

Communications B : Filters and Transmission Lines

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C O N N E X I O N S

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Chapter 1

Transmission Lines

1.1 Distributed Parameters¹

Having learned something about how we generate signals with bipolar and field effect transistors, we now turn our attention to the problem of getting those signals from one place to the next. Ever since Samuel Morse (and the founder of **my alma mater**, Ezra Cornell) demonstrated the first working telegraph, engineers and scientists have been working on the problem of describing and predicting how electrical signals behave as they travel down specific structures called **transmission lines**.

Any electrical structure which carries a signal from one point to another can be considered a transmission line. Be it a long-haul coaxial cable used in the Internet, a twisted pair in a building as part of a local-area network, a cable connecting a PC to a printer, a bus layout on a motherboard, or a metallization layer on an integrated circuit, the fundamental behavior of all of these structures are described by the same basic equations. As computer switching speeds run into the 100s of MHz, into the GHz range, considerations of transmission line behavior are ever more critical, and become a more dominant force in the performance limitations of any system.

For our initial purposes, we will introduce a "generic" transmission line Figure 1.1 ("Generic" Transmission Line), which will incorporate most (but not all) features of real transmission lines. We will then make some rather broad simplifications, which, while rendering our results less applicable to real-life situations, nevertheless **greatly** simplify the solutions, and lead us to insights that we can indeed apply to a broad range of situations.

¹This content is available online at <<http://cnx.org/content/m11373/1.1/>>.

"Generic" Transmission Line

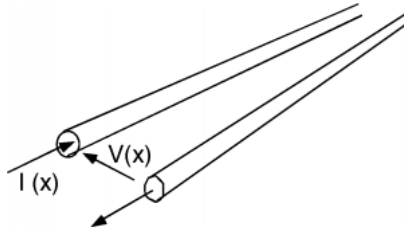


Figure 1.1

The generic line consists of two conductors. We will suppose a potential difference $V(x)$ exists between the two conductors, and that a current $I(x)$ flows down one conductor, and returns via the other. For the time being, we will let the transmission line be "semi-infinite", which means we have access to the line at some point x , but the line then extends out in the x direction to infinity. (Such lines are a bit difficult to handle in the lab!)

In order to be able to describe how $V(x)$ and $I(x)$ behave on this line, we have to make some kind of **model** of the electrical characteristics of the line itself. We can not just make up any model we want however; we have to base the model on physical realities.

Let's start out by just considering one of the conductors and the physical effects of current flowing through that conductor. We know from freshman physics that a current flowing in a wire gives rise to a magnetic field, H (Figure 1.2 (Build Up of Magnetic Field)). Multiply H by μ and you get B , the magnetic flux density, and then integrate B over a plane parallel to the wires and you get Φ , the magnetic flux "linking" the circuit. This is shown in Figure 1.3 (Find the Flux Linkage) for at least part of the surface. The definition of L , the inductance of a circuit element, is just

$$L \equiv \frac{\Phi}{I} \quad (1.1)$$

where Φ is the flux linking the circuit element, and I is the current flowing through it. Our only problem in finding Φ is that the longer a section of wire we take, the more Φ we have for the same I . Thus, we will introduce the concept of a distributed parameter.

Definition 1.1: distributed parameter

A distributed parameter is a parameter which is spread throughout a structure and is not confined to a lumped element such as a coil of wire.

Example

For instance, we will hereby define \mathbf{L} as the **distributed inductance** for the transmission line. It has units of Henrys/meter. If we have a length of transmission line x_0 meters long, and if that line has a distributed inductance of \mathbf{L} H/m, then the inductance L of that length of line is just $L = \mathbf{L}x_0$.

Build Up of Magnetic Field

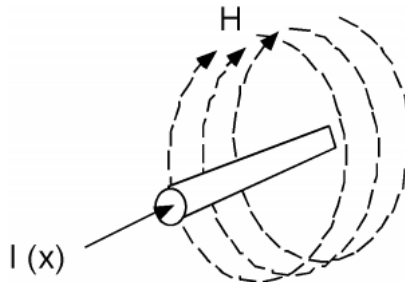
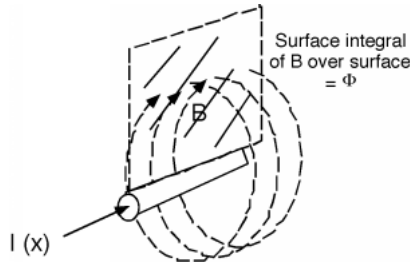
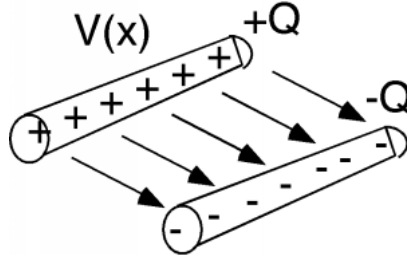


