Fin XIe s. Bernelinus. Abacus. BN, Paris. Ms. lat. 7 193, f° 2.
Fin XIe s. CHARTRES ? Table de calcul. Anonyme. BN, Paris. Ms. lat. 9 377, f° 113.
Fin XIe s. Bernelinus. Abacus. BN, Paris. Ms. lat. 7 193, f° 2.
XIIe s. Bibl. Alessandrina (Rome). Ms. n° 171, f° 1.
XIIe s. Gerlandus. De Abaco. SAINT-VICTOR DE PARIS. BN, Paris. Ms. lat. 15 119, f° 1.
XIIe s. Boecius (sic !). Géométrie. BN, Paris. Ms. lat. 7 185, f° 70.
XIIe s. CHARTRES ? Bernelinus. Abacus. Oxford. Ms. Auct. F.1.9, f° 67v.
XIIe s. Gerlandus. De Abaco. Brit. Museum Add. Ms. 22 414, f° 5.

The Arabic numerals are encoded in ASCII (and Unicode) at positions 48 to 57:

Binary	Dec	Hex	Glyph
0011 0000	48	30	0
0011 0001	49	31	1
0011 0010	50	32	2
0011 0011	51	33	3
0011 0100	52	34	4
0011 0101	53	35	5
0011 0110	54	36	6
0011 0111	55	37	7
0011 1000	56	38	8
0011 1001	57	39	9

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Arithmetic

2008/9 Schools Wikipedia Selection. Related subjects: Mathematics

Arithmetic tables for children, Lausanne, 1835

Arithmetic or **arithmetics** (from the Greek word *αριθμός* = number) is the oldest and most elementary branch of mathematics, used by almost everyone, for tasks ranging from simple day-to-day counting to advanced science and business calculations. In common usage, the word refers to a branch of (or the forerunner of) mathematics which records elementary properties of certain *operations* on numbers. Professional mathematicians sometimes use the term *(higher) arithmetic* when referring to number theory, but this should not be confused with elementary arithmetic.

History

The prehistory of arithmetic is limited to a very small number of small artifacts indicating a clear conception of addition and subtraction, the best-known being the Ishango bone from central Africa, dating from somewhere between 18,000 and 20,000 BC.

It is clear that the Babylonians had solid knowledge of almost all aspects of elementary arithmetic by 1800 BC, although historians can only guess at the methods utilized to generate the arithmetical results - as shown, for instance, in the clay tablet Plimpton 322, which appears to be a list of Pythagorean triples, but with no workings to show how the list was originally produced. Likewise, the Egyptian Rhind Mathematical Papyrus (dating from c. 1650 BC, though evidently a copy of an older text from c. 1850 BC) shows evidence of addition, subtraction, multiplication, and division being used within a unit fraction system.

Nicomachus (c. AD 60 - c. AD 120) summarised the philosophical Pythagorean approach to numbers, and their relationships to each other, in his *Introduction to Arithmetic*. At this time, basic arithmetical operations were highly complicated affairs; it was the method known as the "Method of the Indians" (Latin "Modus Indorum") that became the arithmetic that we know today. Indian arithmetic was much simpler than Greek arithmetic due to the simplicity of the Indian number system, which had a zero and place-value notation. The 7th century Syriac bishop Severus Sebhokt mentioned this method with admiration, stating however that the Method of the Indians was beyond description. The Arabs learned this new method and called it "Hesab" or "Hindu Science". Fibonacci (also known as Leonardo of Pisa) introduced the "Method of the Indians" to Europe in 1202. In his book " Liber Abaci", Fibonacci says that, compared with this new method, all other methods had been mistakes. In the Middle Ages, arithmetic was one of the seven liberal arts taught in universities.

Modern algorithms for arithmetic (both for hand and electronic computation) were made possible by the introduction of Hindu-Arabic numerals and decimal place notation for numbers. Hindu-Arabic numeral based arithmetic was developed by the great Indian mathematicians Aryabhatta, Brahmagupta and Bhāskara I. Aryabhatta tried different place value notations and Brahmagupta added zero to the Indian number system. Brahmagupta developed modern multiplication, division, addition and subtraction based on Hindu-Arabic numerals. Although it is now considered elementary, its simplicity is the culmination of thousands of years of mathematical development. By contrast, the ancient mathematician Archimedes devoted an entire work, The Sand Reckoner, to devising a notation for a certain large integer. The flourishing of algebra in the medieval Islamic world and in Renaissance Europe was an outgrowth of the enormous simplification of computation through decimal notation.

Decimal arithmetic

Decimal notation constructs all real numbers from the basic digits, the first ten non-negative integers 0,1,2,...,9. A decimal numeral consists of a sequence of these basic digits, with the "denomination" of each digit depending on its *position* with respect to the decimal point: for example, 507.36 denotes 5 hundreds (10^2), plus 0 tens (10^1), plus 7 units (10^0), plus 3 tenths (10^{-1}) plus 6 hundredths (10^{-2}). An essential part of this notation (and a major stumbling block in achieving it) was conceiving of zero as a number comparable to the other basic digits.

Algorism comprises all of the rules of performing arithmetic computations using a decimal system for representing numbers in which numbers written using ten symbols having the values 0 through 9 are combined using a place-value system (positional notation), where each symbol has ten times the weight of the one to its right. This notation allows the addition of arbitrary numbers by adding the digits in each place, which is accomplished with a 10 x 10 addition table. (A sum of digits which exceeds 9 must have its 10-digit carried to the next place leftward.) One can make a similar algorithm for multiplying arbitrary numbers because the set of denominations $\{...,10^2,10,1,10^{-1},...\}$ is closed under multiplication. Subtraction and division are achieved by similar, though more complicated algorithms.

Arithmetic operations

The traditional arithmetic operations are addition, subtraction, multiplication and division, although more advanced operations (such as manipulations of percentages, square root, exponentiation, and logarithmic functions) are also sometimes included in this subject. Arithmetic is performed according to an order of operations. Any set of objects upon which all four operations of arithmetic can be performed (except division by zero), and wherein these four operations obey the usual laws, is called a field.

Addition (+)

Addition is the basic operation of arithmetic. In its simplest form, addition combines two numbers, the *addends* or *terms*, into a single number, the *sum*.

Adding more than two numbers can be viewed as repeated addition; this procedure is known as summation and includes ways to add infinitely many numbers in an infinite series; repeated addition of the number one is the most basic form of counting.

Addition is commutative and associative so the order in which the terms are added does not matter. The identity element of addition (the additive identity) is 0, that is, adding zero to any number will yield that same number. Also, the inverse element of addition (the additive inverse) is the opposite of any number, that is, adding the opposite of any number to the number itself will yield the additive identity, 0. For example, the opposite of 7 is (-7), so 7 + (-7) = 0.

Subtraction (−)

Subtraction is essentially the opposite of addition. Subtraction finds the *difference* between two numbers, the *minuend* minus the *subtrahend*. If the minuend is larger than the subtrahend, the difference will be positive; if the minuend is smaller than the subtrahend, the difference will be negative; and if they are equal, the difference will be zero.

Subtraction is neither commutative nor associative. For that reason, it is often helpful to look at subtraction as addition of the minuend and the opposite of the subtrahend, that is $a - b = a + (-b)$. When written as a sum, all the properties of addition hold.

Multiplication (×, ·, or *)

Multiplication is in essence repeated addition, or the sum of a list of identical numbers. Multiplication finds the *product* of two numbers, the *multiplier* and the *multiplicand*, sometimes both simply called *factors*.

Multiplication, as it is really repeated addition, is commutative and associative; further it is distributive over addition and subtraction. The multiplicative identity is 1, that is, multiplying any number by 1 will yield that same number. Also, the multiplicative inverse is the reciprocal of any number, that is, multiplying the reciprocal of any number by the number itself will yield the multiplicative identity, 1.

Division (÷ or /)

Division is essentially the opposite of multiplication. Division finds the *quotient* of two numbers, the *dividend* divided by the *divisor*. Any dividend divided by zero is undefined. For positive numbers, if the dividend is larger than the divisor, the quotient will be greater than one, otherwise it will be less than one (a similar rule applies for negative numbers). The quotient multiplied by the divisor always yields the dividend.

Division is neither commutative nor associative. As it is helpful to look at subtraction as addition, it is helpful to look at division as multiplication of the dividend times the reciprocal of the divisor, that is $a \div b = a \times {}^{1}\!/_{b}$. When written as a product, it will obey all the properties of multiplication.

Examples

Multiplication table

×	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	5	10	15	20	**25**	30	35	40	45	**50**	55	60	65	70	**75**	80	85	90	95	**100**	105	110	115	120	125
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	10	20	30	40	**50**	60	70	80	90	**100**	110	120	130	140	**150**	160	170	180	190	**200**	210	220	230	240	250
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	15	30	45	60	**75**	90	105	120	135	**150**	165	180	195	210	**225**	240	255	270	285	**300**	315	330	345	360	375
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	20	40	60	80	**100**	120	140	160	180	**200**	220	240	260	280	**300**	320	340	360	380	**400**	420	440	460	480	500
21	21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550

23	23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
24	24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	**625**

Number theory

The term *arithmetic* is also used to refer to number theory. This includes the properties of integers related to primality, divisibility, and the solution of equations by integers, as well as modern research which is an outgrowth of this study. It is in this context that one runs across the fundamental theorem of arithmetic and arithmetic functions. *A Course in Arithmetic* by Serre reflects this usage, as do such phrases as *first order arithmetic* or *arithmetical algebraic geometry*. Number theory is also referred to as 'the higher arithmetic', as in the title of H. Davenport's book on the subject.

Arithmetic in education

Primary education in mathematics often places a strong focus on algorithms for the arithmetic of natural numbers, integers, rational numbers (vulgar fractions), and real numbers (using the decimal place-value system). This study is sometimes known as algorism.

The difficulty and unmotivated appearance of these algorithms has long led educators to question this curriculum, advocating the early teaching of more central and intuitive mathematical ideas. One notable movement in this direction was the New Math of the 1960s and '70s, which attempted to teach arithmetic in the spirit of axiomatic development from set theory, an echo of the prevailing trend in higher mathematics.

Since the introduction of the electronic calculator, which can perform the algorithms far more efficiently than humans, an influential school of educators has argued that mechanical mastery of the standard arithmetic algorithms is no longer necessary. In their view, the first years of school mathematics could be more profitably spent on understanding higher-level ideas about what numbers are used for and relationships among number, quantity, measurement, and so on. However, most research mathematicians still consider mastery of the manual algorithms to be a necessary foundation for the study of algebra and computer science. This controversy was central to the " Math Wars" over California's primary school curriculum in the 1990s, and continues today.

Many mathematics texts for K-12 instruction were developed, funded by grants from the United States National Science Foundation based on standards created by the NCTM and given high ratings by United States Department of Education, though condemned by many mathematicians. Some widely adopted texts such as TERC were based on the spirit of research papers which found that instruction of basic arithmetic was harmful to mathematical understanding. Rather than teaching any traditional method of arithmetic, teachers are instructed to instead guide students to invent their own (some critics claim inefficient) methods, instead using such techniques as skip counting, and the heavy use of manipulatives, scissors and paste, and even singing rather than multiplication tables or long division. Although such texts were designed to be a complete curricula, in the face of intense protest and criticism, many districts have chosen to circumvent the intent of such radical approaches by supplementing with traditional texts. Other districts have since adopted traditional mathematics texts and discarded such reform-based approaches as misguided failures.

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Calculator

2008/9 Schools Wikipedia Selection. Related subjects: Mathematics

A **calculator** is a device for performing mathematical calculations, distinguished from a computer generally by a limited problem domain and an interface optimized for interactive calculation rather than programming. Calculators can be hardware or software, and mechanical or electronic, and are often built into devices such as PDAs or mobile phones.

A basic calculator

Modern electronic calculators are generally small (often pocket-sized) and usually inexpensive. In addition to general purpose calculators, there are those designed for specific markets; for example, there are scientific calculators which focus on advanced math like trigonometry and statistics. Modern calculators are more portable than most computers, though most PDAs are comparable in size to handheld calculators.

Overview

In the past, mechanical clerical aids such as abaci, comptometers, Napier's bones, books of mathematical tables, slide rules, or mechanical adding machines were used for numeric work. This semi-manual process of calculation was tedious and error-prone.

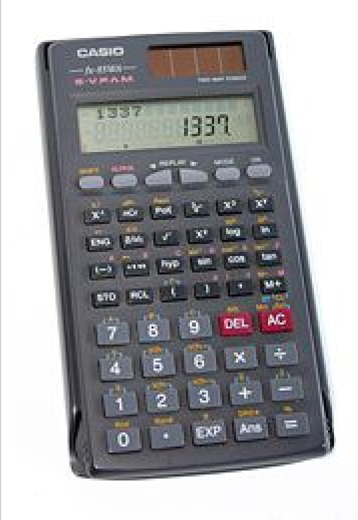
Image:Gosremprom.JPG
An old mechanical calculator.

Modern calculators are electrically powered (usually by battery and/or solar cell) and vary from cheap, give-away, credit-card sized models to sturdy adding machine-like models with built-in printers. They first became popular in the late 1960s as decreasing size and cost of electronics made possible devices for calculations, avoiding the use of scarce and expensive computer resources. By the 1980s, calculator prices had reduced to a point where a basic calculator was affordable to most. By the 1990s they had become common in math classes in schools, with the idea that students could be freed from basic calculations and focus on the concepts.

Computer operating systems as far back as early Unix have included interactive calculator programs such as dc and hoc, and calculator functions are included in almost all PDA-type devices (save a few dedicated address book and dictionary devices).

Electronic calculators

In the past, some calculators were as large as today's computers. The first mechanical calculators were mechanical desktop devices which were replaced by electromechanical desktop calculators, and then by electronic devices using first thermionic valves, then transistors, then hard-wired integrated circuit logic. By the mid-1970s, pocket-sized calculators based on ICs were routinely available, often at prices less than $100, and by the early 1980s the LED displays of 1970s units had been replaced by power-saving liquid crystal displays. Modern electronic calculators range in size from keychain-sized units only a couple of centimeters long all the way up to desktop calculators the size of a textbook, and in complexity from very basic up to graphing calculators capable of video display and sometimes extensive general-purpose programming capability.



A scientific calculator.

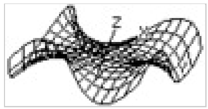
Basic configuration

A simple modern calculator (usually known colloquially as a "four function" calculator, even with the presence of a square root button) might consist of the following parts:

- A power source, such as a battery or a solar panel or both
- A display, usually made from LED lights or liquid crystal (LCD), capable of showing a number of digits (typically 8 or 10)
- Electronic circuitry (often a single chip and some other components)
- A keypad containing:
  - The ten digits, 0 to 9
  - The decimal point
  - The equals sign, to prompt for the answer
  - The four arithmetic functions (addition, subtraction, multiplication and division)
  - A Cancel (or clear) button, to clear the calculation
- On and off buttons
  - Other basic functions, such as square root and percentage (%) (desktop models will sometimes add tax functions and significant digit selectors to simplify work with money)
- A single-number memory, which can be recalled where necessary. It might also have a Cancel Entry button, to clear the numbers entered. (Many scientific calculators have multiple variables available.)

Since the late-1980s, calculators have been installed in other small devices, such as mobile phones, pagers or wrist watches.

Scientific and financial calculators



A TI-89 calculator can produce 3D wire frame graphs such as this graph of $z(x,y) = x^{3y} - y^{3x}$.

More complex *scientific calculators* support trigonometric, statistical and other mathematical functions. The most advanced modern calculators can display graphics, and include features of computer algebra systems. They are also programmable; calculator applications include algebraic equation solvers, financial models and even games. Most calculators of this type can print numbers up to ten digits or decimal places in full on the screen. Scientific notation is used to notate numbers up to a limit chosen by the calculator designer, such as $9.999999999*10^{99}$. If a larger number or a mathematical expression yielding a larger number than this is entered (a common example comes from typing "100!", read as "100 factorial") then the calculator might simply display "error".

"Error" might also be displayed if a function or an operation is undefined mathematically; for example, division by zero or even roots of negative numbers (most scientific calculators do not allow complex numbers, though a few do have a *special function* for working with them). Some, but not most, calculators *do* distinguish between these two types of "error", though when they do, it is not always easy for the user to understand because they are often given as "Error 1" or "Error 2".

Financial calculators are similar in overall design to scientific calculators, but specialize in time value of money calculations and are used in the accounting and real estate professions.

Only a few companies make professional engineering and finance calculators. They include Casio, Sharp, Hewlett-Packard (HP), Victor and Texas Instruments (TI), as well as Chinese manufacturer Karce, who provides OEM calculators for the private label market. Such calculators are examples of embedded systems.

Use in education

In most countries, students use calculators for schoolwork. There was some initial resistance to the idea out of fear that basic arithmetic skills would suffer. There remains disagreement about the importance of the ability to perform calculations by hand or "in the head", with some curricula restricting calculator use until a certain level of proficiency has been obtained, while others concentrate more on teaching estimation techniques and problem-solving. Research suggests that inadequate guidance in the use of calculating tools can restrict the kind of mathematical thinking that students engage in.

There are other concerns - for example, that a pupil could use the calculator in the wrong fashion but believe the answer because that was the result given. Teachers try to combat this by encouraging the student to make an estimate of the result manually and ensuring it roughly agrees with the calculated result. Also, it is possible for a child to type in -1×-1 and obtain the correct answer '1' without realizing the principle involved. In this sense, the calculator becomes a crutch rather than a learning tool, and it can slow down students in exam conditions as they check even the most trivial result on a calculator.

Other concerns on usage

Errors are not restricted to school pupils. Any user could carelessly rely on the calculator's output without double-checking the magnitude of the result — i.e., where the decimal point is positioned. This problem was all but nonexistent in the era of slide rules and pencil-and-paper calculations, when the task of establishing the magnitudes of results had to be done by the user. In addition, algorithmic flaws and rounding techniques can sometimes lead to minor precision errors.

Some fractions such as 2/3 are awkward to display on a calculator display as they are usually rounded to 0.66666667. Also, some fractions such as 1/7 which is 0.14285714285714 can be difficult to recognize in decimal form; as a result, many scientific calculators are able to work in vulgar fractions and/or mixed numbers.

Calculating vs. computing

The fundamental difference between calculators and computers is that computers can be programmed to perform different tasks while calculators are pre-designed with specific functions built in, for example addition, multiplication, logarithms, etc. While computers may be used to handle numbers, they can also manipulate words, images or sounds and other tasks they have been programmed to handle. However, the distinction between the two is quite blurred; some calculators have built-in programming functions, ranging from simple formula entry to full programming languages such as RPL or TI-BASIC. Graphing calculators in particular can, along with PDAs, be viewed as direct descendants of the 1980s pocket computers, essentially calculators with full keyboards and programming capability.

The market for calculators is extremely price-sensitive, to an even greater extent than the personal computer market; typically the user desires the least expensive model having a specific feature set, but does not care much about speed (since speed is constrained by how fast the user can press the buttons). Thus designers of calculators strive to minimize the number of logic elements on the chip, not the number of clock cycles needed to do a computation.

For instance, instead of a hardware multiplier, a calculator might implement floating point mathematics with code in ROM, and compute trigonometric functions with the CORDIC algorithm because CORDIC does not require hardware floating-point. Bit serial logic designs are more common in calculators whereas bit parallel designs dominate general-purpose computers, because a bit serial design minimizes the chip complexity, but takes many more clock cycles. (Again, the line blurs with high-end calculators, which use processor chips associated with computer and embedded systems design, particularly the Z80, MC68000, and ARM architectures, as well as some custom designs specifically made for the calculator market.)

