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ABOUT DESIGN OPTIMIZATION OF CROSS-FLOW HIDRAULIC TURBINES

M. BĂRGLĂZAN
"Politehnica" University from Timişoara,
B-ul Mihai Viteazul nr.1 Timişoara, România

Abstract: *In the present paper there are established the optima values for some geometric parameters of the cross-flow hydraulic turbine. This research is focused on the main machine elements of the turbine, namely, the radial runner and the supply nozzle.*
Also an automated design of the cross-flow hydraulic turbines is obtained through dedicated soft (programs) in which some parameters could be chosen and a lot of other parameters are calculated.

Key wo0rds*: hydraulic turbines, cross-flow turbines, Banki turbines, design and operation optimization.*

1. Nomenclature.

$\vec{u}$ –tangential velocity
$\vec{v}$ –absolute velocity
$\vec{w}$ –relative velocity
ά –angle between $\vec{u}$ and $\vec{v}$
β -angle between $-\vec{u}$ and $\vec{w}$
b –hydrodynamic runner width
s –distance between two neighbor runner blades
g –gravitational acceleration
H_T – hydraulic turbine's head
$ή_h$ – hydraulic efficiency
k_{v1} – absolute velocity entrance coefficient
v_u - absolute velocity's projection on tangential velocity
k –nozzle exit transversal extension coefficient s_0= k D_1

2. Introduction

Nowadays between sustainable development of renewable sources of energy the hydro-energy plays an important and crucial role. Because usually the great hydropower plants are already built it remains to harness the medium and small hydropower sources. Among the different types of hydraulic turbines, the cross-flow type is a real option and viable solution from technical – economical point of view.

The cross-flow hydraulic turbines (CFHT) consist from two main hydrodynamic machine elements namely the runner and the nozzle. The CFHT is in almost all regimes of operation an impulse wheel machine. CFHT has the peculiarity that, like the Pelton hydraulic turbine, there is no connection between the rate of flow of the hydraulic turbine and the rotational speed of the runner but the width of the runner is a function of the rate of flow through the CFHT. In operation the runner is only partially immersed in water, every inter-blade channel, twice in a complete rotation of the runner.
The runner has, an overall given geometry, the shape of a cylindrical annuls in which are inserted the blades. The nozzle – conducting the water to the runner – is a convergent pipe with rectangular cross section and with two plane parallel fixed walls and the other two walls in the shape of cylinder with evolute or spiral logarithmic generation curve (one of them sometimes mobile).

2. Optimization in respect of flow angle.

The design optimization consists of establishing the best geometrical shape of the machine with some geometric and hydrodynamic imposed restrictions.
So for CFHT there are the following restrictions at the runner: a) to be a cylindrical annulus, b) with meridional radial entrance and outlet and c) with double interaction between the water flow and the runner. The supply nozzle geometry must be adapted for these conditions together with the possibility to control the rate of flow through the machine. The ring-shaped runner fig.1 with velocities triangles of the flow like fig.2 introduces the following relations:

$$\left|\vec{u}_1\right| = \left|\vec{u}_4\right| \tag{1}$$

If, in the runner motion, the point 1 reaches the point 4 then $\vec{w}_1$ reaches $\vec{w}_4$ and the directions of these relatives velocities are the same but their orientation is opposite. So:

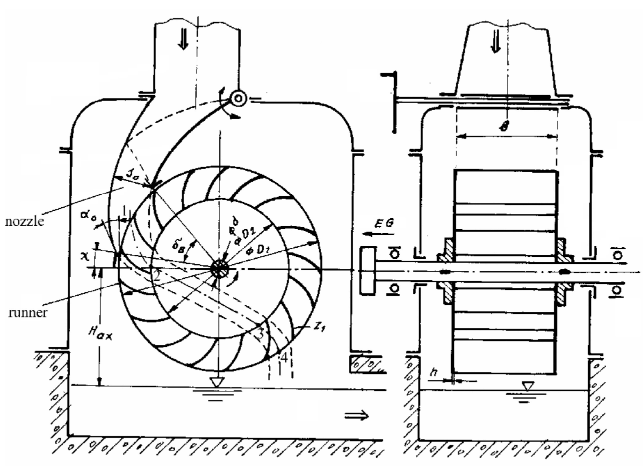

Fig.1. Cross- flow hydraulic turbine (CFHT)