Figure 1.2

Likewise, if we have two conductors separated by some distance, and if there is a potential difference V between the conductors, then there must be some charge $\pm(Q)$ on the two conductors which gives rise to that potential difference. We can imagine a linear charge distribution on the transmission line, ρ (C/m), where we have ρ Coulombs/m on one conductor, and $-\rho$ Coulombs/m on the other conductor. For a line of length x_0 , we would have $Q = \pm(\rho x_0)$ on each section of wire. Whenever you have two charged conductors with a voltage difference between them, you can describe the ratio of the charge to the voltage as a capacitance. The two conductors would have a capacitance

$$\begin{aligned} C &= \frac{Q}{V} \\ &= \frac{\rho x_0}{V} \end{aligned} \quad (1.2)$$

and a distributed capacitance C (F/m) which is just $\frac{\rho}{V}$. A length of line x_0 long would have a capacitance $C = Cx_0$ Farads associated with it Figure 1.4 (Line Capacitance).

Find the Flux Linkage**Figure 1.3****Line Capacitance****Figure 1.4**

Thus, we see that the transmission line has both a distributed inductance \mathbf{L} and a distributed capacitance \mathbf{C} which are tied up with each other. There is really no way in which we can separate one from the other. In other words, we can not have only the capacitance, or only the inductance, there will always be some of each associated with each section of line now matter how small or how big we make it.

We are now ready to build our model. What we want to do is to come up with some arrangement of inductors and capacitors which will represent electrically, the properties of the distributed capacitance and inductance we discussed above. As a length of line gets longer, its capacitance increases, so we had better put the distributed capacitances in parallel with one another, since that is the way capacitors add up. Also, as the line gets longer, its total inductance increases,

so we had better put the distributed inductances in series with one another, for that is the way inductances add up. Figure 1.5 (Distributed Parameter Model) is a representation of the distributed inductance and capacitance of the generic transmission line.

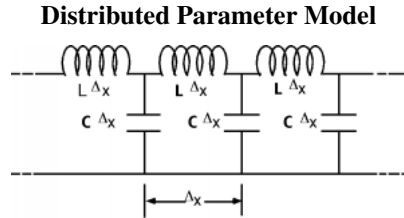


Figure 1.5

We break the line up into sections $\Delta(x)$ long, each one with an inductance $L\Delta(x)$ and a capacitance $C\Delta(x)$. If we halve $\Delta(x)$, we would halve the inductance and capacitance of each section, but we'd have twice as many of them per unit length. Duh! The point is no matter how fine we make $C\Delta(x)$, we still have L s and C s arranged like we see in Figure 1.5 (Distributed Parameter Model), with the two kinds of components intermixed.

We **could** make a more realistic model and realize that all real wires have series resistance associated with them and that whatever we use to keep the two conductors separated will have some leakage conductance associated with it. To account for this we would introduce a series resistance \mathbf{R} (ohms/unit length) and a series conductance \mathbf{G} (ohms/unit length). One section of our line model then looks like Figure 1.6 (Complete Distributed Model).

Complete Distributed Model

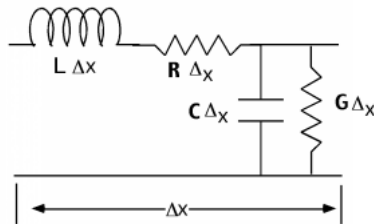


Figure 1.6

Although this is a more realistic model, it leads to much more complicated math. We will start out anyway, ignoring the series resistance \mathbf{R} and the shunt conductance \mathbf{G} . This "approximation" turns out to be pretty good as long as either the line is not too long, or the frequencies of the signals we are sending down the line do not get too high. Without the series resistance or parallel conductance we have what is called an ideal **lossless transmission line**.

1.2 Transmission Line Examples²

As an example, and also because it even has some practical importance, let's look at one kind of transmission line. It is called a **stripline** and it looks like Figure 1.7 (A Stripline). It consists of a flat conductor, located between two ground planes. It is supported by an insulating dielectric with dielectric constant ϵ . This is kind of like the situation you would find on a multi-level PC board, where perhaps the bus lines would be running on an inner layer with ground planes above and below them.