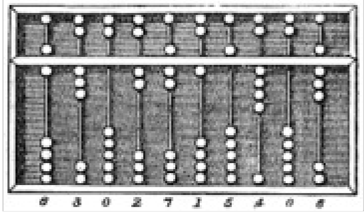
Personal computers and personal digital assistants can perform general calculations in a variety of ways:

- Most computer operating systems, at least those that support some kind of multitasking, include calculator programs, both text mode (such as the Unix bc (1) language) and graphical mode (Mac OS Calculator, Microsoft Calculator, KCalc, Grapher). Most, though not all, imitate the interface of a physical calculator. Some shell programs and interpreted programming languages also provide interactive calculation functions.
- For more complex calculations requiring large amounts of organized data, spreadsheet programs such as Excel or OpenOffice Calc provide calculation and sometimes reporting functions.
- Computer algebra programs such as Mathematica, Maple or Matlab can handle advanced calculations.
- Client-side scripting can be used for calculations, e.g. by entering "`javascript:alert(`*`calculation written in JavaScript`*`)`" in a web browser's address bar (as opposed to "`http://`*`website name`*"). Such calculations can be embedded in a separate Javascript or HTML file as well.
- Online calculators such as the calculator feature of the Google search engine can perform calculations server-side.

History

Origin: the abacus

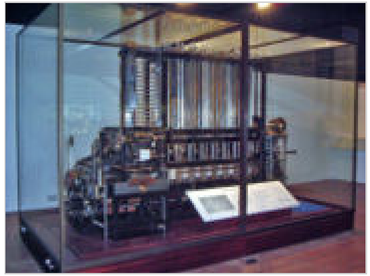
The first calculators were abaci, and were often constructed as a wooden frame with beads sliding on wires. Abacuses were in use centuries before the adoption of the written Arabic numerals system and are still used by some merchants, fishermen and clerks in China and elsewhere.


Chinese abacus.

The 17th century

William Oughtred invents the slide rule in 1622 and is revealed by his student Richard Delamain in 1630. Wilhelm Schickard built the first automatic calculator called the "Calculating Clock" in 1623. Some 20 years later, in 1643, French philosopher Blaise Pascal invented the calculation device later known as the Pascaline, which was used for taxes in France until 1799. The German philosopher G.W.v. Leibniz also produced a calculating machine.

The 19th century


The London Science Museum's working difference engine, built from Charles Babbage's design.

- In 1822 Charles Babbage proposed a mechanical calculator, called a difference engine, which was capable of holding and manipulating seven numbers of 31 decimal digits each. Babbage produced two designs for the difference engine and a further design for a more advanced mechanical programmable computer called an analytical engine. None of these designs were completely built by Babbage. In 1991 the London Science Museum followed Babbage's plans to build a working difference engine using the technology and materials available in the 19th century.
- In 1853 Per Georg Scheutz completed a working difference engine based on Babbage's design. The machine was the size of a piano, and was demonstrated at the Exposition Universelle in Paris in 1855. It was used to create tables of logarithms.
- In 1872, Frank Baldwin in the U.S.A. invented the pin-wheel calculator, which was also independently invented two years later by W.T. Odhner in Sweden. The Odhner models, and similar designs from other companies, sold many thousands into the 1970s.
- In 1875 Martin Wiberg re-designed the Babbage/Scheutz difference engine and built a version that was the size of a sewing machine.
- Dorr E. Felt, in the U.S.A., invented the Comptometer in 1884, the first successful key-driven adding and calculating machine ["key-driven" refers to the fact that just pressing the keys causes the result to be calculated, no separate lever has to be operated]. In 1886 he joined with Robert Tarrant to form the Felt & Tarrant Manufacturing Company which went on to make thousands of Comptometers.
- In 1891 William S. Burroughs began commercial manufacture of his printing adding calculator. Burroughs Corporation became one of the leading companies in the accounting machine and computer businesses.
- The "Millionaire" calculator was introduced in 1893. It allowed direct multiplication by any digit - "one turn of the crank for each figure in the multiplier".

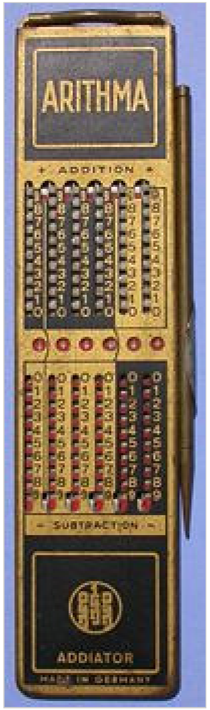
1900s to 1960s

Mechanical calculators reach their zenith

The first half of the 20th century saw the gradual development of the mechanical calculator mechanism.

Mechanical calculator from 1914

The Dalton adding-listing machine introduced in 1902 was the first of its type to use only ten keys, and became the first of many different models of "10-key add-listers" manufactured by many companies.



An Addiator could be used for addition and subtraction.

In 1948 the miniature Curta calculator, that was held in one hand for operation, was introduced after being developed by Curt Herzstark in a Nazi concentration camp. This was an extreme development of the stepped-gear calculating mechanism.

From the early 1900s through the 1960s, mechanical calculators dominated the desktop computing market (see History of computing hardware). Major suppliers in the USA included Friden, Monroe, and SCM/Marchant. (Some comments about European calculators follow below.) These devices were motor-driven, and had movable carriages where results of calculations were displayed by dials. Nearly all keyboards were *full* — each digit that could be entered had its own column of nine keys, 1..9, plus a column-clear key, permitting entry of several digits at once. (See the illustration of a 1914 mechanical calculator.) One could call this parallel entry, by way of contrast with ten-key serial entry that was commonplace in mechanical adding machines, and is now universal in electronic calculators. (Nearly all Friden calculators had a ten-key auxiliary keyboard for entering the multiplier when doing multiplication.) Full keyboards generally had ten columns, although some lower-cost machines had eight. Most machines made by the three companies mentioned did not print their results, although other companies, such as Olivetti, did make printing calculators.

In these machines, Addition and subtraction were performed in a single operation, as on a conventional adding machine, but multiplication and division were accomplished by repeated mechanical additions and subtractions. Friden made a calculator that also provided square roots, basically by doing division, but with added mechanism that automatically incremented the number in the keyboard in a systematic fashion. Friden and Marchant (Model SKA) made calculators with square root. Handheld mechanical calculators such as the 1948 Curta continued to be used until they were displaced by electronic calculators in the 1970s.


Facit NTK (1954)

The Facit, Triumphator, and Walther calculators are typical European machines. Similar-looking machines included the Odhner and Brunsviga. Although these are operated by handcranks, there were motor-driven versions. Most machines that look like these use the Odhner mechanism, or variations of it. The Olivetti Divisumma did all four basic operations of arithmetic, and has a printer. Full-keyboard machines, including motor-driven ones, were also used in Europe for many decades. Some European machines had as many as 20 columns in their full keyboards.

The development of electronic calculators


Triumphator CRN1 (1958)

The first main-frame computers, using firstly vacuum tubes and later transistors in the logic circuits, appeared in the late 1940s and 1950s. This technology was to provide a stepping stone to the development of electronic calculators.

In 1954, IBM, in the U.S.A., demonstrated a large all- transistor calculator and, in 1957, the company released the first *commercial* all-transistor calculator, the IBM 608, though it was housed in several cabinets and cost about $80,000 .

The Casio Computer Co., in Japan, released the Model *14-A* calculator in 1957, which was the world's first all-electric "compact" calculator. It did not use electronic logic but was based on relay technology, and was built into a desk.


Walther WSR160 (1960)

In October 1961, the world's first *all-electronic desktop* calculator, the Bell Punch/Sumlock Comptometer ANITA (**A** **N**ew **I**nspiration **T**o **A**rithmetic/**A**ccounting) was announced. This British designed-and-built machine used vacuum tubes, cold-cathode tubes and Dekatrons in its circuits, with 12 cold-cathode "Nixie"-type tubes for its display. Two models were displayed, The Mk VII for continental Europe and the Mk VIII for Britain and the rest of the world, both for delivery from early 1962. The Mk VII was a slightly earlier design with a more complicated mode of multiplication and was soon dropped in favour of the simpler Mark VIII version. The ANITA had a full keyboard, similar to mechanical Comptometers of the time, a feature that was unique to it and the later Sharp CS-10A among electronic calculators. Bell Punch had been producing key-driven mechanical calculators of the Comptometer type under the names "Plus" and "Sumlock", and had realised in the mid-1950s that the future of calculators lay in electronics. They employed the young graduate Norbert Kitz, who had worked on the early British Pilot ACE computer project, to lead the development. The ANITA sold well since it was the only electronic desktop calculator available, and was silent and quick.

The tube technology of the ANITA was superseded in June 1963, by the U.S. manufactured Friden EC-130, which had an all-transistor design, 13-digit capacity on a 5-inch CRT, and introduced reverse Polish notation (RPN) to the calculator market for a price of $2200, which was about triple the cost of an electromechanical calculator of the time. Like Bell Punch, Friden was a manufacturer of mechanical calculators that had decided that the future lay in electronics. In 1964 more all-transistor elctronic calculators were introduced: Sharp introduced the CS-10A, which weighed 25 kg (55 lb) and cost

500,000 yen (~US$2500), and Industria Macchine Elettroniche of Italy introduced the IME 84, to which several extra keyboard and display units could be connected so that several people could make use of it (but apparently not at the same time).


Olivetti Divisumma 24 (1964)

There followed a series of electronic calculator models from these and other manufacturers, including Canon, Mathatronics, Olivetti, SCM (Smith-Corona-Marchant), Sony, Toshiba, and Wang. The early calculators used hundreds of Germanium transistors, since these were then cheaper than Silicon transistors, on multiple circuit boards. Display types used were CRT, cold-cathode Nixie tubes, and filament lamps. Memory technology was usually based on the delay line memory or the magnetic core memory, though the Toshiba "Toscal" BC-1411 appears to use an early form of dynamic RAM built from discrete components. Already there was a desire for smaller and less power-hungry machines.

The Olivetti Programma 101 was introduced in late 1965; it was a stored program machine which could read and write magnetic cards and displayed results on its built-in printer. Memory, implemented by an acoustic delay line, could be partitioned between program steps, constants, and data registers. Programming allowed conditional testing and programs could also be overlaid by reading from magnetic cards. It is regarded as the first personal computer produced by a company (that is, a desktop electronic calculating machine programmable by non-specialists for personal use). The Olivetti Programma 101 won many industrial design awards.

The *Monroe Epic* programmable calculator came on the market in 1967. A large, printing, desk-top unit, with an attached floor-standing logic tower, it was capable of being programmed to perform many computer-like functions. However, the only *branch* instruction was an implied unconditional branch (GOTO) at the end of the operation stack, returning the program to its starting instruction. Thus, it was not possible to include any conditional branch (IF-THEN-ELSE) logic. During this era, the absence of the conditional branch was sometimes used to distinguish a programmable calculator from a computer.

The first handheld calculator was developed by Texas Instruments in 1967. It could add, multiply, subtract, and divide, and its output device was a paper tape.

1970s to mid-1980s


Old **calculator** LED display.

The electronic calculators of the mid-1960s were large and heavy desktop machines due to their use of hundreds of transistors on several circuit boards with a large power consumption that required an AC power supply. There were great efforts to put the logic required for a calculator into fewer and fewer integrated circuits (chips) and calculator electronics was one of the leading edges of semiconductor development. U.S. semiconductor manufacturers led the world in Large Scale Integration (LSI) semiconductor development, squeezing more and more functions into individual integrated circuits. This led to alliances between Japanese calculator manufacturers and U.S. semiconductor companies: Canon Inc. with Texas Instruments, Hayakawa Electric (later known as Sharp Corporation) with North-American Rockwell Microelectronics, Busicom with Mostek and Intel, and General Instrument with Sanyo.

Pocket calculators

By 1970 a calculator could be made using just a few chips of low power consumption, allowing portable models powered from rechargeable batteries. The first portable calculators appeared in Japan in 1970, and were soon marketed around the world. These included the Sanyo ICC-0081 "Mini Calculator", the Canon Pocketronic, and the Sharp QT-8B "micro Compet". The Canon Pocketronic was a development of the "Cal-Tech" project which had been started at Texas Instruments in 1965 as a research project to produce a portable calculator. The Pocketronic has no traditional display; numerical output is on thermal paper tape. As a result of the "Cal-Tech" project Texas instruments was granted master patents on portable calculators.

Sharp put in great efforts in size and power reduction and introduced in January 1971 the Sharp EL-8, also marketed as the Facit 1111, which was close to being a pocket calculator. It weighed about one pound, had a vacuum fluorescent display, rechargeable NiCad batteries, and initially sold for $395.

However, the efforts in integrated circuit development culminated in the introduction in early 1971 of the first "calculator on a chip", the MK6010 by Mostek, followed by Texas Instruments later in the year. Although these early hand-held calculators were very expensive, these advances in electronics, together with developments in display technology (such as the vacuum fluorescent display, LED, and LCD), lead within a few years to the cheap pocket calculator available to all.

The first truly pocket-sized electronic calculator was the Busicom LE-120A "HANDY", which was marketed early in 1971. Made in Japan, this was also the first calculator to use an LED display, the first hand-held calculator to use a single integrated circuit (then proclaimed as a "calculator on a chip"), the Mostek MK6010, and the first electronic calculator to run off replaceable batteries. Using four AA-size cells the LE-120A measures 4.9x2.8x0.9 in (124x72x24 mm).

The first American-made pocket-sized calculator, the Bowmar 901B (popularly referred to as *The Bowmar Brain*), measuring 5.2×3.0×1.5 in (131×77×37 mm), came out in the fall of 1971, with four functions and an eight-digit red LED display, for $240, while in August 1972 the four-function Sinclair Executive became the first slimline pocket calculator measuring 5.4×2.2×0.35 in (138×56×9 mm) and weighing 2.5 oz (70g). It retailed for around $150 (GB£79). By the end of the decade, similar calculators were priced less than $10 (GB£5).

The first Soviet-made pocket-sized calculator, the "Elektronika B3-04" was developed by the end of 1973 and sold at the beginning of 1974.

One of the first low-cost calculators was the Sinclair Cambridge, launched in August 1973. It retailed for £29.95, or some £5 less in kit form. The Sinclair calculators were successful because they were far cheaper than the competition; however, their design was flawed and their accuracy in some functions was questionable. The scientific programmable models were particularly poor in this respect, with the programmability coming at a heavy price in transcendental accuracy.

Meanwhile Hewlett Packard (HP) had been developing its own pocket calculator. Launched in early 1972 it was unlike the other basic four-function pocket calculators then available in that it was the first pocket calculator with *scientific* functions that could replace a slide rule. The $395 HP-35, along with all later HP engineering calculators, used reverse Polish notation (RPN), also called postfix notation. A calculation like "8 plus 5" is, using RPN, performed by pressing "8", "Enter↑", "5", and "+"; instead of the algebraic infix notation: "8", "+", "5", "=").

The first Soviet *scientific* pocket-sized calculator the "B3-18" was completed by the end of 1975.

In 1973, Texas Instruments(TI) introduced the SR-10, (*SR* signifying slide rule) an *algebraic entry* pocket calculator for $150. It was followed the next year by the SR-50 which added log and trig functions to compete with the HP-35, and in 1977 the mass-marketed TI-30 line which is still produced.

The first *programmable* pocket calculator was the HP-65, in 1974; it had a capacity of 100 instructions, and could store and retrieve programs with a built-in magnetic card reader. A year later the HP-25C introduced *continuous memory*, i.e. programs and data were retained in CMOS memory during power-off. In 1979, HP released the first *alphanumeric*, programmable, *expandable* calculator, the HP-41C. It could be expanded with RAM (memory) and ROM (software) modules, as well as peripherals like bar code readers, microcassette and floppy disk drives, paper-roll thermal printers, and miscellaneous communication interfaces (RS-232, HP-IL, HP-IB).

The first Soviet programmable calculator Elektronika " B3-21" was developed by the end of 1977 and sold at the beginning of 1978. The successor of B3-21, the Elektronika B3-34 wasn't backward compatible with B3-21, even if it kept the reverse Polish notation (RPN). Thus B3-34 defined a new command set, which later was used in all programmable soviet calculators. There are hundreds of developed programs for science, business and even games for these machines. The Elektronika MK-52 calculator (using the extended B3-34 command set, and featuring internal EEPROM memory for storing programs and external interface for EEPROM cards and other periphery) was used in soviet spacecraft program (for Soyuz TM-7 flight) as a backup of the board computer.

Mechanical calculators continued to be sold, though in rapidly decreasing numbers, into the early 1970s, with many of the manufacturers closing down or being taken over. Comptometer type calculators were often retained for much longer to be used for adding and listing duties, especially in accounting, since a trained and skilled operator could enter all the digits of a number in one movement of the hands on a Comptometer quicker than was possible serially with a 10-key electronic calculator. The spread of the computer rather than the simple electronic calculator put an end to the Comptometer. Also, by the end of the 1970s, the slide rule had become obsolete.

Technical improvements

Through the 1970s the hand-held electronic calculator underwent rapid development. The red LED and blue/green vacuum-fluorescent displays consumed a lot of power and the calculators either had a short battery life (often measured in hours, so rechargeable Nickel-Cadmium batteries were common) or were large so that they could take larger, higher capacity batteries. In the early 1970s Liquid crystal displays (LCDs) were in their infancy and there was a great deal of concern that they only had a short operating lifetime. Busicom introduced the Busicom *LE-120A "HANDY"* calculator, the first pocket-sized calculator and the first with an LED display, and announced the Busicom *LC* with LCD display. However, there were problems with this display and the calculator never went on sale. The first successful calculators with LCDs were manufactured by Rockwell International and sold from 1972 by other companies under such names as: Dataking *LC-800*, Harden *DT/12*, Ibico *086*, Lloyds *40*, Lloyds *100*, Prismatic *500* (aka *P500*), Rapid Data *Rapidman 1208LC*. The LCDs were an early form with the numbers appearing as silver against a dark background. To present a high-contrast display these models illuminated the LCD using a filament lamp and solid plastic light guide, which negated the low power consumption of the display. These models appear to have been sold only for a year or two.

A more successful series of calculators using the reflective LCD display was launched in 1972 by Sharp Inc with the Sharp *EL-805*, which was a slim pocket calculator. This, and another few similar models, used Sharp's "COS" (Crystal on Substrate) technology. This used a glass-like circuit board which was also an integral part of the LCD. In operation the user looked through this "circuit board" at the numbers being displayed. The "COS" technology may have been too expensive since it was only used in a few models before Sharp reverted to conventional circuit boards, though all the models with the reflective LCD displays are often referred to as "COS".

In the mid-1970s the first calculators appeared with the now "normal" LCDs with dark numerals against a grey background, though the early ones often had a yellow filter over them to cut out damaging UV rays. The big advantage of the LCD is that it is passive and reflects light, which requires much less power than generating light. This led the way to the first credit-card-sized calculators, such as the Casio *Mini Card LC-78* of 1978, which could run for months of normal use on a couple of button cells.

There were also improvements to the electronics inside the calculators. All of the logic functions of a calculator had been squeezed into the first "Calculator on a chip" integrated circuits in 1971, but this was leading edge technology of the time and yields were low and costs were high. Many calculators continued to use two or more integrated circuits (ICs), especially the scientific and the programmable ones, into the late 1970s.

The power consumption of the integrated circuits was also reduced, especially with the introduction of CMOS technology. Appearing in the Sharp "EL-801" in 1972, the transistors in the logic cells of CMOS ICs only used any apreciable power when they changed state. The LED and VFD displays had often required additional driver transistors or ICs, whereas the LCD displays were more amenable to being driven directly by the calculator IC itself.

With this low power consumption came the possibility of using solar cells as the power source, realised around 1978 by such calculators as the Royal *Solar 1*, Sharp *EL-8026*, and Teal *Photon*.