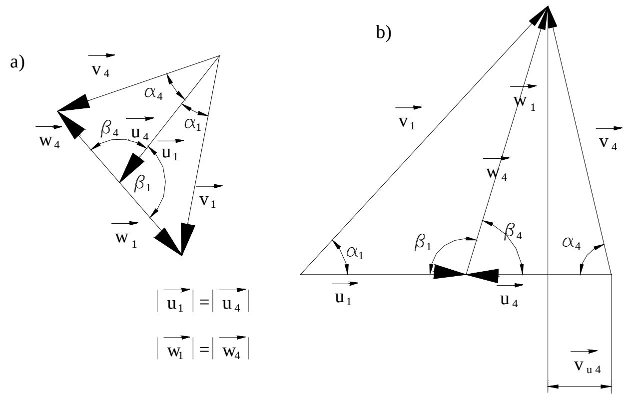

Fig.2. Velocities triangles in different points of the CFHT

$$\hat{\beta}_1 + \hat{\beta}_4 = \pi \tag{2}$$

Because it is a small distance between points 2 and 3 (fig.2), it can be accepted

$$\vec{v}_2 = \vec{v}_3 \tag{3}$$

and from the above mentioned conditions

$$|\vec{u}_2| = |\vec{u}_3| \tag{4}$$

$$\hat{\alpha}_2 = \hat{\alpha}_3 \tag{5}$$

$$\hat{\beta}_2 = \hat{\beta}_3 \tag{6}$$

For the same reasons, of relation (2), it is:

$$\hat{\beta}_2 + \hat{\beta}_3 = \pi \tag{7}$$

Combining relations (6) and (7) results:

$$\hat{\beta}_2 = \hat{\beta}_3 = \frac{\pi}{2} \tag{8}$$

Applying the equation of continuity between the sections 1 and 4:

$$b_1 \cdot w_1 \cdot s_1 = b_4 \cdot w_4 \cdot s_4 \tag{9}$$

But the imposed restrictions introduce:

$$b_1 = b_4 \tag{10}$$

$$s_1 = s_4 \tag{11}$$

And it results:

$$w_1 = w_4 \tag{12}$$

Using the fundamental equation of turbomachines (EFT) after [1]:

$$\eta_h \cdot g \cdot H_T = u_1 \cdot v_{u1} - u_4 \cdot v_{u4} = u_1 \cdot (v_{u1} - v_{u4}) \tag{13}$$

The entrance velocity of the flow in the runner may be expressed through similitude with:

$$v_1 = k_{v1} \cdot \sqrt{2 \cdot g \cdot H_T} \tag{14}$$

In it velocity coefficient is $k_{v1} = 0{,}96...0{,}98$ for a hydrodynamic shaped nozzle and with the turbine head, H_T, corresponding to the entrance section 1.

From the velocity triangle, fig.2, results:

$$v_{u4} = 2 \cdot u_1 - v_{u1} \tag{15}$$

Introducing relation (14) in (15) it is obtained:

$$\eta_h = 4 \cdot k_{v1}^2 \cdot \frac{u_1}{v_1} \cdot \left(\cos\alpha_1 - \frac{u_1}{v_1} \right) \tag{16}$$

The optimum efficiency in respect of the velocities ratio is:

$$\frac{\partial \eta_h}{\partial \left(\frac{u_1}{v_1} \right)} = 0 \tag{17}$$

namely, $u_1 = \dfrac{v_1 \cdot \cos\alpha_1}{2}$ (18)

and because

$$\frac{\partial^2 \eta_h}{\partial \left(\frac{u_1}{v_1} \right)^2} = -8 \cdot k_{v1}^2 < 0 \tag{19}$$

this optimum is a maximum. The maximum efficiency is

$$\eta_{h\,max} = k_{v1}^2 \cdot \cos^2 \alpha_1 \tag{20}$$

Discussion:

1) For inviscid flow $k_{v1} = 1$ and for tangential entrance $\alpha_1 = 0$. It results the maximum efficiency $\eta_{h\,max} = 1$. These theoretical approaches are impossible because the real flow in the nozzle is with hydraulic losses and the tangential entrance implies zero rate of flow through the runner.
2) The real angles are $\alpha_1 = 10...20^o$ depending on specific speed and it is associated with central angles extension of the nozzle exit reflected at the CFHT axis. After [1] and [2] $\delta_B = 30...100^o$. Using the formula:

$$\delta_B = 114{,}59115 \cdot \frac{k}{\sin\alpha_1} \tag{21}$$

The table 1 summarize some results of δ_B for the usual values of α_1 and k.

Table 1

α_1 \ k	0,1	0,15
10^o	65^o,99064901	98^o,98597352
15^o	44^o,27477854	66^o,41216781
20^o	33^o,50433047	50^o,25649571

In literature the values mostly met are $\alpha_1 = 16^o$, k = 0,1 and it results $\delta_B \cong 42^o$.

3) The formula (18) is a generalization of the same kind of formula obtained through optimization of the Pelton type hydraulic turbine. It is interesting to remark that formula (18) gives every time:

$$u_1 \le 0{,}5 \cdot v_1 \tag{22}$$

but in the literature you will find recommended values like:

$$u_1 = 0{,}481 \cdot v_1 \tag{23}$$

in [2] and

$$u_1 = 0{,}518 \cdot v_1 \tag{24}$$

in [3], in conclusion, smaller and greater than half of the absolute velocity.

3. Optimization in respect of the blade angle.

The problem is posed in respect of the entrance blade angle of the runner and the rotational speed of the runner. From relations (13) and (14) it is obtained:

$$\eta_h \cdot \frac{v_1^2}{2 \cdot k_{v1}^2} = 2 \cdot u_1 \cdot (v_1 - u_1) \tag{25}$$

From the velocity triangle, fig 2:

$$v_{u1} - u_1 = w_1 \cdot \cos\beta_4 = -w_1 \cdot \cos\beta_1 \tag{26}$$

and that:

$$v_1^2 = u_1^2 + w_1^2 - 2 \cdot u_1 \cdot w_1 \cdot \cos\beta_1 \tag{27}$$

The optimum is:

$$\frac{\partial \eta_h}{\partial u_1} = -4 \cdot k_{v1}^2 \cdot \frac{w_1 \cdot (w_1^2 - u_1^2) \cdot \cos\beta_1}{(u_1^2 + w_1^2 - 2 \cdot u_1 \cdot w_1 \cdot \cos\beta_1)^2} = 0 \tag{28}$$

The technically acceptable solution is:

$$w_1 = u_1 \tag{29}$$

Which gives:

$$\eta_{h\,max} = \frac{2 \cdot k_{v1}^2 \cdot \cos\beta_1}{\cos\beta_1 - 1} \tag{30}$$

Further optimization in respect of the angle β_1 offers:

$$\frac{\partial \eta_{h\,max}}{\partial \beta_1} = \frac{2 \cdot k_{v1}^2 \cdot \sin\beta_1}{(\cos\beta_1 - 1)^2} = 0 \tag{31}$$

With the solutions:

$$\sin\beta_1 = 0 \qquad \beta_1 = 0, \pi, 2\cdot\pi, \ldots \tag{32}$$

Which produce maximum only for uneven of π because:

$$\frac{\partial^2 \eta_{h\,max}}{\partial \beta_1^2} = -2 \cdot k_{v1}^2 \cdot \frac{2 + \cos\beta_1}{(\cos\beta_1 - 1)^2} \tag{33}$$

and

$$\left.\frac{\partial^2 \eta_{h\,max}}{\partial \beta_1^2}\right|_{\beta_1 = \pi} = -\frac{k_{v1}^2}{2} < 0 \tag{34}$$

so that $\left.\eta_{h\,max\,max}\right|_{\beta_1 = \pi} = k_{v1}^2$ (35)