²This content is available online at <<http://cnx.org/content/m1046/2.10/>>.

A Stripline

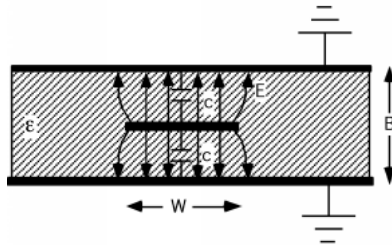


Figure 1.7

Between the center conductor and the ground plane, there will be some capacitance, C . If we can assume that the electric field is more or less confined to the regions between the strip conductor and the ground plane (which occurs when the ratio of $\frac{W}{B}$ is not too small) then for either capacitor (assuming unit length into the picture) we will get a value

$$C = \frac{\epsilon W}{\frac{B}{2}} \quad (1.3)$$

since the value of a capacitor is just the dielectric constant times the area of the plates, divided by the spacing of the plates.

Looking quickly at Figure 1.7 (A Stripline) you might think the two capacitors are in series, but you would be wrong! Note that each capacitor has one lead connected to the center conductor and the other lead connected to ground, and so the two capacitors are in fact, in parallel, and hence their capacitances add. Thus, for the capacitance per unit length for this line, we can write:

$$C = \frac{4\epsilon W}{B} \quad (1.4)$$

It can be shown (although we won't do it here) that for **any** transmission line where the electric and magnetic fields are perpendicular to one another (called **TEM** or **transverse electromagnetic**) the speed of propagation of the wave down the line is just

$$\begin{aligned} v_p &= \frac{c}{\sqrt{\frac{\epsilon}{\epsilon_0}}} \\ &= \frac{3 \times 10^8 \frac{m}{s}}{\sqrt{\epsilon_r}} \end{aligned} \quad (1.5)$$

Where ϵ_r is called the **relative dielectric constant** for the material. Well, we also know that

$$v_p = \frac{1}{\sqrt{LC}} \quad (1.6)$$

From which we can write

$$\begin{aligned} \mathbf{L} &= \frac{1}{v_p^2 \mathbf{C}} \\ &= \frac{B}{v_p^2 4\epsilon W} \end{aligned} \quad (1.7)$$

We can now insert this value for \mathbf{L} into the expression for Z_0 , the impedance of the line.

$$\begin{aligned} Z_0 &= \sqrt{\frac{\mathbf{L}}{\mathbf{C}}} \\ &= \sqrt{\frac{\frac{B}{v_p^2 4\epsilon W}}{\frac{4\epsilon W}{B}}} \\ &= \frac{B}{4\epsilon W v_p} \\ &= \frac{B}{4\epsilon W \frac{c}{\sqrt{\epsilon_r}}} \end{aligned} \quad (1.8)$$

And so, we have derived an equation for the impedance Z_0 of the line in terms of the dimensions W and B , the dielectric constant of the insulating material, ϵ , and c , the speed of light. How good is this expression, and in particular how good is our assumption that the electric field is all confined to the region under the conductor? Not so great actually Figure 1.8 (Exact and Approximate Impedance For a Stripline).

Exact and Approximate Impedance For a Stripline

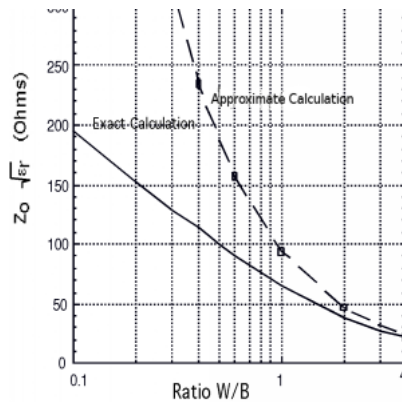


Figure 1.8: Exact and approximate Z_0 for a stripline

Figure 1.8 (Exact and Approximate Impedance For a Stripline) shows the results from using (1.8) and a more exact calculation, which takes into account the fringing fields. As you can see we have to get the ratio $\frac{W}{B}$ up to about 4 or so before the two match. But at least we get the right behavior and we're not totally out of the ball park.

1.3 Telegrapher's Equations³

Let's look at just one little section of the line, and define some voltages and currents Figure 1.9 (Applying Kirchoff's Laws).

³This content is available online at <<http://cnx.org/content/m11374/1.1/>>.