A pocket calculator for everyone

At the beginning of the 1970s hand-held electronic calculators were very expensive, costing two or three weeks' wages, and so were a luxury item. The high price was due to their construction requiring many mechanical and electronic components which were expensive to produce, and production runs were not very large. Many companies saw that there were good profits to be made in the calculator business with the margin on these high prices. However, the cost of calculators fell as components and their production techniques improved, and the effect of economies of scale were felt.

By 1976 the cost of the cheapest 4-function pocket calculator had dropped to a few dollars, about one twentieth of the cost five years earlier. The consequences of this were that the pocket calculator was affordable, and that it was now difficult for the manufacturers to make a profit out of calculators, leading to many companies dropping out of the business or closing down altogether. The companies that survived making calculators tended to be those with high outputs of higher quality calculators, or producing high-specification scientific and programmable calculators.

Mid-1980s to present

The first calculator capable of symbolic computation was the HP-28, released in 1987. It was able to, for example, solve quadratic equations symbolically. The first graphing calculator was the Casio fx7000G released in 1985.

The two leading manufacturers, HP and TI, released increasingly feature-laden calculators during the 1980s and 1990s. At the turn of the millennium, the line between a graphing calculator and a handheld computer was not always clear, as some very advanced calculators such as the TI-89, the Voyage 200 and HP-49G could differentiate and integrate functions, solve differential equations, run word processing and PIM software, and connect by wire or IR to other calculators/computers.

The HP 12c financial calculator is still produced. It was introduced in 1981 and is still being made with few changes. The HP 12c featured the reverse Polish notation mode of data entry. In 2003 several new models were released, including an improved version of the HP 12c, the "HP 12c platinum edition" which added more memory, more built-in functions, and the addition of the algebraic mode of data entry.

The CASIO CM-602 Mini Electronic Calculator provided basic functions in the 1970s

Online calculators are programs designed to work just like a normal calculator does. Usually the keyboard (or the mouse clicking a virtual numpad) is used, but other means of input (e.g. slide bars) are possible.

Thanks to the Internet, many new types of calculators are possible for calculations that would otherwise be much more difficult or impossible, such as for real time currency exchange rates, loan rates and statistics.

Patents

- – *Complex computer* – G. R. Stibitz, Bell Laboratories, 1954 (filed 1941, refiled 1944), electromechanical (relay) device that could calculate complex numbers, record, and print results by teletype
- – *Miniature electronic calculator* – J. S. Kilby, Texas Instruments, 1974 (originally filed 1967), handheld (3 lb, 1.4 kg) battery operated electronic device with thermal printer
    - The Japanese Patent Office granted a patent in June 1978 to Texas Instruments (TI) based on US patent 3819921, notwithstanding objections from 12 Japanese calculator manufacturers. This gave TI the right to claim royalties retroactively to the original publication of the Japanese patent application in August 1974. A TI spokesman said that it would actively seek what was due, either in cash or technology cross-licensing agreements. Nineteen other countries, including the United Kingdom, had already granted a similar patent to Texas Instruments. – *New Scientist*, 17 Aug. 1978 p455, and *Practical Electronics* (British publication), October 1978 p1094.
- – *Floating Point Calculator With RAM Shift Register* - 1977 (originally filed GB Mar 1971, US Jul 1971), very early single chip calculator claim.
- – *Extended Numerical Keyboard with Structured Data-Entry Capability* – J. H. Redin, 1997 (originally filed 1996), Usage of Verbal Numerals as a way to enter a number.

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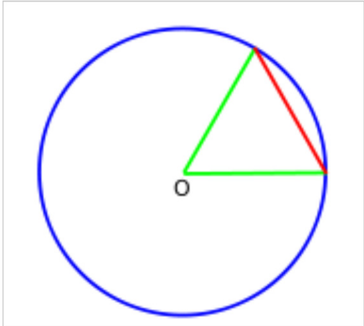
Degree (angle)

2008/9 Schools Wikipedia Selection. Related subjects: Mathematics

A **degree** (in full, a **degree of arc**, **arc degree**, or **arcdegree**), usually denoted by ° (the degree symbol), is a measurement of plane angle, representing $^1\!/_{360}$ of a full rotation; one degree is equivalent to $\pi/180$ radians. When that angle is with respect to a reference meridian, it indicates a location along a great circle of a sphere, such as Earth (see Geographic coordinate system), Mars, or the celestial sphere.

History



A circle with an equilateral chord (red). One sixtieth of this arc is a degree. Six such chords complete the circle

Selecting 360 as the number of degrees (*i.e.,* smallest practical sub-arcs) in a circle was probably based on the fact that 360 is approximately the number of days in a year. Its use is often said to originate from the methods of the ancient Babylonians. Ancient astronomers noticed that the stars in the sky, which circle the celestial pole every day, seem to advance in that circle by approximately one-360th of a circle, *i.e.,* one degree, each day. (Primitive calendars, such as the Persian Calendar, used 360 days for a year.) Its application to measuring angles in geometry can possibly be traced to Thales who popularized geometry among the Greeks and lived in Anatolia (modern western Turkey) among people who had dealings with Egypt and Babylon.

The earliest trigonometry, used by the Babylonian astronomers and their Greek successors, was based on chords of a circle. A chord of length equal to the radius made a natural base quantity. One sixtieth of this, using their standard sexagesimal divisions, was a degree; while six such chords completed the full circle.

Another motivation for choosing the number 360 is that it is readily divisible: 360 has 24 divisors (including 1 and 360), including every number from 1 to 10 except 7. For the number of degrees in a circle to be divisible by every number from 1 to 10, there would need to be 2520 degrees in a circle, which is a much less convenient number.

> Divisors of 360 are 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, and 360.

India

The division of the circle into 360 parts also occurred in ancient India, as evidenced in the Rig Veda:

> Twelve spokes, one wheel, navels three.
> Who can comprehend this?
> On it are placed together

three hundred and sixty like pegs.
They shake not in the least.
(Dirghatama, Rig Veda 1.164.48)

Subdivisions

For many practical purposes, a degree is a small enough angle that whole degrees provide sufficient precision. When this is not the case, as in astronomy or for latitudes and longitudes on the Earth, degree measurements may be written with decimal places, but the traditional sexagesimal unit subdivision is commonly seen. One degree is divided into 60 *minutes (of arc)*, and one minute into 60 *seconds (of arc)*. These units, also called the *arcminute* and *arcsecond*, are respectively represented as a single and double prime, or if necessary by a single and double quotation mark: for example, 40.1875° = 40° 11′ 15″ (or 40° 11' 15").

If still more accuracy is required, decimal divisions of the second are normally used, rather than *thirds* of $^1/_{60}$ second, *fourths* of $^1/_{60}$ of a third, and so on. These (rarely used) subdivisions were noted by writing the Roman numeral for the number of sixtieths in superscript: 1^{I} for a "prime" (minute of arc), 1^{II} for a second, 1^{III} for a third, 1^{IV} for a fourth, etc. Hence the modern symbols for the minute and second of arc.

Alternative units

In most mathematical work beyond practical geometry, angles are typically measured in radians rather than degrees. This is for a variety of reasons; for example, the trigonometric functions have simpler and more "natural" properties when their arguments are expressed in radians. These considerations outweigh the convenient divisibility of the number 360. One complete circle (360°) is equal to 2π radians, so 180° is equal to π radians, or equivalently, the degree is a mathematical constant $° = ^{\pi}/_{180}$.

With the invention of the metric system, based on powers of ten, there was an attempt to define a "decimal degree" (**grad** or **gon**), so that the number of decimal degrees in a right angle would be 100 *gon*, and there would be 400 *gon* in a circle. Although this idea did not gain much momentum, most scientific calculators used to support it.

An angular mil which is most used in military applications has at least three specific variants.

In computer games which depict a three-dimensional virtual world, the need for very fast computations resulted in the adoption of a binary, 256 degree system. In this system, a right angle is 64 degrees, angles can be represented in a single byte, and all trigonometric functions are implemented as small lookup tables. These units are sometimes called "binary radians" ("brads") or "binary degrees".

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Elementary algebra

2008/9 Schools Wikipedia Selection. Related subjects: Mathematics

Elementary algebra is a fundamental and relatively basic form of algebra taught to students who are presumed to have little or no formal knowledge of mathematics beyond arithmetic. While in arithmetic only numbers and their arithmetical operations (such as +, −, ×, ÷) occur, in algebra one also uses symbols (such as x and y, or a and b) to denote numbers. These are called variables. This is useful because:

- It allows the generalization of arithmetical equations (and inequalities) to be stated as laws (such as $a + b = b + a$ for all a and b), and thus is the first step to the systematic study of the properties of the real number system.
- It allows reference to numbers which are not known. In the context of a problem, a variable may represent a certain value of which is uncertain, but may be solved through the formulation and manipulation of equations.
- It allows the exploration of mathematical relationships between quantities (such as "if you sell x tickets, then your profit will be $3x - 10$ dollars").

These three are the main strands of elementary algebra, which should be distinguished from abstract algebra, a more advanced topic generally taught to college students.

In elementary algebra, an " expression" may contain numbers, variables and arithmetical operations. These are usually written (by convention) with 'higher-power' terms on the left (see polynomial); a few examples are:

$$x + 3$$

$$y^2 + 2x - 3$$

$$z^7 + a(b + x^3) + 42/y - \pi.$$

In more advanced algebra, an expression may also include elementary functions.

An "equation" is the claim that two expressions are equal. Some equations are true for all values of the involved variables (such as $a + b = b + a$); such equations are called " identities". "Conditional" equations are true for only some values of the involved variables: $x^2 - 1 = 4$. The values of the variables which make the equation true are called the "solutions" of the equation.

Laws of elementary algebra

- Commutative property of addition

  $$a + b = b + a.$$

  - Subtraction is the reverse of addition.
  - To subtract is the same as to add a negative number:

    $$a - b = a + (-b).$$

    Example: if $5 + x = 3$ then $x = -2$.

- Commutative property of multiplication

  $$a \times b = b \times a$$

  - Division is the reverse of multiplication.
  - To divide is the same as to multiply by a reciprocal:

    $$\frac{a}{b} = a\left(\frac{1}{b}\right).$$

- Exponentiation is not a commutative operation.
  - Therefore exponentiation has a pair of reverse operations: logarithm and exponentiation with fractional exponents (e.g. square roots).
    - Examples: if $3^x = 10$ then $x = \log_3 10$. If $x^2 = 10$ then $x = 10^{1/2}$.
  - The square roots of negative numbers do not exist in the real number system. (See: complex number system)
- Associative property of addition: $(a + b) + c = a + (b + c)$.
- Associative property of multiplication: $(ab)c = a(bc)$.
- Distributive property of multiplication with respect to addition: $c(a + b) = ca + cb$.
- Distributive property of exponentiation with respect to multiplication: $(ab)^c = a^c b^c$.
- How to combine exponents: $a^b a^c = a^{b+c}$.
- Power to a power property of exponents: $(a^b)^c = a^{bc}$.

Laws of equality

- If $a = b$ and $b = c$, then $a = c$ (transitivity of equality).
- $a = a$ (reflexivity of equality).
- If $a = b$ then $b = a$ (symmetry of equality).

Other laws

- If $a = b$ and $c = d$ then $a + c = b + d$.

  - If $a = b$ then $a + c = b + c$ for any c (addition property of equality).
- If $a = b$ and $c = d$ then $ac = bd$.
  - If $a = b$ then $ac = bc$ for any c (multiplication property of equality).
- If two symbols are equal, then one can be substituted for the other at will (substitution principle).
- If $a > b$ and $b > c$ then $a > c$ (transitivity of inequality).
- If $a > b$ then $a + c > b + c$ for any c.
- If $a > b$ and $c > 0$ then $ac > bc$.
- If $a > b$ and $c < 0$ then $ac < bc$.

Examples

Linear equations in one variable

The simplest equations to solve are linear equations that have only one variable. They contain only constant numbers and a single variable without an exponent. For example:

$$2x + 4 = 12.$$

The central technique is add, subtract, multiply, or divide both sides of the equation by the same number in order to isolate the variable on one side of the equation. Once the variable is isolated, the other side of the equation is the value of the variable. For example, by subtracting 4 from both sides in the equation above:

$$2x + 4 - 4 = 12 - 4$$

which simplifies to:

$$2x = 8.$$

Dividing both sides by 2:

$$\frac{2x}{2} = \frac{8}{2}$$

simplifies to the solution:

$$x = 4.$$

The general case,

$$ax + b = c$$

follows the same format for the solution:

$$x = \frac{c - b}{a}$$

Quadratic equations

Quadratic equations can be expressed in the form $ax^2 + bx + c = 0$, where a is not zero (if it were zero, then the equation would not be quadratic but linear). Because of this a quadratic equation must contain the term ax^2, which is known as the quadratic term. Hence $a \neq 0$, and so we may divide by a and rearrange the equation into the standard form

$$x^2 + px = q$$

where $p = b/a$ and $q = -c/a$. Solving this, by a process known as completing the square, leads to the quadratic formula.

Quadratic equations can also be solved using factorization (the reverse process of which is expansion, but for two linear terms is sometimes denoted foiling). As an example of factoring:

$$x^2 + 3x - 10 = 0.$$

Which is the same thing as

$$(x + 5)(x - 2) = 0.$$

It follows from the zero-product property that either $x = 2$ or $x = -5$ are the solutions, since precisely one of the factors must be equal to zero. All quadratic equations will have two solutions in the complex number system, but need not have any in the real number system. For example,

$$x^2 + 1 = 0$$

has no real number solution since no real number squared equals -1. Sometimes a quadratic equation has a root of multiplicity 2, such as:

$$(x + 1)^2 = 0.$$

For this equation, −1 is a root of multiplicity 2.

System of linear equations

In the case of a system of linear equations, like, for instance, two equations in two variables, it is often possible to find the solutions of both variables that satisfy both equations.

First method of finding a solution

An example of a system of linear equations could be the following:

$$\begin{cases} 4x + 2y = 14 \\ 2x - y = 1. \end{cases}$$

Multiplying the terms in the second equation by 2:

$$4x + 2y = 14$$
$$4x - 2y = 2.$$

Adding the two equations together to get:

$$8x = 16$$

which simplifies to

$$x = 2.$$

Since the fact that $x = 2$ is known, it is then possible to deduce that $y = 3$ by either of the original two equations (by using *2* instead of *x*) The full solution to this problem is then

$$\begin{cases} x = 2 \\ y = 3. \end{cases}$$

Note that this is not the only way to solve this specific system; *y* could have been solved before *x*.

Second method of finding a solution

Another way of solving the same system of linear equations is by substitution.

$$\begin{cases}4x+2y=14\\2x-y=1.\end{cases}$$

An equivalent for *y* can be deduced by using one of the two equations. Using the second equation:

$$2x-y=1$$

Subtracting *2x* from each side of the equation:

$$2x-2x-y=1-2x$$
$$-y=1-2x$$

and multiplying by -1:

$$y=2x-1.$$

Using this *y* value in the first equation in the original system:

$$4x+2(2x-1)=14$$
$$4x+4x-2=14$$
$$8x-2=14$$

Adding *2* on each side of the equation:

$$8x-2+2=14+2$$
$$8x=16$$

which simplifies to

$$x=2$$

Using this value in one of the equations, the same solution as in the previous method is obtained.

$$\begin{cases} x = 2 \\ y = 3. \end{cases}$$

Note that this is not the only way to solve this specific system; in this case as well, *y* could have been solved before *x*.

Other types of Systems of Linear Equations

Unsolvable Systems

In the above example, it is possible to find a solution. However, there are also systems of equations which do not have a solution. An obvious example would be:

$$\begin{cases} x + y = 1 \\ 0x + 0y = 2 \end{cases}$$

The second equation in the system has no possible solution. Therefore, this system can't be solved. However, not all incompatible systems are recognized at first sight. As an example, the following system is studied:

$$\begin{cases} 4x + 2y = 12 \\ -2x - y = -4 \end{cases}$$

When trying to solve this (for example, by using the method of substitution above), the second equation, after adding $-2x$ on both sides and multiplying by -1, results in:

$$y = -2x + 4$$

And using this value for *y* in the first equation:

$$\begin{aligned} &4x + 2(-2x + 4) = 12 \\ &4x - 4x + 8 = 12 \\ &8 = 12 \end{aligned}$$

No variables are left, and the equality is not true. This means that the first equation can't provide a solution for the value for *y* obtained in the second equation.

Undetermined Systems

There are also systems which have multiple or infinite solutions, in opposition to a system with a unique solution (meaning, two unique values for *x* and *y*) For example:

$$\begin{cases} 4x + 2y = 12 \\ -2x - y = -6 \end{cases}$$

Isolating *y* in the second equation:

$$y = -2x + 6$$

And using this value in the first equation in the system:

$$\begin{array}{l} 4x + 2(-2x + 6) = 12 \\ 4x - 4x + 12 = 12 \\ 12 = 12 \end{array}$$

The equality is true, but it does not provide a value for *x*. Indeed, one can easily verify (by just filling in some values of *x*) that for any *x* there is a solution as long as $y = -2x + 6$. There are infinite solutions for this system.

Over and underdetermined Systems

Systems with more variables than the number of linear equations do not have a unique solution. An example of such a system is

$$\begin{cases} x + 2y = 10 \\ y - z = 2 \end{cases}$$

Such a system is called underdetermined; when trying to find a solution, one or more variables can only be expressed in relation to the other variables, but cannot be determined numerically. Incidentally, a system with a greater number of equations than variables, in which necessarily some equations are sums or multiples of others, is called overdetermined.

Relation between Solvability and Multiplicity

Given any system of linear equations, there is a relation between multiplicity and solvability.
If one equation is a multiple of the other (or, more generally, a sum of multiples of the other equations), then the system of linear equations is undetermined, meaning that the system has infinitely many solutions. Example:

$$\begin{cases} x + y = 2 \\ 2x + 2y = 4 \end{cases}$$

When the multiplicity is only partial (meaning that for example, only the left hand sides of the equations are multiples, while the right hand sides are not or not by the *same* number) then the system is unsolvable. For example, in

$$\begin{cases} x + y = 2 \\ 4x + 4y = 1 \end{cases}$$

the second equation yields that $x + y = 1/4$ which is in contradiction with the first equation. Such a system is also called *inconsistent* in the language of linear algebra. When trying to solve a system of linear equations it is generally a good idea to check if one equation is a multiple of the other. If this is precisely so, the solution cannot be uniquely determined. If this is only partially so, the solution does not exist.
This, however, does not mean that the equations must be multiples of each other to have a solution, as shown in the sections above; in other words: multiplicity in a system of linear equations is **not** a necessary condition for solvability.

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See also

Elementary arithmetic

2008/9 Schools Wikipedia Selection. Related subjects: Mathematics

Elementary arithmetic is the most basic kind of mathematics: it concerns the operations of addition, subtraction, multiplication, and division. Most people learn elementary arithmetic in elementary school.

Elementary arithmetic starts with the natural numbers and the Arabic numerals used to represent them. It requires the memorization of addition tables and multiplication tables for adding and multiplying pairs of digits. Knowing these tables, a person can perform certain well-known procedures for adding and multiplying natural numbers. Other algorithms are used for subtraction and division. **Mental arithmetic** is elementary arithmetic performed in the head, for example to know that 100 − 37 = 63 without use of paper. It is an everyday skill. Extended forms of mental calculation may involve calculating extremely large numbers, but this is a skill not usually taught at the elementary level.

Elementary arithmetic then moves on to fractions, decimals, and negative numbers, which can be represented on a number line.

Nowadays people routinely use electronic calculators, cash registers, and computers to perform their elementary arithmetic for them. Earlier calculating tools included slide rules (for multiplication, division, logs and trig), tables of logarithms, nomographs, and mechanical calculators.