Discussion:

1) Once again $\beta_1 = \pi$ is technically impossible but it is an asymptotical goal.
2) Also the radial flow at the outlet from the runner, fig.3, introduces the relation:

$$u_4 = w_4 \cdot \cos\beta_4 \tag{36}$$

And in accord with fig.3:

$$tg\,\beta_4 = 2 \cdot tg\,\alpha_1 = -tg\,\beta_1 \tag{37}$$

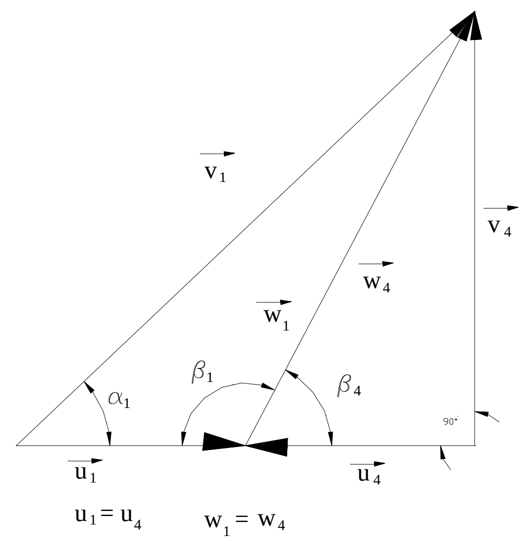

Fig. 3. Velocities triangles in different points of the CFHT

3) The connection between the angles α_1 and β_1 are not a consequence of relation like (18) as it is argued in reference [2], but it is obtained with the condition of radial centrifugal direction at the flow exit (see, point 4)).
4) Knowing from the shock-less second entrance of the flow in the runner that $\beta_2 = \beta_3 = 90^o$ and that $\beta_1 \to 180^o$ it results blades generated from a cylindrical surfaces with a quite great geometrical angle of deviation. These means that the energy transfer could not be obtained through hydrodynamic lift but may be explained through momentum equation. In consequence it is necessary to have a lot of blades possibly of constant thickness at not hydrodynamic shaped profiles to realize more inter-blades channels which direct the flow.

In accord with relation (37) the values $\alpha_1(\beta_1)$ are exposed in table 2.

Table 2

α_1 [°]	10	15	16	17	20
β_1 [°]	19,425	28,186	29,834	31,444	36,052

5. Designing programs

Based on optimization criteria established in the above chapters it was realized some soft (programs) for a personal computers.

Program CFHT-1 is an introductory soft which offers in function of the entrances data the range and variants of cross-flow hydraulic main parameters. The program offers some possibilities to fit the initial data with the requirements of the hydro-unit and hydro-power-plant. So, for a given turbine head H_T, and stereo-mechanical power P_s, it are calculated all the variants of synchronous speeds of the hydro-unit which are in the range of CFHT, namely, with specific speeds between 50 and 150. Example are presented in table 3.

Table 3

H_T <m>	Q<m^3/s>	P_s<kW>	n <rot/min>	n_s <rot/min>
8	0,12746	8	600	147,1
			500	122,6
			428,5	105,1
			375	91,9
			333,3	81,7
			300	73,5
			272,7	66,9
			250	61,2
			230,7	56,6
			214,2	52,5

With initial data turbine head H_t = 8 m and rate of flow Q =0,1 m^3/s the above studied case gives the results from table 4.