The question of whether or not calculators should be used, and whether traditional mathematics manual computation methods should still be taught in elementary school has provoked heated controversy as many standards-based mathematics texts deliberately omit some or most standard computation methods. The 1989 NCTM standards led to curricula which de-emphasized or omitted much of what was considered to be elementary arithmetic in elementary school, and replaced it with emphasis on topics traditionally studied in college such as algebra, statistics and problem solving, and non-standard computation methods unfamiliar to most adults.

In ancient times, the abacus was used to perform elementary arithmetic, and still is in many parts of Asia. A skilled user can be as fast with an abacus as with a calculator, which may require batteries.

In the 14th century Arabic numerals were introduced to Europe by Leonardo Pisano. These numerals were more efficient for performing calculations than Roman numerals, because of the positional system.

The digits

0 , zero, represents absence of objects to be counted.
1 , one. This is one stick: I

2 , two. This is two sticks: I I
3 , three. This is three sticks: I I I
4 , four. This is four sticks: I I I I
5 , five. This is five sticks: I I I I I
6 , six. This is six sticks: I I I I I I
7 , seven. This is seven sticks: I I I I I I I
8 , eight. This is eight sticks: I I I I I I I I
9 , nine. This is nine sticks: I I I I I I I I I
There are as many digits as fingers on the hands: the word "digit" can also mean finger. But if counting the digits on both hands, the first digit would be one and the last digit would not be counted as "zero" but as " ten": 10 , made up of the digits one and zero. The number 10 is the first two-digit number. This is ten sticks: I I I I I I I I I I

If a number has more than one digit, then the rightmost digit, said to be the last digit, is called the "ones-digit". The digit immediately to its left is the "tens-digit". The digit immediately to the left of the tens-digit is the "hundreds-digit". The digit immediately to the left of the hundreds-digit is the "thousands-digit".

Addition

+	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

What does it mean to add two natural numbers? Suppose you have two bags, one bag holding five apples and a second bag holding three apples. Grabbing a third, empty bag, move all the apples from the first and second bags into the third bag. The third bag now holds eight apples. This illustrates the combination of three apples and five apples is eight apples; or more generally: "three plus five is eight" or "three plus five equals eight" or "eight is the sum of three and five". Numbers are abstract, and the addition of a group of three things to a group of five things will yield a group of eight things. Addition is a regrouping: two sets of objects which were counted separately are put into a single group and counted together: the count of the new group is the "sum" of the separate counts of the two original groups.

Symbolically, addition is represented by the " plus sign": +. So the statement "three plus five equals eight" can be written symbolically as $3 + 5 = 8$. The order in which two numbers are added does not matter, so $3 + 5 = 5 + 3 = 8$. This is the commutative property of addition.

To add a pair of digits using the table, find the intersection of the row of the first digit with the column of the second digit: the row and the column intersect at a square containing the sum of the two digits. Some pairs of digits add up to two-digit numbers, with the tens-digit always being a 1. In the addition algorithm the tens-digit of the sum of a pair of digits is called the " carry digit".

Addition algorithm

For simplicity, consider only numbers with three digits or less. To add a pair of numbers (written in Arabic numerals), write the second number under the first one, so that digits line up in columns: the rightmost column will contain the ones-digit of the second number under the ones-digit of the first number. This rightmost column is the ones-column. The column immediately to its left is the tens-column. The tens-column will have the tens-digit of the second number (if it has one) under the tens-digit of the first number (if it has one). The column immediately to the left of the tens-column is the hundreds-column. The hundreds-column will line up the hundreds-digit of the second number (if there is one) under the hundreds-digit of the first number (if there is one).

After the second number has been written down under the first one so that digits line up in their correct columns, draw a line under the second (bottom) number. Start with the ones-column: the ones-column should contain a pair of digits: the ones-digit of the first number and, under it, the ones-digit of the second number. Find the sum of these two digits: write this sum under the line and in the ones-column. If the sum has two digits, then write down only the ones-digit of the sum. Write the "carry digit" above the top digit of the next column: in this case the next column is the tens-column, so write a 1 above the tens-digit of the first number.

If both first and second number each have only one digit then their sum is given in the addition table, and the addition algorithm is unnecessary.

Then comes the tens-column. The tens-column might contain two digits: the tens-digit of the first number and the tens-digit of the second number. If one of the numbers has a missing tens-digit then the tens-digit for this number can be considered to be a zero. Add the tens-digits of the two numbers. Then, if there is a carry digit, add it to this sum. If the sum was 18 then adding the carry digit to it will yield 19. If the sum of the tens-digits (plus carry digit, if there is one) is less than ten then write it in the tens-column under the line. If the sum has two digits then write its last digit in the tens-column under the line, and carry its first digit (which should be a one) over to the next column: in this case the hundreds column.

If none of the two numbers has a hundreds-digit then if there is no carry digit then the addition algorithm has finished. If there is a carry digit (carried over from the tens-column) then write it in the hundreds-column under the line, and the algorithm is finished. When the algorithm finishes, the number under the line is the sum of the two numbers.

If at least one of the numbers has a hundreds-digit then if one of the numbers has a missing hundreds-digit then write a zero digit in its place. Add the two hundreds-digits, and to their sum add the carry digit if there is one. Then write the sum of the hundreds-column under the line, also in the hundreds column. If the sum has two digits then write down the last digit of the sum in the hundreds-column and write the carry digit to its left: on the thousands-column.

Example

Say one wants to find the sum of the numbers 653 and 274. Write the second number under the first one, with digits aligned in columns, like so:

```
6 5 3
2 7 4
```

Then draw a line under the second number and start with the ones-column. The ones-digit of the first number is 3 and of the second number is 4. The sum of three and four is seven, so write a seven in the ones-column under the line:

```
6 5 3
2 7 4
-----
    7
```

Next, the tens-column. The tens-digit of the first number is 5, and the tens-digit of the second number is 7, and five plus seven is twelve: 12, which has two digits, so write its last digit, 2, in the tens-column under the line, and write the carry digit on the hundreds-column above the first number:

```
1
6 5 3
2 7 4
-----
  2 7
```

Next, the hundreds-column. The hundreds-digit of the first number is 6, while the hundreds-digit of the second number is 2. The sum of six and two is eight, but there is a carry digit, which added to eight is equal to nine. Write the nine under the line in the hundreds-column:

```
1
6 5 3
2 7 4
-----
9 2 7
```

No digits (and no columns) have been left unadded, so the algorithm finishes, and

653 + 274 = 927.

Successorship and size

The result of the addition of one to a number is the *successor* of that number. Examples:
the successor of zero is one,
the successor of one is two,
the successor of two is three,
the successor of ten is eleven.
Every natural number has a successor.

The predecessor of the successor of a number is the number itself. For example, five is the successor of four therefore four is the predecessor of five. Every natural number except zero has a predecessor.

If a number is the successor of another number, then the first number is said to be *larger than* the other number. If a number is larger than another number, and if the other number is larger than a third number, then the first number is also larger than the third number. Example: five is larger than four, and four is larger than three, therefore five is larger than three. But six is larger than five, therefore six is also larger than three. But seven is larger than six, therefore seven is also larger than three... therefore eight is larger than three... therefore nine is larger than three, etc.

If two non-zero natural numbers are added together, then their sum is larger than either one of them. Example: three plus five equals eight, therefore eight is larger than three (8>3) and eight is larger than five (8>5). The symbol for "larger than" is >.

If a number is larger than another one, then the other is *smaller than* the first one. Examples: three is smaller than eight (3<8) and five is smaller than eight (5<8). The symbol for smaller than is <. A number cannot be at the same time larger and smaller than another number. Neither can a number be at the same time larger than and equal to another number. Given a pair of natural numbers, one and only one of the following cases must be true:

- the first number is larger than the second one,
- the first number is equal to the second one,
- the first number is smaller than the second one.

Counting

To count a group of objects means to assign a natural number to each one of the objects, as if it were a label for that object, such that a natural number is never assigned to an object unless its predecessor was already assigned to another object, with the exception that zero is not assigned to any object: the smallest natural number to be assigned is one, and the largest natural number assigned depends on the size of the group. It is called *the count* and it is equal to the number of objects in that group.

The process of counting a group is the following:
Step 1: Let "the count" be equal to zero. "The count" is a variable quantity, which though beginning with a value of zero, will soon have its value changed several times.
Step 2: Find at least one object in the group which has not been labeled with a natural number. If no such object can be found (if they have all been labeled) then the counting is finished. Otherwise choose one of the unlabeled objects.
Step 3: Increase the count by one. That is, replace the value of the count by its successor.
Step 4: Assign the new value of the count, as a label, to the unlabeled object chosen in Step 2.
Step 5: Go back to Step 2.

When the counting is finished, the last value of the count will be the final count. This count is equal to the number of objects in the group.

Often, when counting objects, one does not keep track of what numerical label corresponds to which object: one only keeps track of the subgroup of objects which have already been labeled, so as to be able to identify unlabeled objects necessary for Step 2. However, if one is counting persons, then one can ask the persons who are being counted to each keep track of the number which the person's self has been assigned. After the count has finished it is possible to ask the group of persons to file up in a line, in order of increasing numerical label. What the persons would do during the process of lining up would be something like this: each pair of persons who are unsure of their positions in the line ask each other what their numbers are: the person whose number is smaller should stand on the left side and the one with the larger number on the right side of the other person. Thus, pairs of persons compare their numbers and their positions, and commute their positions as necessary, and through repetition of such conditional commutations they become ordered.

Algorithms for Subtraction

There are several methods to accomplish subtraction. Traditional mathematics taught elementary school children to subtract using methods suitable for hand calculation. The particular method used varies from country from country, and within a country, different methods are in fashion at different times. Standards-based mathematics are distinguished generally by the lack of preference for any standard method, replaced by guiding 2nd grade children to invent their own methods of computation, such as using properties of negative numbers in the case of TERC.

American schools currently teach a method of subtraction using borrowing and a system of markings called crutches. Although a method of borrowing had been known and published in textbooks prior, apparently the crutches are the invention of William A. Browell who used them in a study in November of 1937 . This system caught on rapidly, displacing the other methods of subtraction in use in America at that time.

European children are taught, and some older Americans employ, a method of subtraction called the Austrian method, also known as the additions method. There is no borrowing in this method. There are also crutches (markings to aid the memory) which [probably] vary according to country.

In the method of borrowing, a subtraction such as 86 - 39 will accomplish the one's place subtraction of 9 from 6 by borrowing a 10 from 80 and adding it to the 6. The problem is thus transformed into (70+16)-39, effectively. This is indicated by striking through the 8, writing a small 7 above it, and writing a small 1 above the 6. These markings are called *crutches*. The 9 is then subtracted from 16, leaving 7, and the 30 from the 70, leaving 40, or 47 as the result.

In the additions method, a 10 is borrowed to make the 6 into 16, in preparation for the subtraction of 9, just as in the borrowing method. However, the 10 is not taken by reducing minuend, rather one augments the subtrahend. Effectively, the problem is transformed into (80+16)-(39+10). Typically a crutch of a small one is marked just below the subtrahend digit as a reminder. Then the operations proceed: 9 from 16 is 7; and 40 (that is, 30+10) from 80 is 40, or 47 as the result.

The additions method seem to be taught in two variations, which differ only in psychology. Continuing the example of 86-39, the first variation attempts to subtract 9 from 6, and then 9 from 16, borrowing a 10 by marking near the digit of the subtrahend in the next column. The second variation attempts to find a digit which, when added to 9 gives 6, and recognizing that is not possible, gives 16, and carrying the 10 of the 16 as a one marking near the same digit as in the first method. The markings are the same, it is just a matter of preference as to how one explains its appearance.

As a final caution, the borrowing method gets a bit complicated in cases such as 100-87, where a borrow cannot be made immediately, and must be obtained by reaching across several columns. In this case, the minuend is effectively rewriten as 90+10, by taking a one hundred from the hundreds, making ten tens from it, and immediately borrowing that down to 9 tens in the tens column and finally placing a ten in the one's column.

There are several other methods, some of which are particularly advantageous to machine calculation. For example, digital computers employ the method of two's complement. Of great importance is the counting up method by which change is made. Suppose an amount P is given to pay the required amount Q, with P greater than Q. Rather than performing the subtraction P-Q and counting out that amount in change, money is counted out starting at Q and continuing until reaching P. Curiously, although the amount counted out must equal the result of the subtraction P-Q, the subtraction was never really done and the value of P-Q might still be unknown to the change-maker.

1 Subtraction in the United States: An Historial Perspective, Susan Ross, Mary Pratt-Cotter, The Mathematics Educator, Vol. 8, No. 1.

Browell, W. A. (1939). Learning as reorganization: An experimental study in third-grade arithmetic, Duke University Press.

See also:

- Method of complements
- Subtraction without borrowing

Multiplication

×	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9

2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

When two numbers are multiplied together, the result is called a *product*. The two numbers being multiplied together are called *factors*.

What does it mean to multiply two natural numbers? Suppose there are five red bags, each one containing three apples. Now grabbing an empty green bag, move all the apples from all five red bags into the green bag. Now the green bag will have fifteen apples. Thus the product of five and three is fifteen. This can also be stated as "five times three is fifteen" or "five times three equals fifteen" or "fifteen is the product of five and three". Multiplication can be seen to be a form of repeated addition: the first factor indicates how many times the second factor should be added onto itself; the final sum being the product.

Symbolically, multiplication is represented by the *multiplication sign*: $\times$. So the statement "five times three equals fifteen" can be written symbolically as

$$5 \times 3 = 15.$$

In some countries, and in more advanced arithmetic, other multiplication signs are used, e.g. $5 \cdot 3$. In some situations, especially in algebra, where numbers can be symbolized with letters, the multiplication symbol may be omitted; e.g *xy* means $x \times y$. The order in which two numbers are multiplied does not matter, so that, for example, three times four equals four times three. This is the commutative property of multiplication.

To multiply a pair of digits using the table, find the intersection of the row of the first digit with the column of the second digit: the row and the column intersect at a square containing the product of the two digits. Most pairs of digits produce two-digit numbers. In the multiplication algorithm the tens-digit of the product of a pair of digits is called the " carry digit".

Multiplication algorithm for a single-digit factor

Consider a multiplication where one of the factors has only one digit, whereas the other factor has an arbitrary quantity of digits. Write down the multi-digit factor, then write the single-digit factor under the last digit of the multi-digit factor. Draw a horizontal line under the single-digit factor. Henceforth, the single-digit factor will be called the "multiplier" and the multi-digit factor will be called the "multiplicand".

Suppose for simplicity that the multiplicand has three digits. The first digit is the hundreds-digit, the middle digit is the tens-digit, and the last, rightmost, digit is the ones-digit. The multiplier only has a ones-digit. The ones-digits of the multiplicand and multiplier form a column: the ones-column.

Start with the ones-column: the ones-column should contain a pair of digits: the ones-digit of the multiplicand and, under it, the ones-digit of the multiplier. Find the product of these two digits: write this product under the line and in the ones-column. If the product has two digits, then write down only the ones-digit of the product. Write the "carry digit" as a superscript of the yet-unwritten digit in the next column and under the line: in this case the next column is the tens-column, so write the carry digit as the superscript of the yet-unwritten tens-digit of the product (under the line).

If both first and second number each have only one digit then their product is given in the multiplication table, and the multiplication algorithm is unnecessary.

Then comes the tens-column. The tens-column so far contains only one digit: the tens-digit of the multiplicand (though it might contain a carry digit under the line). Find the product of the multiplier and the tens-digits of the multiplicand. Then, if there is a carry digit (superscripted, under the line and in the tens-column), add it to this product. If the resulting sum is less than ten then write it in the tens-column under the line. If the sum has two digits then write its last digit in the tens-column under the line, and carry its first digit over to the next column: in this case the hundreds column.

If the multiplicand does not have a hundreds-digit then if there is no carry digit then the multiplication algorithm has finished. If there is a carry digit (carried over from the tens-column) then write it in the hundreds-column under the line, and the algorithm is finished. When the algorithm finishes, the number under the line is the product of the two numbers.

If the multiplicand has a hundreds-digit... find the product of the multiplier and the hundreds-digit of the multiplicand, and to this product add the carry digit if there is one. Then write the resulting sum of the hundreds-column under the line, also in the hundreds column. If the sum has two digits then write down the last digit of the sum in the hundreds-column and write the carry digit to its left: on the thousands-column.

Example

Say one wants to find the product of the numbers 3 and 729. Write the single-digit multiplier under the multi-digit multiplicand, with the multiplier under the ones-digit of the multiplicand, like so:

```
7 2 9
    3
```

Then draw a line under the multiplier and start with the ones-column. The ones-digit of the multiplicand is 9 and the multiplier is 3. The product of three and nine is 27, so write a seven in the ones-column under the line, and write the carry-digit 2 as a superscript of the yet-unwritten tens-digit of the product under the line:

```
7 2 9
```

_ _ 3

^{2} 7

Next, the tens-column. The tens-digit of the multiplicand is 2, the multiplier is 3, and three times two is six. Add the carry-digit, 2, to the product 6 to obtain 8. Eight has only one digit: no carry-digit, so write in the tens-column under the line:

7 2 9

_ _ 3

8^{2} 7

Next, the hundreds-column. The hundreds-digit of the multiplicand is 7, while the multiplier is 3. The product of three and seven is 21, and there is no previous carry-digit (carried over from the tens-column). The product 21 has two digits: write its last digit in the hundreds-column under the line, then carry its first digit over to the thousands-column. Since the multiplicand has no thousands-digit, then write this carry-digit in the thousands-column under the line (not superscripted):

7 2 9

_ _ _ 3

2 1 8^{2} 7

No digits of the multiplicand have been left unmultiplied, so the algorithm finishes, and

$$3 \times 729 = 2187 .$$

Multiplication algorithm for multi-digit factors

Given a pair of factors, each one having two or more digits, write both factors down, one under the other one, so that digits line up in columns.

For simplicity consider a pair of three-digits numbers. Write the last digit of the second number under the last digit of the first number, forming the ones-column. Immediately to the left of the ones-column will be the tens-column: the top of this column will have the second digit of the first number, and below it will be the second digit of the second number. Immediately to the left of the tens-column will be the hundreds-column: the top of this column will have the first digit of the first number and below it will be the first digit of the second number. After having written down both factors, draw a line under the second factor.

The multiplication will consist of two parts. The first part will consist of several multiplications involving one-digit multipliers. The operation of each one of such multiplications was already described in the previous multiplication algorithm, so this algorithm will not describe each one individually, but will only describe how the several multiplications with one-digit multipliers shall be coördinated. The second part will add up all the subproducts of the first part, and the resulting sum will be the product.

First part. Let the first factor be called the multiplicand. Let each digit of the second factor be called a multiplier. Let the ones-digit of the second factor be called the "ones-multiplier". Let the tens-digit of the second factor be called the "tens-multiplier". Let the hundreds-digit of the second factor be called the "hundreds-multiplier".

Start with the ones-column. Find the product of the ones-multiplier and the multiplicand and write it down in a row under the line, aligning the digits of the product in the previously-defined columns. If the product has four digits, then the first digit will be the beginning of the thousands-column. Let this product be called the "ones-row".

Then the tens-column. Find the product of the tens-multiplier and the multiplicand and write it down in a row — call it the "tens-row" — under the ones-row, *but shifted one column to the left*. That is, the ones-digit of the tens-row will be in the tens-column of the ones-row; the tens-digit of the tens-row will be under the hundreds-digit of the ones-row; the hundreds-digit of the tens-row will be under the thousands-digit of the ones-row. If the tens-row has four digits, then the first digit will be the beginning of the ten-thousands-column.

Next, the hundreds-column. Find the product of the hundreds-multiplier and the multiplicand and write it down in a row — call it the "hundreds-row" — under the tens-row, but shifted one more column to the left. That is, the ones-digit of the hundreds-row will be in the hundreds-column; the tens-digit of the hundreds-row will be in the thousands-column; the hundreds-digit of the hundreds-row will be in the ten-thousands-column. If the hundreds-row has four digits, then the first digit will be the beginning of the hundred-thousands-column.

After having down the ones-row, tens-row, and hundreds-row, draw a horizontal line under the hundreds-row. The multiplications are over.

Second part. Now the multiplication has a pair of lines. The first one under the pair of factors, and the second one under the three rows of subproducts. Under the second line there will be six columns, which from right to left are the following: ones-column, tens-column, hundreds-column, thousands-column, ten-thousands-column, and hundred-thousands-column.

Between the first and second lines, the ones-column will contain only one digit, located in the ones-row: it is the ones-digit of the ones-row. Copy this digit by rewriting it in the ones-column under the second line.

Between the first and second lines, the tens-column will contain a pair of digits located in the ones-row and the tens-row: the tens-digit of the ones-row and the ones-digit of the tens-row. Add these digits up and if the sum has just one digit then write this digit in the tens-column under the second line. If the sum has two digits then the first digit is a carry-digit: write the last digit down in the tens-column under the second line and carry the first digit over to the hundreds-column, writing it as a superscript to the yet-unwritten hundreds-digit under the second line.

Between the first and second lines, the hundreds-column will contain three digits: the hundreds-digit of the ones-row, the tens-digit of the tens-row, and the ones-digit of the hundreds-row. Find the sum of these three digits, then if there is a carry-digit from the tens-column (written in superscript under the second line in the hundreds-column) then add this carry-digit as well. If the resulting sum has one digit then write it down under the second line in the hundreds-column; if it has two digits then write the last digit down under the line in the hundreds-column, and carry over the first digit to the thousands-column, writing it as a superscript to the yet-unwritten thousands-digit under the line.

Between the first and second lines, the thousands-column will contain either two or three digits: the hundreds-digit of the tens-row, the tens-digit of the hundreds-row, and (possibly) the thousands-digit of the ones-row. Find the sum of these digits, then if there is a carry-digit from the hundreds-column (written in superscript under the second line in the thousands-column) then add this carry-digit as well. If the resulting sum has one digit then write it down under the second line in the thousands-column; if it has two digits then write the last digit down under the line in the thousands-column, and carry the first digit over to the ten-thousands-column, writing it as a superscript to the yet-unwritten ten-thousands-digit under the line.

Between the first and second lines, the ten-thousands-column will contain either one or two digits: the hundreds-digit of the hundreds-column and (possibly) the thousands-digit of the tens-column. Find the sum of these digits (if the one in the tens-row is missing think of it as a zero), and if there is a carry-digit from the thousands-column (written in superscript under the second line in the ten-thousands-column) then add this carry-digit as well. If the resulting sum has one digit then write it down under the second line in the ten-thousands-column; if it has two digits then write the last digit down under the line in the ten-thousands-column, and carry the first digit over to the hundred-thousands-column, writing it as a superscript to the yet-unwritten ten-thousands digit under the line. However, if the hundreds-row has no thousands-digit then do not write this carry-digit as a superscript, but in normal size, in the position of the hundred-thousands-digit under the second line, and the multiplication algorithm is over.

If the hundreds-row does have a thousands-digit, then add to it the carry-digit from the previous row (if there is no carry-digit then think of it as a zero) and write the single-digit sum in the hundred-thousands-column under the second line.

The number under the second line is the sought-after product of the pair of factors above the first line.

Example

Let our objective be to find the product of 789 and 345. Write the 345 under the 789 in three columns, and draw a horizontal line under them:

7 8 9
3 4 5

First part. Start with the ones-column. The multiplicand is 789 and the ones-multiplier is 5. Perform the multiplication in a row under the line:

  7 8 9
_ 3 4 5
$3\ 9^4\ 4^4\ 5$

Then the tens-column. The multiplicand is 789 and the tens-multiplier is 4. Perform the multiplication in the tens-row, under the previous subproduct in the ones-row, but shifted one column to the left:

$$\begin{array}{cccccc} & & 7 & 8 & 9 \\ \_ & \_ & \underline{3} & \underline{4} & \underline{5} \\ & 3 & 9^4 & 4^4 & 5 \\ 3 & 1^3 & 5^3 & 6 & \end{array}$$

Next, the hundreds-column. The multiplicand is once again 789, and the hundreds-multiplier is 3. Perform the multiplication in the hundreds-row, under the previous subproduct in the tens-row, but shifted one (more) column to the left. Then draw a horizontal line under the hundreds-row:

$$\begin{array}{cccccc} & & & 7 & 8 & 9 \\ \_ & \_ & \_ & \underline{3} & \underline{4} & \underline{5} \\ & & 3 & 9^4 & 4^4 & 5 \\ & 3 & 1^3 & 5^3 & 6 & \\ \underline{2} & \underline{3^2} & \underline{6^2} & \underline{7} & \_ & \_ \end{array}$$

Second part. Now add the subproducts between the first and second lines, but ignoring any superscripted carry-digits located between the first and second lines.

$$\begin{array}{cccccc} & & & 7 & 8 & 9 \\ \_ & \_ & \_ & \underline{3} & \underline{4} & \underline{5} \\ & & 3 & 9^4 & 4^4 & 5 \\ & 3 & 1^3 & 5^3 & 6 & \\ \underline{2} & \underline{3^2} & \underline{6^2} & \underline{7} & \_ & \_ \\ 2 & 7^1 & 2^2 & 2^1 & 0 & 5 \end{array}$$

The answer is

$$789 \times 345 = 272205.$$

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Fraction (mathematics)

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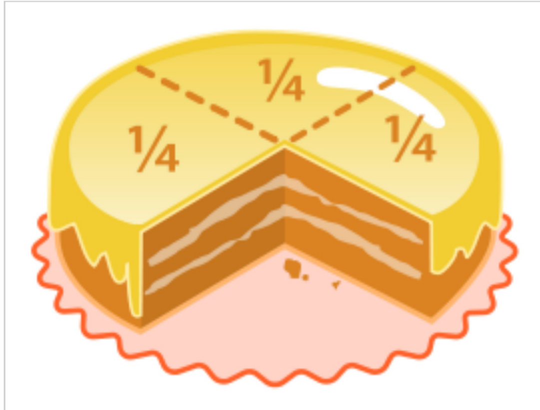


A cake with one quarter removed. The remaining three quarters are shown.

In mathematics, a **fraction** (from the Latin *fractus*, broken) is a concept of a proportional relation between an object part and the object whole. Each fraction consists of a denominator (bottom) and a numerator (top), representing (respectively) the number of equal parts that an object is divided into, and the number of those parts indicated for the particular fraction.

For example, the fraction $^{3}/_{4}$ could be used to represent three equal parts of a whole object, were it divided into four equal parts. Because it is impossible to divide something into zero equal parts, zero can never be the denominator of a fraction (see division by zero). A fraction with equal numerator and denominator is equal to one (e.g. $^{5}/_{5} = 1$) and the fraction form is rarely, if ever, given as a final result.

A fraction is an example of a specific type of ratio, in which the two numbers are related in a part-to-whole relationship, rather than as a comparative relation between two separate quantities. A fraction is a quotient of numbers, the quantity obtained when the numerator is divided by the denominator. Thus $^{3}/_{4}$ represents three divided by four, in decimals 0.75, as a percentage 75%. The three equal parts of the cake are 75% of the whole cake.

In higher mathematics, a fraction is viewed as an element of a field of fractions.

Historically, any number that did not represent a whole was called a "fraction". The numbers that we now call "decimals" were originally called "decimal fractions"; the numbers we now call "fractions" were called "vulgar fractions", the word "vulgar" meaning "commonplace".

The numerator and denominator of a fraction may be separated by a slanting line called a **solidus** or **slash**, for example $^{3}/_{4}$, or may be written above and below a horizontal line called a **vinculum**, thus: $\frac{3}{4}$.

The solidus may be omitted from the slanting style (e.g. $^{3}{}_{4}$) where space is short and the meaning is obvious from context, for example in road signs in some countries.

Fractions are used most often when the denominator is relatively small. It is easier to multiply 32 by $^{3}/_{16}$ than to do the same calculation using the fraction's decimal equivalent (0.1875). It is also more accurate to multiply 15 by $^{1}/_{3}$, for example, than it is to multiply 15 by a decimal approximation of one third. To change a fraction to a decimal, divide the numerator by the denominator, and round off to the desired accuracy.

Fractions are also rational numbers, in which means that the denominator and the numerator are integers.

The word is also used in related expressions, such as *continued fraction* and *algebraic fraction—see Special cases below.*

Forms of fractions

Vulgar, proper, and improper fractions

A **vulgar fraction** (or **common fraction**) is a rational number written as one integer (the *numerator*) divided by a non-zero integer (the *denominator*), for example, $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{4}{3}$.

A vulgar fraction is said to be a **proper fraction** if the absolute value of the numerator is less than the absolute value of the denominator—that is, if the absolute value of the entire fraction is less than 1 (e.g. $\frac{4}{9}$)—but an **improper fraction** (US, British or Australian) or **top-heavy fraction** (British only) if the absolute value of the numerator is greater than or equal to the absolute value of the denominator (e.g. $\frac{9}{7}$).

Mixed numbers

A **mixed number** is the sum of a whole number and a proper fraction. For instance, in referring to two entire cakes and three quarters of another cake, the whole and fractional parts of the number are written next to each other: 2 + $\frac{3}{4}$ =2$\frac{3}{4}$.

An improper fraction can be thought of as another way to write a mixed number; in the "2$\frac{3}{4}$" example above, imagine that the two entire cakes are each divided into quarters. Each entire cake contributes $\frac{4}{4}$ to the total, so $\frac{4}{4}+\frac{4}{4}+\frac{3}{4}=\frac{11}{4}$ is another way of writing 2$\frac{3}{4}$.

A mixed number can be converted to an improper fraction in three steps:

1. Multiply the whole part by the denominator of the fractional part.
2. Add the numerator of the fractional part to that product.
3. The resulting sum is the numerator of the new (improper) fraction, and the new denominator is the same as that of the fractional part of the mixed number.

Similarly, an improper fraction can be converted to a mixed number:

1. Divide the numerator by the denominator.
2. The quotient (without remainder) becomes the whole part and the remainder becomes the numerator of the fractional part.
3. The new denominator is the same as that of the original improper fraction.

Equivalent fractions

Multiplying the numerator and denominator of a fraction by the same (non-zero) number results in a new fraction that is said to be **equivalent** to the original fraction. The word *equivalent* means that the two fractions have the same value. This is true because for any number *n*, multiplying by $\frac{n}{n}$ is really multiplying by one, and any number multiplied by one has the same value as the original number. For instance, consider the fraction $\frac{1}{2}$: when the numerator and denominator are both multiplied by 2, the result is $\frac{2}{4}$, which has the same value (0.5) as $\frac{1}{2}$. To picture this visually, imagine cutting the example cake into four pieces; two of the pieces together ($\frac{2}{4}$) make up half the cake ($\frac{1}{2}$).

For example: $\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{9}$ and $\frac{100}{300}$ are all equivalent fractions.

Dividing the numerator and denominator of a fraction by the same non-zero number will also yield an equivalent fraction. this is called **reducing** or **simplifying** the fraction. A fraction in which the numerator and denominator have no factors in common (other than 1) is said to be **irreducible** or in its **lowest** or **simplest** terms. For instance, $\frac{3}{9}$ is not in lowest terms because both 3 and 9 can be exactly divided by 3. In contrast, $\frac{3}{8}$ *is* in lowest terms—the only number that is a factor of both 3 and 8 is 1.

Reciprocals and the "invisible denominator"

The **reciprocal** of a fraction is another fraction with the numerator and denominator reversed. The reciprocal of $\frac{3}{7}$, for instance, is $\frac{7}{3}$.

Because any number divided by 1 results in the same number, it is possible to write any whole number as a fraction by using 1 as the denominator: 17 = $\frac{17}{1}$ (1 is sometimes referred to as the "invisible denominator"). Therefore, except for zero, every fraction or whole number has a reciprocal. The reciprocal of 17 would be $\frac{1}{17}$.

Complex fractions

A complex fraction (or compound fraction) is a fraction in which the numerator and denominator contain a fraction. For example, $\frac{\frac{1}{2}}{\frac{1}{3}}$ is a complex fraction. To simplify a complex fraction, divide the numerator by the denominator, as with any other fraction: $\frac{\frac{1}{2}}{\frac{1}{3}} = \frac{3}{2}$.

Arithmetic with fractions

Fractions, like whole numbers, obey the commutative, associative, and distributive laws, and the rule against division by zero.

Comparing fractions

Comparing fractions with the same denominator only requires comparing the numerators.

$\frac{3}{4} > \frac{2}{4}$ as 3 > 2.

In order to compare fractions with different denominators, these are converted to a common denominator: to compare $\frac{a}{b}$ and $\frac{c}{d}$, these are converted to $\frac{ad}{bd}$ and $\frac{bc}{bd}$, where bd is the product of the denominators, and then the numerators ad and bc are compared.

$\frac{2}{3}$? $\frac{1}{2}$ gives $\frac{4}{6} > \frac{3}{6}$

This method is also known as the "cross-multiply" method which can be explained by multiplying the top and bottom numbers crosswise. The product of the denominators is used as a common (but not necessary the least common) denominator.

$\frac{5}{18}$? $\frac{4}{17}$

Multiply 17 by 5 and 18 by 4. Place the products of the equations on top of the denominators. The highest number identifies the largest fraction. Therefore $\frac{5}{18} > \frac{4}{17}$ as 17 × 5 = 85 is greater than 18 × 4 = 72.

In order to work with smaller numbers, the least common denominator is used instead of the product. The fractions are converted to fractions with the least common denominator, and then the numerators are compared.

$\frac{5}{6}$? $\frac{3}{4}$ gives $\frac{10}{12} > \frac{8}{12}$

Some standards-based mathematics texts such as Connected Mathematics omit instruction of least common denominators entirely. That text presents the use of "fraction strips" (a strip of paper folded into fractions) or "benchmark fractions" such as one-half against which a fraction such as two-fifths may be compared. While such methods may be useful to build conceptual understanding, they are controversial as they are not effective beyond the elementary school level, and such texts are often supplemented by teachers with the standard method.

Addition

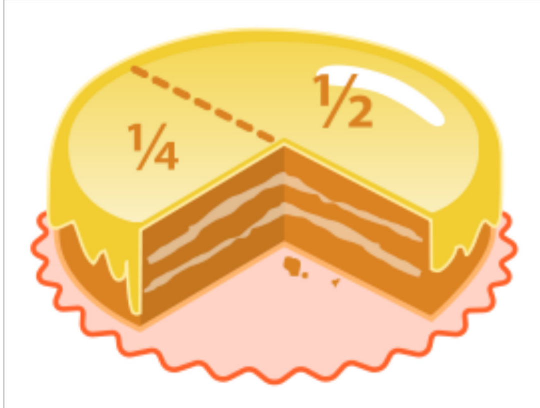
The first rule of addition is that only like quantities can be added; for example, various quantities of quarters. Unlike quantities, such as adding thirds to quarters, must first be converted to like quantities as described below:

Adding like quantities

Imagine a pocket containing two quarters, and another pocket containing three quarters; in total, there are five quarters. Since four quarters is equivalent to one (dollar), this can be represented as follows:

$$\tfrac{2}{4} + \tfrac{3}{4} = \tfrac{5}{4} = 1\tfrac{1}{4}.$$

Adding unlike quantities



If $\frac{1}{2}$ of a cake is to be added to $\frac{1}{4}$ of a cake, the pieces need to be converted into comparable quantities, such as cake-eighths or cake-quarters.

To add fractions containing unlike quantities (e.g. quarters and thirds), it is necessary to convert all amounts to like quantities. It is easy to work out the type of fraction to convert to; simply multiply together the two denominators (bottom number) of each fraction.

For adding quarters to thirds, both types of fraction are converted to $\tfrac{1}{4} \times \tfrac{1}{3} = \tfrac{1}{12}$ (twelfths).

Consider adding the following two quantities:

$$\tfrac{3}{4} + \tfrac{2}{3}$$

First, convert $\tfrac{3}{4}$ into twelfths by multiplying both the numerator and denominator by three: $\tfrac{3}{4} \times \tfrac{3}{3} = \tfrac{9}{12}$. Note that $\tfrac{3}{3}$ is equivalent to 1, which shows that $\tfrac{3}{4}$ is equivalent to the resulting $\tfrac{9}{12}$

Secondly, convert $\tfrac{2}{3}$ into twelfths by multiplying both the numerator and denominator by four: $\tfrac{2}{3} \times \tfrac{4}{4} = \tfrac{8}{12}$. Note that $\tfrac{4}{4}$ is equivalent to 1, which shows that $\tfrac{2}{3}$ is equivalent to the resulting $\tfrac{8}{12}$

Now it can be seen that:

$$\tfrac{3}{4} + \tfrac{2}{3}$$

is equivalent to:

$$\tfrac{9}{12} + \tfrac{8}{12} = \tfrac{17}{12} = 1\tfrac{5}{12}$$

This method always works, but sometimes there is a smaller denominator that can be used (a least common denominator). For example, to add $\tfrac{3}{4}$ and $\tfrac{5}{12}$ the denominator 48 can be used (the product of 4 and 12), but the smaller denominator 12 may also be used, being the least common multiple of 4 and 12.

$$\frac{3}{4} + \frac{5}{12} = \frac{9}{12} + \frac{5}{12} = \frac{14}{12} = \frac{7}{6} = 1\frac{1}{6}$$

Subtraction

The process for subtracting fractions is, in essence, the same as that of adding them: find a common denominator, and change each fraction to an equivalent fraction with the chosen common denominator. The resulting fraction will have that denominator, and its numerator will be the result of subtracting the numerators of the original fractions. For instance,

$$\frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$$

Multiplication

When multiplying or dividing, it may be possible to choose to cancel down crosswise multiples that share a common factor. For example:

$^{\cancel{2}}\,{}^{1}/_{\cancel{7}}\,_{1}$ X $^{\cancel{7}}\,{}^{1}/_{\cancel{8}}\,_{4}$ = ${}^{1}/_{1}$ X ${}^{1}/_{4}$. The following will explain how to complete this equation.

Multiplication by whole numbers

Considering the cake example above, if you have a quarter of the cake and you multiply the amount by three, then you end up with three quarters. We can write this numerically as follows:

$$3 \times \frac{1}{4} = \frac{3}{4}$$

As another example, suppose that five people work for three hours out of a seven hour day (ie. for three sevenths of the work day). In total, they will have worked for 15 hours (5 x 3 hours each), or 15 sevenths of a day. Since 7 sevenths of a day is a whole day and 14 sevenths is two days, then in total, they will have worked for 2 days and a seventh of a day. Numerically:

$$5 \times \frac{3}{7} = \frac{15}{7} = 2\frac{1}{7}$$

Multiplication by fractions

Considering the cake example above, if you have a quarter of the cake and you multiply the amount by a third, then you end up with a twelfth of the cake. In other words, a third of a quarter (or a third times a quarter) is a twelfth. Why? Because we are splitting each quarter into three pieces, and four quarters times three makes 12 parts (or twelfths). We can write this numerically as follows:

$$\tfrac{1}{3} \times \tfrac{1}{4} = \tfrac{1}{12}$$

As another example, suppose that five people do an equal amount work that *totals* three hours out of a seven hour day. Each person will have done a fifth of the work, so they will have worked for a fifth of three sevenths of a day. Numerically:

$$\tfrac{1}{5} \times \tfrac{3}{7} = \tfrac{3}{35}$$

General rule

You may have noticed that when we multiply fractions, we multiply the two *numerators* (the top numbers) to make the new numerator, and multiply the two *denominators* (the bottom numbers) to make the new denominator. For example:

$$\tfrac{5}{6} \times \tfrac{7}{8} = \tfrac{5 \times 7}{6 \times 8} = \tfrac{35}{48}$$

Multiplication by mixed numbers

When multiplying mixed numbers, it's best to convert the whole part of the mixed number into a fraction. For example:

$$3 \times 2\tfrac{3}{4} = 3 \times \left(\tfrac{8}{4} + \tfrac{3}{4}\right) = 3 \times \tfrac{11}{4} = \tfrac{33}{4} = 8\tfrac{1}{4}$$

In other words, $2\tfrac{3}{4}$ is the same as $\left(\tfrac{8}{4} + \tfrac{3}{4}\right)$, making 11 quarters in total (because 2 cakes, each split into quarters makes 8 quarters total) and 33 quarters is $8\tfrac{1}{4}$, since 8 cakes, each made of quarters, is 32 quarters in total.