Table 4

H_T<m>	Q <m^3/s>	P_s <kW>	n <rot/min>	n_s <rot/min>
8	0,1	6,27616	600	130,3
			500	108,6
			428,5	93,1
			375	81,4
			333,3	72,4
			300	65,1
			272,7	59,2
			250	54,3
			230	50,1

For a hydro-unit with speed amplifier between the hydraulic turbine and the electric generator, the imposed non synchronous speed of rotation gives the following results presented in table 5.

Table 5

H_T <m>	Q <m^3/s>	P_s <kW>	n <rot/min>	n_s <rot/min>	Initial Data
8	0,102	8	400	98,065	H_T, P_s, n
8	0,1	6,276	400	86,859	H_T, Q, n

With the established main parameters from the program CFHT – 1 it is possible to proceed to the CFHT -2 program. This program makes complete design calculus of the turbine runner, nozzle and the whole hydraulic machine in different variants, following the steps described in [1]. A special attention is given to the influence of runner blades thickness, nozzle geometry, nozzle position around the runner, angle of access of the flow in the runner and operation regimes with pure double or single interaction or a mixed one for the flow. Some results for a specific hydro-power plant are summarized in table 6.

The program CFHT -3 gives the geometry of the entrance nozzle in the variants with spiral logarithmic or evolute walls both prolonged with circular arc walls.

6. Conclusions

1. The CFHT optimization in respect of rotational speed, flow angle by entrance and blades angles by entrance in the runner is possible.
2. The theoretical obtained values are only asymptotic values for the real engineering problem.
3. The one – dimensional model used here is quite good confirmed by measurements.
4. The programs and soft developed, for the best parameters and the design of the CFHT, are friendly user tool in the cross-flow hydraulic turbines development for hydro-energetic purposes.
5. The position of the nozzle around the external cylindrical surface of the runner (by the hydro-units with horizontal shaft) is accepted but it may be another optimization problem, not investigated here.

Table 6

Parameters	Values
Turbine head	H_T = 8 m
Rate of flow	Q = 0,1019732 m^3/s
Stereo-mechanical power	P_s = 6893, 238 w
Speed of rotation	n = 400 rot/min
Specific speed	n_s = 98,01181 rot/min
Access angle of the flow	$\alpha_1 = 15{,}17341^{o}$
External diameter	D_1 = 0,2828427 m
Internal diameter	D_2 = 0,1821843 m
Diameters ratio	D_2/D_1 = 0,6441188
Absolute velocity of the flow	v_1 = 12,27564 m/s
Tangential velocity in point 1	u_1 = 5,923844 m/s
Blade angle by entrance	$\beta_1 = 151{,}525^{o}$
Internal blade angle	$\beta_2 = \beta_3 = 90^{o}$
Outlet blade angle	$\beta_4 = 28{,}475^{o}$
Overall efficiency	η_T = 0,86165
Hydraulic efficiency	η_h = 0,8946044
Number of blades	z = 28
Width of the runner	b_r = 0,3136 m
Shaft diameter	D = 0,02756861 m
Nozzle opening	s_0 = 0,02828427 m
Blades circular radius	ρ_s = 0,0470676 m

Parameters	Values
Central angle of the blades	$\gamma = 76{,}20832^o$
Disposal radius of the centers of the blades	$\rho_1 = 0{,}1881661 m$
Width of the nozzle	b = 0,2936952 m
Generating radius of the evolute	$\rho_e = 0{,}03701582 m$
Runner central angle of the nozzle exit	$\delta_B = 43{,}78045^o$
Number of blades covered by nozzle exit	$z_B = 3{,}405146$
Angular position of the upper nozzle edge	$\varphi_{inf} = 15{,}07585^o$
Angular position of the lower nozzle edge	$\varphi_{sup} = 58{,}8563^o$
Rotor head	$H_r = 7{,}156835$ m
Rotor head of the first interaction	$H_{r\,1} = 5{,}672189$ m
Rotor head of the second interaction	$H_{r\,2} = 1{,}484646$ m
Turbine axis position above the water level	$h_{ax} = 0{,}2414214$ m

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Prof.dr.ing. Mircea POPOVICIU

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