Division

To divide by a fraction, simply multiply by the reciprocal of that fraction.

$$5 \div \tfrac{1}{2} = 5 \times \tfrac{2}{1} = 5 \times 2 = 10$$

$$\tfrac{2}{3} \div \tfrac{2}{5} = \tfrac{2}{3} \times \tfrac{5}{2} = \tfrac{10}{6} = \tfrac{5}{3}$$

To understand why this works, consider the following:

Question, does

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

Given/Accepted

I. Any number divided by itself is one (e.g. $\frac{d}{d} = \frac{1}{1}$)
II. When a number is multiplied by one it does not change (e.g. $\frac{a}{b} \times \frac{1}{1} = \frac{a}{b} \times \frac{d}{d} = \frac{a}{b}$)
III. If two fractions have common denominators, then the numerators may be divided to find the quotient (e.g. $\frac{ad}{bd} \div \frac{bc}{bd} = ad \div bc$)

Proof

1. $\frac{a}{b} \div \frac{c}{d}$, Problem
2. $\frac{ad}{bd} \div \frac{bc}{bd}$, Multiplied the first fraction by $\frac{d}{d}$ and the second fraction by $\frac{b}{b}$, which is the same as multiplying by one, and as accepted above (I & II) does not change the value of the fraction
Note: These values of one were chosen so the fractions would have a common denominator; **bd** is the common denominator.
3. $\frac{ad}{bd} \div \frac{bc}{bd} = ad \div bc$, From what was given in (III)
4. $ad \div bc = \frac{ad}{bc}$, Changed notation
5. $\frac{ad}{bc} = \frac{a}{b} \times \frac{d}{c}$, Can be seen
6. $\frac{a}{b} \times \frac{d}{c}$, Solution

About 4,000 years ago Egyptians divided with fractions using slightly different methods, using least common multiples with unit fractions.

Converting repeating decimals to fractions

Decimal numbers, while arguably more useful to work with when performing calculations, lack the same kind of precision that regular fractions (as they are explained in this article) have. Sometimes an infinite number of decimals is required to convey the same kind of precision. Thus, it is often useful to convert repeating decimals into fractions.

For most repeating patterns, a simple division of the pattern by the same number of nines as numbers it has will suffice. For example (the pattern is highlighted in bold):

0.**5**55… = 5/9
0.**264**264264… = 264/999
0.**6291**62916291… = 6291/9999

In case zeros precede the pattern, the nines are suffixed by the same number of zeros:

0.0**5**55… = 5/90
0.000**392**392392… = 392/999000
0.00**12**1212… = 12/9900

In case a non-repeating set of decimals precede the pattern (such as 0.1523**987**987987…), we must equate it as the sum of the non-repeating and repeating parts:

0.1523 + 0.0000987987987…

Then, convert both of these to fractions. Since the first part is not repeating, it is not converted according to the pattern given above:

1523/10000 + 987/9990000

We add these fractions by expressing both with a common divisor...

1521477/9990000 + 987/9990000

And add them.

1522464/9990000

Finally, we simplify it:

31718/208125

Special cases

A **unit fraction** is a vulgar fraction with a numerator of 1, e.g. $\frac{1}{7}$.

An **Egyptian fraction** is the sum of distinct unit fractions, e.g. $\frac{1}{2} + \frac{1}{3}$.

A **dyadic fraction** is a vulgar fraction in which the denominator is a power of two, e.g. $\frac{1}{8}$.

An expression that has the form of a fraction but actually represents division by or into an irrational number is sometimes called an "irrational fraction". A common example is $\frac{\pi}{2}$, the radian measure of a right angle.

Rational numbers are the quotient field of integers. Rational functions are functions evaluated in the form of a fraction, where the numerator and denominator are polynomials. These rational expressions are the quotient field of the polynomials (over some integral domain).

A **continued fraction** is an expression such as $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \ldots}}$, where the a_i are integers. This is **not** an element of a quotient field.

The term **partial fraction** is used in algebra, when decomposing rational expressions (a fraction with an algebraic expression in the denominator). The goal is to write the rational expression as the sum of other rational expressions with denominators of lesser degree. For example, the rational expression $\frac{2x}{(x^2-1)}$ can be rewritten as the sum of two fractions: $\frac{1}{(x+1)}$ and $\frac{1}{(x-1)}$.

Pedagogical tools

In primary schools, fractions have been demonstrated through Cuisenaire rods.

Parents of children learning fractions should also be aware that arithmetic is often taught very differently with reform mathematics. Many texts do not give instruction of standard methods which may use the least common denominator, to compare or add fractions. Some introduce newly developed concepts such as "fraction strips" and benchmark fractions (1/2, 1/4, 3/4 and 1/10) which are unfamiliar to parents or mathematicians. Some are concerned that such methods will not prepare students for mathematics in college or high school. If this is the case, parents may ask their schools to supplement their children's learning with standard methods or switch to texts which give instruction in traditional methods. Fraction arithmetic is normally taught and mastered from late elementary to middle or junior high school. However, some texts such as the Connected Mathematics do not discuss division of fractions at all even through 8th grade in CMP

See also the external links below.

History

The earliest known use of decimal fractions is ca. 2800 BC as Ancient Indus Valley units of measurement. The Egyptians used Egyptian fractions ca. 1000 BC. The Greeks used unit fractions and later continued fractions and followers of the Greek philosopher Pythagoras, ca. 530 BC, discovered that the square root of two cannot be expressed as a fraction. In 150 BC Jain mathematicians in India wrote the "Sthananga Sutra", which contains work on the theory of numbers, arithmetical operations, operations with fractions.

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Multiplication

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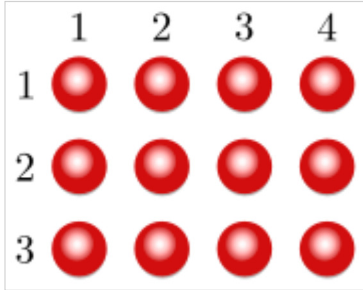


3 × 4 = 12, so twelve dots can be arranged in three rows of four (or four columns of three).

Multiplication of whole numbers is the mathematical operation of adding together multiple copies of the same number. For example, four multiplied by three is twelve, since three sets of four make twelve:

$$4 + 4 + 4 = 12.$$

Multiplication can also be viewed as counting objects arranged in a rectangle, or finding the area of rectangle whose sides have given lengths.

Multiplication is one of four main operations in elementary arithmetic, and most people learn basic multiplication algorithms in elementary school. The inverse of multiplication is division.

Multiplication is generalized to many kinds of numbers and to more abstract constructs such as matrices.

Notation and terminology

Multiplication is written using the multiplication sign "×" between the terms; that is, in infix notation. The result is expressed with an equals sign. For example,

$2 \times 3 = 6$ (verbally, "two times three equals six")

$$3 \times 4 = 12$$

$$2 \times 3 \times 5 = 30$$

$$2 \times 2 \times 2 \times 2 \times 2 = 32$$

There are several other common notations for multiplication:

- Multiplication is sometimes denoted by either a middle dot or a period:
  $$5 \cdot 2 \quad \text{or} \quad 5\,.\,2$$
  The middle dot is standard in the United States, the United Kingdom, and other countries where the period is used as a decimal point. In some countries

that use a comma as a decimal point, the period is used for multiplication instead.
- The asterisk (e.g. 5 * 2) is often used with computers because it appears on every keyboard. This usage originated in the FORTRAN programming language.
- In algebra, multiplication involving variables is often written as a juxtaposition (e.g. xy for x times y or $5x$ for five times x). This notation can also be used for numbers that are surrounded by parentheses (e.g. 5(2) or (5)(2) for five times two).

The numbers to be multiplied are generally called the "factors" or "multiplicands". When thinking of multiplication as repeated addition, the number to be repeated is called the "multiplicand", while the number of repetitions is called the "multiplier". In algebra, a number that is multiplied by a variable or expression (i.e. the 3 in $3xy^2$) is called a coefficient.

The result of a multiplication is called a product, and is a multiple of each factor. For example 15 is the product of 3 and 5, and is both a multiple of 3 and a multiple of 5.

Computation

The standard methods for multiplying numbers using pencil and paper require a multiplication table of memorized or consulted products of small numbers (typically any two numbers from 0 to 9), however one method, the peasant multiplication algorithm, does not. Many mathematics curricula developed according to the 1989 standards of the NCTM do not teach standard arithmetic methods, instead guiding students to invent their own methods of computation. Though widely adopted by many school districts in nations such as the United States, they have encountered resistance from some parents and mathematicians, and some districts have since abandoned such curricula in favour of traditional mathematics.

Multiplying numbers to more than a couple of decimal places by hand is tedious and error prone. Common logarithms were invented to simplify such calculations. The slide rule allowed numbers to be quickly multiplied to about three places of accuracy. Beginning in the early twentieth century, mechanical calculators, such as the Marchant, automated multiplication of up to 10 digit numbers. Modern electronic computers and calculators have greatly reduced the need for multiplication by hand.

Historical algorithms

Methods of multiplication were documented in the Egyptian, Greece, Babylonian, Indus valley, and Chinese civilizations.

Egyptians

The Egyptian method of multiplication of integers and fractions, documented in the Ahmes Papyrus, was by successive additions and doubling. For instance, to find the product of 13 and 21 one had to double 21 three times, obtaining $1 \times 21 = 21$, $2 \times 21 = 42$, $4 \times 21 = 84$, $8 \times 21 = 168$. The full product could then be found by adding the appropriate terms found in the doubling sequence:

13 × 21 = (1 + 4 + 8) × 21 = (1 × 21) + (4 × 21) + (8 × 21) = 21 + 84 + 168 = 273.

Babylonians

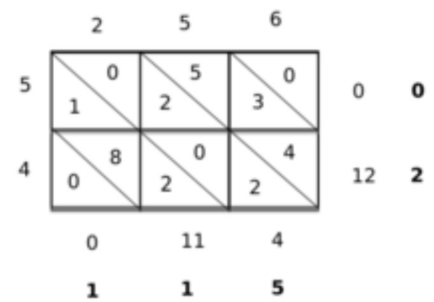
The Babylonians used a sexagesimal positional number system, analogous to the modern day decimal system. Thus, Babylonian multiplication was very similar to modern decimal multiplication. Because of the relative difficulty of remembering 60 × 60 different products, Babylonian mathematicians employed multiplication tables. These tables consisted of a list of the first twenty multiples of a certain *principal number n*: *n*, 2*n*, ..., 20*n*; followed by the multiples of 10*n*: 30*n* 40*n*, and 50*n*. Then to compute any sexagesimal product, say 53*n*, one only needed to add 50*n* and 3*n* computed from the table.

Chinese

In the books, Chou Pei Suan Ching dated prior to 300 B.C., and the Nine Chapters on the Mathematical Art, multiplication calculations were written out in words, although the early Chinese mathematicians employed an abacus in hand calculations involving addition and multiplication.

Indus Valley

The early Hindu mathematicians of the Indus valley region used a variety of intuitive tricks to perform multiplication. Most calculations were performed on small slate hand tablets, using chalk tables. One technique was that of *lattice multiplication* (or *gelosia multiplication*). Here a table was drawn up with the rows and columns labelled by the multiplicands. Each box of the table was divided diagonally into two, as a triangular lattice. The entries of the table held the partial products, written as decimal numbers. The product could then be formed by summing down the diagonals of the lattice.



Product of 45 and 256. Note the order of the numerals in 45 is reversed down the left column. The carry step of the multiplication can be performed at the final stage of the calculation (in bold), returning the final product of 45 × 256 = 11520.

Modern method

The modern method of multiplication based on the Hindu-Arabic numeral system was first described by Brahmagupta. Brahmagupta gave rules for addition, subtraction, multiplication and division. Henry Burchard Fine, then professor of Mathematics at Princeton University, wrote the following:

> *The Indians are the inventors not only of the positional decimal system itself, but of most of the processes involved in elementary reckoning with the system. Addition and subtraction they performed quite as they are performed nowadays; multiplication they effected in many ways, ours among them, but division they did cumbrously.*

Products of sequences

Capital pi notation

The product of a sequence of terms can be written with the product symbol, which derives from the capital letter Π (Pi) in the Greek alphabet. Unicode position U+220F (∏) is defined a *n*-ary product for this purpose, distinct from U+03A0 (Π), the letter. This is defined as:

$$\prod_{i=m}^{n} x_i := x_m \cdot x_{m+1} \cdot x_{m+2} \cdot \cdots \cdot x_{n-1} \cdot x_n.$$

The subscript gives the symbol for a dummy variable (*i* in our case) and its lower value (*m*); the superscript gives its upper value. So for example:

$$\prod_{i=2}^{6} \left(1 + \frac{1}{i}\right) = \left(1 + \frac{1}{2}\right) \cdot \left(1 + \frac{1}{3}\right) \cdot \left(1 + \frac{1}{4}\right) \cdot \left(1 + \frac{1}{5}\right) \cdot \left(1 + \frac{1}{6}\right) = \frac{7}{2}.$$

In case $m = n$, the value of the product is the same as that of the single factor x_m. If $m > n$, the product is the empty product, with the value 1.

Infinite products

One may also consider products of infinitely many terms; these are called infinite products. Notationally, we would replace *n* above by the lemniscate (infinity symbol) ∞. In the reals, the product of such a series is defined as the limit of the product of the first *n* terms, as *n* grows without bound. That is, by definition,

$$\prod_{i=m}^{\infty} x_i = \lim_{n \to \infty} \prod_{i=m}^{n} x_i.$$

One can similarly replace *m* with negative infinity, and define:

$$\prod_{i=-\infty}^{\infty} x_i = \left(\lim_{m \to -\infty} \prod_{i=m}^{0} x_i\right) \cdot \left(\lim_{n \to \infty} \prod_{i=1}^{n} x_i\right),$$

provided both limits exist.

Interpretation

Cartesian product

The definition of multiplication as repeated addition provides a way to arrive at a set-theoretic interpretation of multiplication of cardinal numbers. In the expression

$$a \cdot n = \underbrace{a + \cdots + a}_{n},$$

if the n copies of a are to be combined in disjoint union then clearly they must be made disjoint; an obvious way to do this is to use either a or n as the indexing set for the other. Then, the members of $a \cdot n$ are exactly those of the Cartesian product $a \times n$. The properties of the multiplicative operation as applying to natural numbers then follow trivially from the corresponding properties of the Cartesian product.

Properties

For integers, fractions, real and complex numbers, multiplication has certain properties:

Commutative property
: The order in which two numbers are multiplied does not matter.
: $x \cdot y = y \cdot x$.

Associative property
: Problems solely involving multiplication are invariant with respect to order of operations.
: $(x \cdot y)\cdot z = x\cdot(y \cdot z)$.

Distributive property
: Holds with respect to addition over multiplication. This identity is of prime importance in simplifying algebraic expressions.
: $x\cdot(y + z) = x\cdot y + x\cdot z$.

Identity element
: of multiplication is 1; anything multiplied by one is itself. This is known as the identity property
: $x \cdot 1 = x$.

Zero element
: Anything multiplied by zero is zero. This is known as the zero property of multiplication.
: $x \cdot 0 = 0$

Inverse property
: Every number x, except zero, has a **multiplicative inverse**, $1/x$, such that $x\cdot(1/x) = 1$.

Order preservation

Multiplication by a positive number preserves order: if $a > 0$, then if $b > c$ then $a \cdot b > a \cdot c$. Multiplication by a negative number **reverses** order: if $a < 0$, then if $b > c$ then $a \cdot b < a \cdot c$.

- Negative one times any number is equal to the negative of that number.

$$(-1) \cdot x = (-x)$$

- Negative one times negative one is positive one.

$$(-1) \cdot (-1) = 1$$

Other mathematical systems that include a multiplication operation may not have all these properties. For example, multiplication is not, in general, commutative for matrices and quaternions.

Proofs

Not all of these properties are independent; some are a consequence of the others. A property that can be proven from the others is the zero property of multiplication. It is proved by means of the distributive property. We assume all the usual properties of addition and subtraction, and $-x$ means the same as

.

$$\begin{aligned} x \cdot 0 & \\ &= (x \cdot 0) + x - x \\ &= (x \cdot 0) + (x \cdot 1) - x \\ &= x \cdot (0 + 1) - x \\ &= (x \cdot 1) - x \\ &= x - x \\ &= 0. \end{aligned}$$

So we have proved:

$$x \cdot 0 = 0.$$

The identity $(-1) \cdot x = (-x)$ can also be proved using the distributive property:

$$\begin{aligned} (-1) \cdot x & \\ &= (-1) \cdot x + x - x \end{aligned}$$

$$
\begin{aligned}
&= (-1) \cdot x + 1 \cdot x - x \\
&= (-1 + 1) \cdot x - x \\
&= 0 \cdot x - x \\
&= 0 - x \\
&= -x
\end{aligned}
$$

The proof that $(-1) \cdot (-1) = 1$ is now easy:

$$
\begin{aligned}
&(-1) \cdot (-1) \\
&\quad = -(-1) \\
&\quad = 1.
\end{aligned}
$$

Multiplication with Peano's axioms

In the book *Arithmetices principia, nova methodo exposita*, Giuseppe Peano proposed a new system for multiplication based on his axioms for natural numbers.

- a×1=a
- a×b'=(a×b)+a

Here, b' represents the successor of b, or the natural number which *follows* b. With his other nine axioms, it is possible to prove common rules of multiplication, such as the distributive or associative properties.

Multiplication with set theory

It is possible, though difficult, to create a recursive definition of multiplication with set theory. Such a system usually relies on the peano definition of multiplication.

Multiplication in group theory

It is easy to show that there is a group for multiplication- the non-zero rational numbers. Multiplication with the non-zero numbers satisfies

- **Closure** - For all a and b in the group, a×b is in the group.
- **Associativity** - This is just the associative property: (a×b)×c=a×(b×c)

- **Identity** - This follows straight from the peano definition. Anything multiplied by one is itself.
- **Inverse** - All non-zero numbers have a multiplicative inverse.

Multiplication also is an abelian group, since it follows the commutative property.

a×b=b×a

Multiplication of different kinds of numbers

Numbers can *count* (3 apples), *order* (the 3rd apple), or *measure* (3.5 feet high); as the history of mathematics has progressed from counting on our fingers to modelling quantuum mechanics, multiplication has been generalized to more complicated and abstract types of numbers, and to things that aren't numbers (such as matrices) or don't look much like numbers (such as quaternions).

- **Integers** N × M is the sum of M copies of N when N and M are positive whole numbers. This gives the number of things in an array N wide and M high. Generalization to negative numbers can be done by (N × -M) = - (N × M).

- **Rationals** Generalization to fractions A/B × C/D is by multiplying the numerators and denominators respectively: A/B × C/D = (A × B) / (C × D). This gives the area of a rectangle A/B high and C/D wide, and is the same as the number of things in an array when the rational numbers happen to be whole numbers.

- **Reals** $x \times y$ is the limit of the products of the corresponding terms in certain sequences of rationals that converge to x and y, respectively, and is significant in Calculus. This gives the area of a rectangle x high and y wide. See above.

- **Complex** Considering complex numbers *z1* and *z2* as ordered pairs or real numbers (a1, b1) and (a2, b2), the product *z1* × *z2* is (a1 × a2 - b1 × b2, a1 × b2 + a2 × b1). This is the same as for reals, a1 × a2, when the *imaginary parts* b1 and b2 are zero.

- **Further generalizations** See above and Multiplicative Group, which for example includes matrix multiplication. A very general, and abstract, concept of multiplication is as the "multiplicatively denoted" (second) binary operation in a ring. An example of a ring which is not any of the above number systems is polynomial rings (you can add and multiply polynomials, but polynomials are not numbers in any usual sense.)

- **Division** Often division x / y is the same as multiplication by an inverse, x × (1/y). Multiplication for some types of "numbers" may have corresponding division, without inverses; in an Integral domain x may have no inverse "1/x" but x/y may be defined. In a Division ring there are inverses but they are not commutative (since 1/x X 1/y is not the same as 1/y X 1/x, x/y may be ambiguous).

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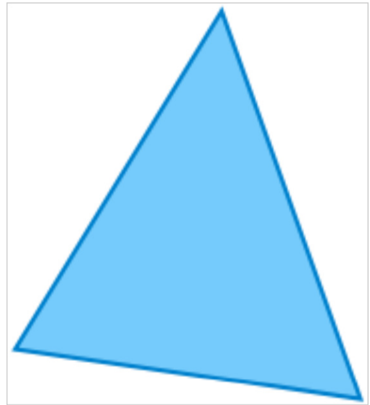
Triangle

2008/9 Schools Wikipedia Selection. Related subjects: Mathematics

A **triangle** is one of the basic shapes of geometry: a polygon with three corners or vertices and three sides or edges which are line segments.

In Euclidean geometry any three non- collinear points determine a unique triangle and a unique plane (i.e. two-dimensional Cartesian space).

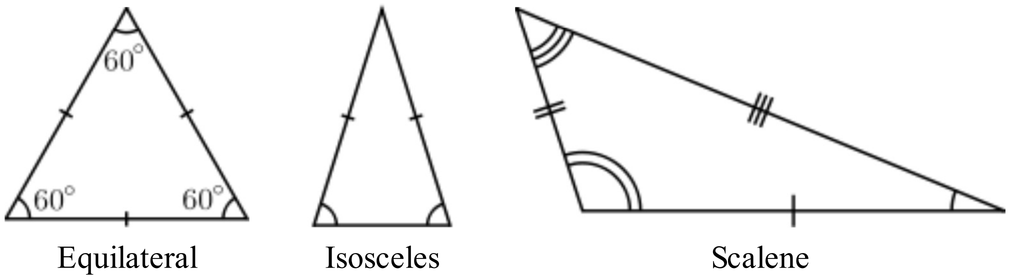
Types of triangles



A triangle.

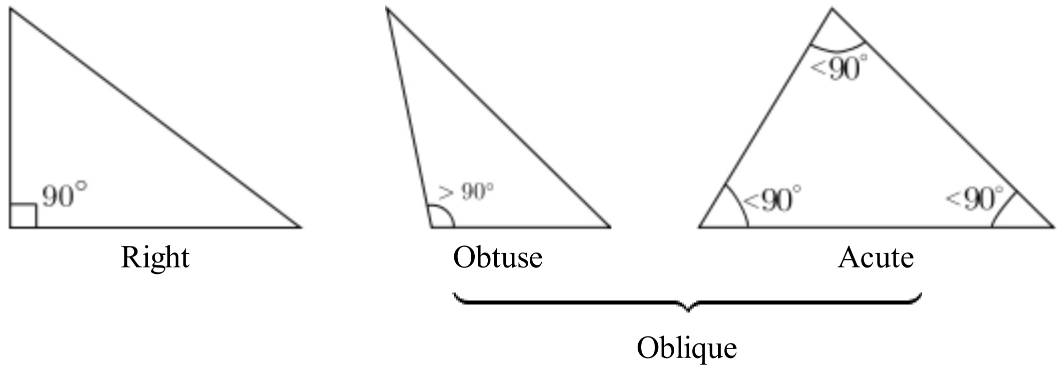
Triangles can be classified according to the relative lengths of their sides:

- In an **equilateral triangle**, all sides are of equal length. An equilateral triangle is also an equiangular polygon, i.e. all its internal angles are equal—namely, 60°; it is a regular polygon.
- In an **isosceles triangle**, two sides are of equal length (originally and conventionally limited to *exactly* two). An isosceles triangle also has two equal angles: the angles opposite the two equal sides.
- In a **scalene triangle**, all sides have different lengths. The internal angles in a scalene triangle are all different.



Triangles can also be classified according to their internal angles, described below using degrees of arc:

- A **right triangle** (or **right-angled triangle**, formerly called a **rectangled triangle**) has one 90° internal angle (a right angle). The side opposite to the right angle is the hypotenuse; it is the longest side in the right triangle. The other two sides are the *legs* or **catheti** (singular: **cathetus**) of the triangle.
- An **oblique triangle** has no internal angle equal to 90°.
- An **obtuse triangle** is an oblique triangle with one internal angle larger than 90° (an obtuse angle).
- An **acute triangle** is an oblique triangle with internal angles all smaller than 90° (three acute angles). An equilateral triangle is an acute triangle, but not all acute triangles are equilateral triangles.



Basic facts

Elementary facts about triangles were presented by Euclid in books 1-4 of his *Elements* around 300 BCE. A triangle is a polygon and a 2- simplex (see polytope). All triangles are two- dimensional.

The angles of a triangle add up to 180 degrees. An exterior angle of a triangle (an angle that is adjacent and supplementary to an internal angle) is always equal to the two angles of a triangle that it is not adjacent/supplementary to. Like all convex polygons, the exterior angles of a triangle add up to 360 degrees.

The sum of the lengths of any two sides of a triangle always exceeds the length of the third side. That is the triangle inequality. (In the special case of equality, two of the angles have collapsed to size zero, and the triangle has degenerated to a line segment.)

Two triangles are said to be *similar* if and only if the angles of one are equal to the corresponding angles of the other. In this case, the lengths of their corresponding sides are proportional. This occurs for example when two triangles share an angle and the sides opposite to that angle are parallel.

A few basic postulates and theorems about similar triangles:

- Two triangles are similar if at least two corresponding angles are equal.
- If two corresponding sides of two triangles are in proportion, and their included angles are equal, the triangles are similar.
- If three sides of two triangles are in proportion, the triangles are similar.

For two triangles to be congruent, each of their corresponding angles and sides must be equal (6 total). A few basic postulates and theorems about congruent triangles:

- SAS Postulate: If two sides and the included angles of two triangles are correspondingly equal, the two triangles are congruent.
- SSS Postulate: If every side of two triangles are correspondingly equal, the triangles are congruent.

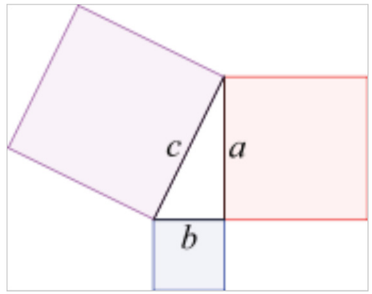
- ASA Postulate: If two angles and the included sides of two triangles are correspondingly equal, the two triangles are congruent.
- AAS Theorem: If two angles and any side of two triangles are correspondingly equal, the two triangles are congruent.
- Hypotenuse-Leg Theorem: If the hypotenuses and one leg of two right triangles are correspondingly equal, the triangles are congruent.

Using right triangles and the concept of similarity, the trigonometric functions sine and cosine can be defined. These are functions of an angle which are investigated in trigonometry.

In Euclidean geometry, the sum of the internal angles of a triangle is equal to 180°. This allows determination of the third angle of any triangle as soon as two angles are known.

A central theorem is the Pythagorean theorem, which states in any right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the two other sides. If the hypotenuse has length c, and the legs have lengths a and b, then the theorem states that

$$a^2 + b^2 = c^2$$



The Pythagorean theorem

The converse is true: if the lengths of the sides of a triangle satisfy the above equation, then the triangle is a right triangle.

Some other facts about right triangles:

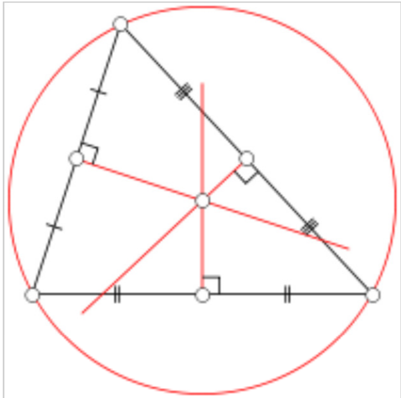
- The acute angles of a right triangle are complementary.
- If the legs of a right triangle are equal, then the angles opposite the legs are equal, acute and complementary, and thus are both 45 degrees. By the Pythagorean theorem, the length of the hypotenuse is the square root of two times the length of a leg.
- In a 30-60 right triangle, in which the acute angles measure 30 and 60 degrees, the hypotenuse is twice the length of the shorter side.
- In all right triangles, the median on the hypotenuse is the half of the hypotenuse.

For all triangles, angles and sides are related by the law of cosines and law of sines.

Points, lines and circles associated with a triangle

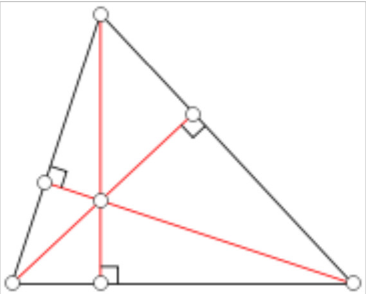
There are hundreds of different constructions that find a special point inside a triangle, satisfying some unique property: see the references section for a catalogue of them. Often they are constructed by finding three lines associated in a symmetrical way with the three sides (or vertices) and then proving that the three lines meet in a single point: an important tool for proving the existence of these is Ceva's theorem, which gives a criterion for determining when three such lines are concurrent. Similarly, lines associated with a triangle are often constructed by proving that three symmetrically constructed points are collinear: here Menelaus' theorem gives a useful general criterion. In this section just a few of the most commonly-encountered constructions are explained.

A perpendicular bisector of a triangle is a straight line passing through the midpoint of a side and being perpendicular to it, i.e. forming a right angle with it. The three perpendicular bisectors meet in a single point, the triangle's circumcenter; this point is the centre of the circumcircle, the circle passing through all three vertices. The diameter of this circle can be found from the law of sines stated above.



The circumcenter is the centre of a circle passing through the three vertices of the triangle.

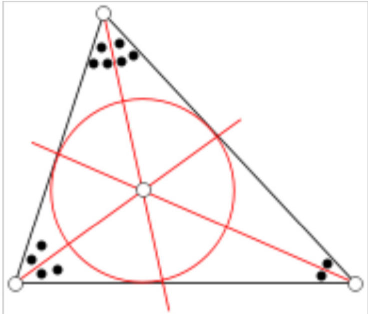
Thales' theorem implies that if the circumcenter is located on one side of the triangle, then the opposite angle is a right one. More is true: if the circumcenter is located inside the triangle, then the triangle is acute; if the circumcenter is located outside the triangle, then the triangle is obtuse.



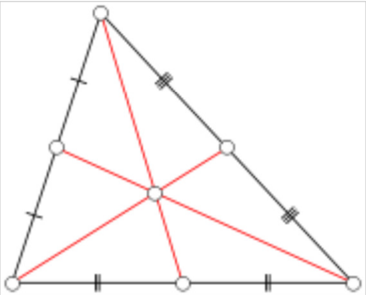
The intersection of the altitudes is the orthocenter.

An altitude of a triangle is a straight line through a vertex and perpendicular to (i.e. forming a right angle with) the opposite side. This opposite side is called the *base* of the altitude, and the point where the altitude intersects the base (or its extension) is called the *foot* of the altitude. The length of the altitude is the distance between the base and the vertex. The three altitudes intersect in a single point, called the orthocenter of the triangle. The orthocenter lies inside the triangle if and only if the triangle is acute. The three vertices together with the orthocenter are said to form an orthocentric system.

An angle bisector of a triangle is a straight line through a vertex which cuts the corresponding angle in half. The three angle bisectors intersect in a single point, the incenter, the centre of the triangle's incircle. The incircle is the circle which lies inside the triangle and touches all three sides. There are three other important circles, the excircles; they lie outside the triangle and touch one side as well as the extensions of the other two. The centers of the in- and excircles form an orthocentric system.

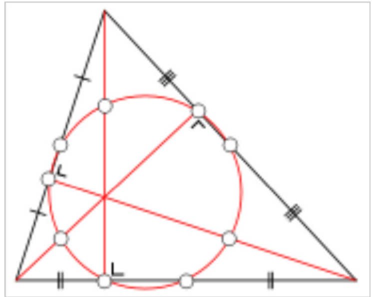


The intersection of the angle bisectors finds the centre of the incircle.



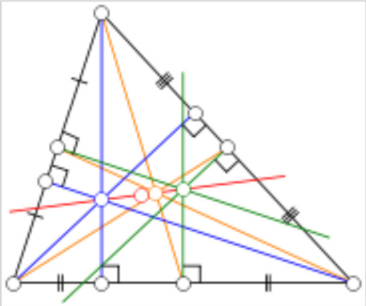
The barycenter is the centre of gravity.

A median of a triangle is a straight line through a vertex and the midpoint of the opposite side, and divides the triangle into two equal areas. The three medians intersect in a single point, the triangle's centroid. This is also the triangle's centre of gravity: if the triangle were made out of wood, say, you could balance it on its centroid, or on any line through the centroid. The centroid cuts every median in the ratio 2:1, i.e. the distance between a vertex and the centroid is twice as large as the distance between the centroid and the midpoint of the opposite side.



Nine-point circle demonstrates a symmetry where six points lie on the edge of the triangle.

The midpoints of the three sides and the feet of the three altitudes all lie on a single circle, the triangle's nine-point circle. The remaining three points for which it is named are the midpoints of the portion of altitude between the vertices and the orthocenter. The radius of the nine-point circle is half that of the circumcircle. It touches the incircle (at the Feuerbach point) and the three excircles.



Euler's line is a straight line through the centroid (orange), orthocenter (blue), circumcenter (green) and centre of the nine-point circle (red).

The centroid (yellow), orthocenter (blue), circumcenter (green) and barycenter of the nine-point circle (red point) all lie on a single line, known as Euler's line (red line). The centre of the nine-point circle lies at the midpoint between the orthocenter and the circumcenter, and the distance between the centroid and the circumcenter is half that between the centroid and the orthocenter.

The centre of the incircle is not in general located on Euler's line.

If one reflects a median at the angle bisector that passes through the same vertex, one obtains a symmedian. The three symmedians intersect in a single point, the symmedian point of the triangle.

Computing the area of a triangle

Calculating the area of a triangle is an elementary problem encountered often in many different situations. The best known, and simplest formula is

$$S = \frac{1}{2}bh$$

where *S* is area, *b* is the length of the base of the triangle, and *h* is the height or altitude of the triangle. The term 'base' denotes any side, and 'height' denotes the length of a perpendicular from the point opposite the side onto the side itself.

Although simple, this formula is only useful if the height can be readily found. For example, the surveyor of a triangular field measures the length of each side, and can find the area from his results without having to construct a 'height'. Various methods may be used in practice, depending on what is known about the triangle. The following is a selection of frequently used formulae for the area of a triangle.

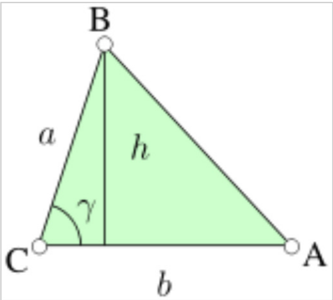
Using vectors

The area of a parallelogram can be calculated using vectors. Let vectors *AB* and *AC* point respectively from A to B and from A to C. The area of parallelogram ABDC is then |*AB* × *AC*|, which is the magnitude of the cross product of vectors *AB* and *AC*. |*AB* × *AC*| is equal to |*h* × *AC*|, where *h* represents the altitude *h* as a vector.

The area of triangle ABC is half of this, or $S = \frac{1}{2}|AB \times AC|$.

The area of triangle ABC can also be expressed in term of dot products as follows:

$$\frac{1}{2}\sqrt{(\mathbf{AB}\cdot\mathbf{AB})(\mathbf{AC}\cdot\mathbf{AC})-(\mathbf{AB}\cdot\mathbf{AC})^2} = \frac{1}{2}\sqrt{|\mathbf{AB}|^2|\mathbf{AC}|^2-(\mathbf{AB}\cdot\mathbf{AC})^2}\,.$$



Applying trigonometry to find the altitude *h*.

Using trigonometry

The height of a triangle can be found through an application of trigonometry. Using the labelling as in the image on the left, the altitude is $h = a \sin \gamma$. Substituting this in the formula $S = \frac{1}{2}bh$ derived above, the area of the triangle can be expressed as:

$$S = \frac{1}{2}ab\sin\gamma = \frac{1}{2}bc\sin\alpha = \frac{1}{2}ca\sin\beta.$$

Furthermore, since $\sin \alpha = \sin(\pi - \alpha) = \sin(\beta + \gamma)$, and similarly for the other two angles:

$$S = \frac{1}{2}ab\sin(\alpha+\beta) = \frac{1}{2}bc\sin(\beta+\gamma) = \frac{1}{2}ca\sin(\gamma+\alpha).$$

Using coordinates

If vertex A is located at the origin (0, 0) of a Cartesian coordinate system and the coordinates of the other two vertices are given by $B = (x_B, y_B)$ and $C = (x_C, y_C)$, then the area *S* can be computed as ½ times the absolute value of the determinant

$$S = \frac{1}{2}\left|\det\begin{pmatrix}x_B & x_C \\ y_B & y_C\end{pmatrix}\right| = \frac{1}{2}|x_B y_C - x_C y_B|.$$

For three general vertices, the equation is:

$$S = \frac{1}{2}\left|\det\begin{pmatrix} x_A & x_B & x_C \\ y_A & y_B & y_C \\ 1 & 1 & 1 \end{pmatrix}\right| = \frac{1}{2}\left|x_A y_C - x_A y_B + x_B y_A - x_B y_C + x_C y_B - x_C y_A\right|.$$

In three dimensions, the area of a general triangle {A = (x_A, y_A, z_A), B = (x_B, y_B, z_B) and C = (x_C, y_C, z_C)} is the Pythagorean sum of the areas of the respective projections on the three principal planes (i.e. $x = 0$, $y = 0$ and $z = 0$):

$$S = \frac{1}{2}\sqrt{\left(\det\begin{pmatrix} x_A & x_B & x_C \\ y_A & y_B & y_C \\ 1 & 1 & 1 \end{pmatrix}\right)^2 + \left(\det\begin{pmatrix} y_A & y_B & y_C \\ z_A & z_B & z_C \\ 1 & 1 & 1 \end{pmatrix}\right)^2 + \left(\det\begin{pmatrix} z_A & z_B & z_C \\ x_A & x_B & x_C \\ 1 & 1 & 1 \end{pmatrix}\right)^2}.$$

Using Heron's formula

The shape of the triangle is determined by the lengths of the sides alone. Therefore the area *S* also can be derived from the lengths of the sides. By Heron's formula:

$$S = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = ½ (a + b + c)$ is the **semiperimeter**, or half of the triangle's perimeter.

An equivalent way of writing Heron's formula is

$$S = \frac{1}{4}\sqrt{(a^2+b^2+c^2)^2 - 2(a^4+b^4+c^4)}.$$

Non-planar triangles

A non-planar triangle is a triangle which is not contained in a (flat) plane. Examples of non-planar triangles in noneuclidean geometries are spherical triangles in spherical geometry and hyperbolic triangles in hyperbolic geometry.

While all regular, planar (two dimensional) triangles contain angles that add up to 180°, there are cases in which the angles of a triangle can be greater than or less than 180°. In curved figures, a triangle on a negatively curved figure ("saddle") will have its angles add up to less than 180° while a triangle on a positively curved figure ("sphere") will have its angles add up to more than 180°. Thus, if one were to draw a giant triangle on the surface of the Earth, one would find that the sum of its angles were greater than 180°.

Retrieved from " http://en.wikipedia.org/wiki/Triangle"

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Trigonometry

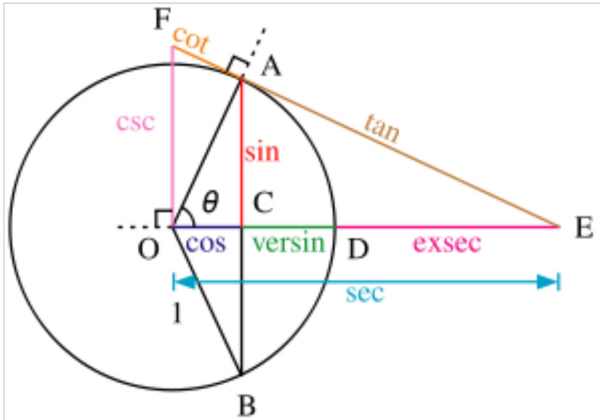
2008/9 Schools Wikipedia Selection. Related subjects: Mathematics

Trigonometry (from Greek *trigōnon* "triangle" + *metron* "measure") is a branch of mathematics that deals with triangles, particularly those plane triangles in which one angle has 90 degrees (**right triangles**). Trigonometry deals with relationships between the sides and the angles of triangles and with the trigonometric functions, which describe those relationships.

Trigonometry has applications in both pure mathematics and in applied mathematics, where it is essential in many branches of science and technology. It is usually taught in secondary schools either as a separate course or as part of a precalculus course. Trigonometry is informally called “trig.”

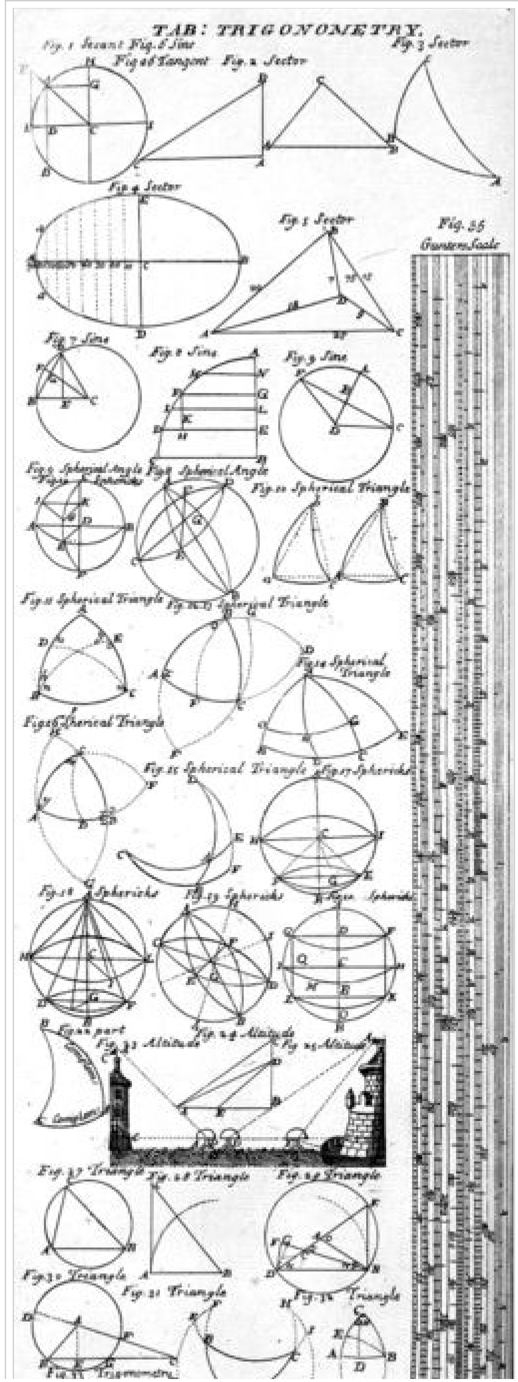
A branch of trigonometry, called spherical trigonometry, studies triangles on spheres, and is important in astronomy and navigation.

The Canadarm2 robotic manipulator on the International Space Station is operated by controlling the angles of its joints. Calculating the final position of the astronaut at the end of the arm requires repeated use of the trigonometric functions of those angles.



All of the trigonometric functions of an angle θ can be constructed geometrically in terms of a unit circle centered at O.

History



Trigonometry was probably developed for use in sailing as a navigation method used with astronomy. The origins of trigonometry can be traced to the civilizations of ancient Egypt, Mesopotamia and the Indus Valley, more than 4000 years ago. The common practice of measuring angles in degrees, minutes and seconds comes from the Babylonian's base sixty system of numeration. The Sulba Sutras written in India, between 800 BC and 500 BC, correctly computes the sine of π/4 (45°) as 1/√2 in a procedure for circling the square (the opposite of squaring the circle).

The first recorded use of trigonometry came from the Hellenistic mathematician Hipparchus circa 150 BC, who compiled a trigonometric table using the sine for solving triangles. Ptolemy further developed trigonometric calculations circa 100 AD.

The ancient Sinhalese in Sri Lanka, when constructing reservoirs in the Anuradhapura kingdom, used trigonometry to calculate the gradient of the water flow. Archeological research also provides evidence of trigonometry used in other unique hydrological structures dating back to 4 BC.

The Indian mathematician Aryabhata in 499, gave tables of half chords which are now known as sine tables, along with cosine tables. He used *zya* for sine, *kotizya* for cosine, and *otkram zya* for inverse sine, and also introduced the versine. Another Indian mathematician, Brahmagupta in 628, used an interpolation formula to compute values of sines, up to the second order of the Newton- Stirling interpolation formula.

In the 10th century, the Persian mathematician and astronomer Abul Wáfa introduced the tangent function and improved methods of calculating trigonometry tables. He established the angle addition identities, e.g. sin (a + b), and discovered the sine formula for spherical geometry:

$$\frac{\sin(A)}{\sin(a)} = \frac{\sin(B)}{\sin(b)} = \frac{\sin(C)}{\sin(c)}.$$

Also in the late 10th and early 11th centuries, the Egyptian astronomer Ibn Yunus performed many careful trigonometric calculations and demonstrated the formula

$$\cos(a)\cos(b) = \frac{\cos(a+b)+\cos(a-b)}{2}.$$

Indian mathematicians were the pioneers of variable computations algebra for use in astronomical calculations along with trigonometry. Lagadha (circa 1350-1200 BC) is the first person thought to have used geometry and trigonometry for astronomy, in his *Vedanga Jyotisha*.

Persian mathematician Omar Khayyám (1048-1131) combined trigonometry and approximation theory to provide methods of solving algebraic equations by geometrical means. Khayyam solved the cubic equation $x^3 + 200x = 20x^2 + 2000$ and found a positive root of this cubic by considering the intersection of a rectangular hyperbola and a circle. An approximate numerical solution was then found by interpolation in trigonometric tables.

Detailed methods for constructing a table of sines for any angle were given by the Indian mathematician Bhaskara in 1150, along with some sine and cosine formulae. Bhaskara also developed spherical trigonometry.

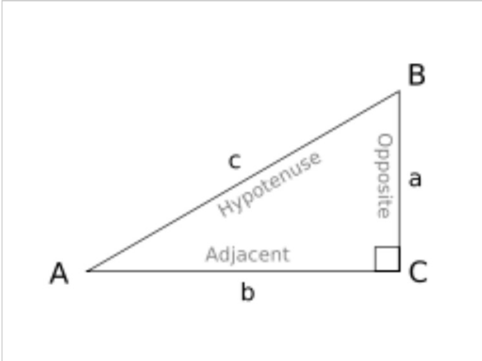
The 13th century Persian mathematician Nasir al-Din Tusi, along with Bhaskara, was probably the first to treat trigonometry as a distinct mathematical discipline. Nasir al-Din Tusi in his *Treatise on the Quadrilateral* was the first to list the six distinct cases of a right angled triangle in spherical trigonometry.

In the 14th century, Persian mathematician al-Kashi and Timurid mathematician Ulugh Beg (grandson of Timur) produced tables of trigonometric functions as part of their studies of astronomy.

The mathematician Bartholemaeus Pitiscus published an influential work on trigonometry in 1595 which may have coined the word "trigonometry".

Overview

If one angle of a right triangle is 90 degrees and one of the other angles is known, the third is thereby fixed, because the three angles of any triangle add up to 180 degrees. The two acute angles therefore add up to 90 degrees: they are complementary angles. The shape of a right triangle is completely determined, up to similarity, by the angles. This means that once one of the other angles is known, the ratios of the various sides are always the same regardless of the overall size of the triangle. These ratios are given by the following trigonometric functions of the known angle *A*:



In this right triangle:

- The **sine** function (sin), defined as the ratio of the side opposite the angle to the hypotenuse.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

- The **cosine** function (cos), defined as the ratio of the adjacent leg to the hypotenuse.

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

- The **tangent** function (tan), defined as the ratio of the opposite leg to the adjacent leg.

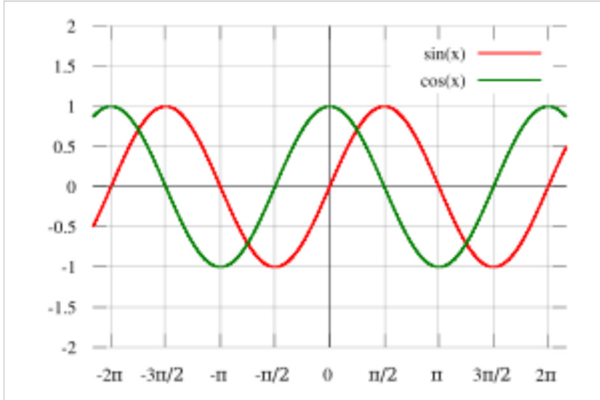
$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin A}{\cos A}$$

The **hypotenuse** is the side opposite to the 90 degree angle in a right triangle; it is the longest side of the triangle, and one of the two sides adjacent to angle A. The **adjacent leg** is the other side that is adjacent to angle A. The **opposite side** is the side that is opposite to angle A. The terms **perpendicular** and **base** are sometimes used for the opposite and adjacent sides respectively. Many people find it easy to remember what sides of the right triangle are equal to sine, cosine, or tangent, by memorizing the word SOH-CAH-TOA (see below under Mnemonics).

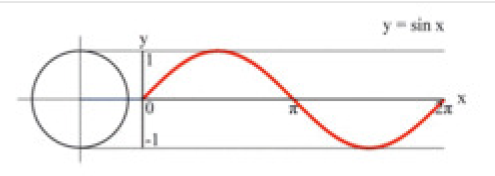
The reciprocals of these functions are named the **cosecant** (csc or cosec), **secant** (sec) and **cotangent** (cot), respectively. The inverse functions are called the **arcsine**, **arccosine**, and **arctangent**, respectively. There are arithmetic relations between these functions, which are known as trigonometric identities.

With these functions one can answer virtually all questions about arbitrary triangles by using the law of sines and the law of cosines. These laws can be used to compute the remaining angles and sides of any triangle as soon as two sides and an angle or two angles and a side or three sides are known. These laws are useful in all branches of geometry, since every polygon may be described as a finite combination of triangles.

Extending the definitions



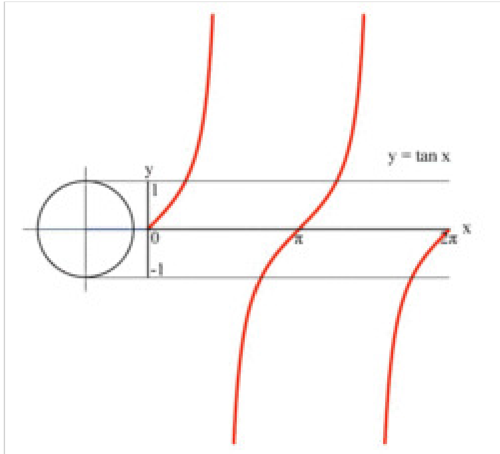
Graphs of the functions sin(x) and cos(x), where the angle x is measured in radians.



Graphing process of y = sin(x) using a unit circle.

http://cd3wd.com wikipedia-for-schools http://gutenberg.org page: 103 of 109.

The above definitions apply to angles between 0 and 90 degrees (0 and π/2 radians) only. Using the unit circle, one can extend them to all positive and negative arguments (see trigonometric function). The trigonometric functions are periodic, with a period of 360 degrees or 2π radians. That means their values repeat at those intervals.



Graphing process of $y = \tan(x)$ using a unit circle.

The trigonometric functions can be defined in other ways besides the geometrical definitions above, using tools from calculus and infinite series. With these definitions the trigonometric functions can be defined for complex numbers. The complex function **cis** is particularly useful

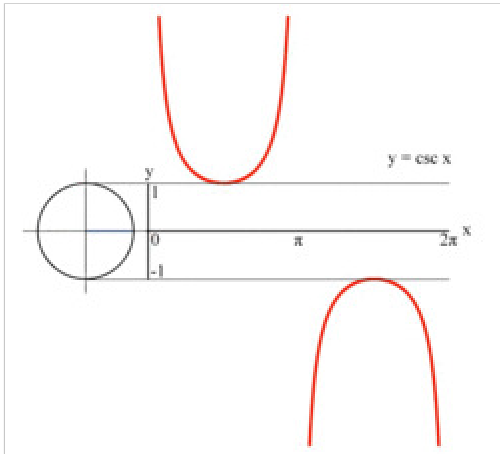
$$\operatorname{cis}(x) = \cos x + i \sin x = e^{ix}$$

See Euler's and De Moivre's formulas.

Mnemonics

Students often use mnemonics to remember facts and relationships in trigonometry. For example, the *sine*, *cosine*, and *tangent* ratios in a right triangle can be remembered by representing them as strings of letters, as in SOH-CAH-TOA.

Sine = **O**pposite ÷ **H**ypotenuse
Cosine = **A**djacent ÷ **H**ypotenuse
Tangent = **O**pposite ÷ **A**djacent



Graphing process of $y = \csc(x)$ using a unit circle.

Alternatively, one can devise sentences which consist of words beginning with the letters to be remembered. For example, to recall that Tan = Opposite/Adjacent, the letters T-O-A must be remembered. Any memorable phrase constructed of words beginning with the letters T-O-A will serve.

Another type of mnemonic describes facts in a simple, memorable way, such as "Plus to the right, minus to the left; positive height, negative depth," which refers to trigonometric functions generated by a revolving line.

Calculating trigonometric functions

Trigonometric functions were among the earliest uses for mathematical tables. Such tables were incorporated into mathematics textbooks and students were taught to look up values and how to interpolate between the values listed to get higher accuracy. Slide rules had special scales for trigonometric functions.

Today scientific calculators have buttons for calculating the main trigonometric functions (sin, cos, tan and sometimes cis) and their inverses. Most allow a choice of angle measurement methods, degrees, radians and, sometimes, Grad. Most computer programming languages provide function libraries that include the trigonometric functions. The floating point unit hardware incorporated into the microprocessor chips used in most personal computers have built in instructions for calculating trigonometric functions.

Applications of trigonometry

There are an enormous number of applications of trigonometry and trigonometric functions. For instance, the technique of triangulation is used in astronomy to measure the distance to nearby stars, in geography to measure distances between landmarks, and in satellite navigation systems. The sine and cosine functions are fundamental to the theory of periodic functions such as those that describe sound and light waves.

Fields which make use of trigonometry or trigonometric functions include astronomy (especially, for locating the apparent positions of celestial objects, in which spherical trigonometry is essential) and hence navigation (on the oceans, in aircraft, and in space), music theory, acoustics, optics, analysis of financial markets, electronics, probability theory, statistics, biology, medical imaging (CAT scans and ultrasound), pharmacy, chemistry, number theory (and hence cryptology), seismology, meteorology, oceanography, many physical sciences, land surveying and geodesy, architecture, phonetics, economics, electrical engineering, mechanical engineering, civil engineering, computer graphics, cartography, crystallography and game development.

Common formulae

Marine sextants like this are used to measure the angle of the sun or stars with respect to the horizon. Using trigonometry and a marine chronometer, the position of the ship can then be determined from several such measurements.

Certain equations involving trigonometric functions are true for all angles and are known as *trigonometric identities.* Many express important geometric relationships. For example, the Pythagorean identities are an expression of the Pythagorean Theorem. Here are some of the more commonly used identities, as well as the most important formulae connecting angles and sides of an arbitrary triangle. For more identities see trigonometric identity.

Trigonometric identities

Pythagorean identities

$$\sin^2 A + \cos^2 A = 1$$
$$\tan^2 A + 1 = \sec^2 A$$
$$1 + \cot^2 A = \csc^2 A$$

Trigonometry

Sum and product identities

Sum to product:

$$\sin A \pm \sin B = 2\sin\left(\frac{A \pm B}{2}\right)\cos\left(\frac{A \mp B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)$$

$$\cos A - \cos B = -2\sin\left(\frac{A + B}{2}\right)\sin\left(\frac{A - B}{2}\right)$$

Product to sum:

$$\cos A\ \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$\sin A\ \sin B = -\frac{1}{2}[\cos(A+B) - \cos(A-B)]$$

$$\cos A\ \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$$

$$\sin A\ \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$$

Sine, cosine, and tangent of a sum

Detailed, diagramed proofs of the first two of these formulas are available for download as a four-page PDF document at Image:Sine Cos Proofs.pdf.

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double-angle identities

$$\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\cot A}{\cot^2 A - 1} = \frac{2}{\cot A - \tan A}$$

Half-angle identities

Note that $\pm$ is correct, it means it may be either one, depending on the value of *A/2*.

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$$

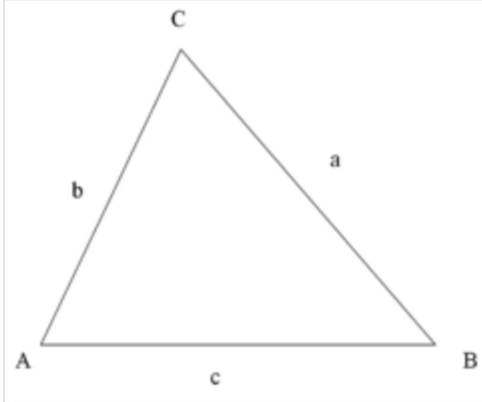
Stereographic (or parametric) identities

$$\sin A = \frac{2T}{1+T^2}$$
$$\cos A = \frac{1-T^2}{1+T^2}$$

where $T = \tan \frac{A}{2}$.

Triangle identities

In the following identities, *A*, *B* and *C* are the angles of a triangle and *a*, *b* and *c* are the lengths of sides of the triangle opposite the respective angles.



Laws of Sines and Cosines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

Law of sines

The **law of sines** (also know as the "sine rule") for an arbitrary triangle states:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

where *R* is the radius of the circumcircle of the triangle.

Law of cosines

The **law of cosines** (also known as the cosine formula, or the "cos rule") is an extension of the Pythagorean theorem to arbitrary triangles:

$$c^2 = a^2 + b^2 - 2ab\cos C,$$

or equivalently:

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

Law of tangents

The **law of tangents**:

$$\frac{a+b}{a-b} = \frac{\tan\left[\frac{1}{2}(A+B)\right]}{\tan\left[\frac{1}{2}(A-B)\right]}$$

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