Applying Kirchoff's Laws

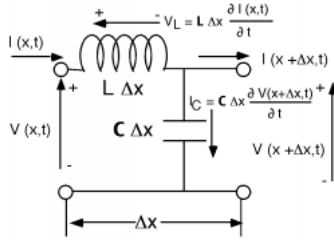


Figure 1.9

For the section of line $\Delta(x)$ long, the voltage at its input is just $V(x,t)$ and the voltage at the output is $V(x+\Delta(x),t)$. Likewise, we have a current $I(x,t)$ entering the section, and another current $I(x+\Delta(x),t)$ leaving the section of line. Note that both the voltage and the current are functions of **time** as well as position.

The voltage drop across the inductor is just:

$$V_L = \mathbf{L}\Delta(x) \frac{\partial I(x,t)}{\partial t} \quad (1.9)$$

Likewise, the current flowing down through the capacitor is

$$I_C = \mathbf{C}\Delta(x) \frac{\partial V(x+\Delta(x),t)}{\partial t} \quad (1.10)$$

Now we do a KVL⁴ around the outside of the section of line and we get

$$V(x,t) - V_L - V(x+\Delta(x),t) = 0 \quad (1.11)$$

Substituting (1.9) for V_L and taking it over to the RHS we have

$$V(x,t) - V(x+\Delta(x),t) = \mathbf{L}\Delta(x) \frac{\partial I(x,t)}{\partial t} \quad (1.12)$$

Let's multiply by -1, and bring the $\Delta(x)$ over to the left hand side.

$$\frac{V(x+\Delta(x),t) - V(x,t)}{\Delta(x)} = - \left(\mathbf{L} \frac{\partial I(x,t)}{\partial t} \right) \quad (1.13)$$

⁴"Electric Circuits and Interconnection Laws": Section Kirchoff's Voltage Law (KVL)
<http://cnx.org/content/m0014/latest/#voltage>

We take the limit as $\Delta(x) \rightarrow 0$ and the LHS becomes a derivative:

$$\frac{\partial V(x,t)}{\partial x} = - \left(\mathbf{L} \frac{\partial I(x,t)}{\partial t} \right) \quad (1.14)$$

Now we can do a KCL⁵ at the node where the inductor and capacitor come together.

$$I(x,t) - \mathbf{C}\Delta(x) \frac{\partial I(x+\Delta(x),t)}{\partial t} - V(x+\Delta(x),t) = 0 \quad (1.15)$$

And upon rearrangement:

$$\frac{I(x+\Delta(x),t) - I(x,t)}{\Delta(x)} = - \left(\mathbf{C} \frac{\partial V(x+\Delta(x),t)}{\partial t} \right) \quad (1.16)$$

Now when we let $\Delta(x) \rightarrow 0$, the left hand side again becomes a derivative, and on the right hand side, $V(x+\Delta(x),t) \rightarrow V(x,t)$, so we have:

$$\frac{\partial I(x,t)}{\partial x} = - \left(\mathbf{C} \frac{\partial V(x,t)}{\partial t} \right) \quad (1.17)$$

(1.14) and (1.17) are so important we will write them out again together:

$$\frac{\partial V(x,t)}{\partial x} = - \left(\mathbf{L} \frac{\partial I(x,t)}{\partial t} \right) \quad (1.18)$$

$$\frac{\partial I(x,t)}{\partial x} = - \left(\mathbf{C} \frac{\partial V(x,t)}{\partial t} \right) \quad (1.19)$$

These are called the **telegrapher's equations** and they are all we really need to derive how electrical signals behave as they move along on transmission lines. Note what they say. The first one says that at some point x along the line, the incremental voltage drop that we experience as we move down the line is just the distributed inductance \mathbf{L} times the time derivative of the current flowing in the line at that point. The second equation simply tells us that the loss of current as we go down the line is proportional to the distributed capacitance \mathbf{C} times the time rate of change of the voltage on the line. As you should be easily aware, what we have here are a pair of **coupled linear differential equations in time and position** for $V(x,t)$ and $I(x,t)$

⁵"Kirchhoff's Laws": Section Kirchhoff's Current Law
<http://cnx.org/content/m0015/latest/#current>

1.4 Telegrapher's Equation in Real Lines⁶

So far, the transmission lines we have looked at have been "ideal". That is they have been lossless and dispersionless. Lest you leave the course with a false idea of how things **really** work, we should go back to our model and try to get things adjusted just a bit.

As you can probably imagine, a real transmission line is going to have some series resistance, associated with the real losses in the copper wire. There may also be some shunt conductance, if the insulating material holding the two conductors has some leakage current. We will need to include these effects along with the distributed inductance and capacitance which we have already talked about. Fixing up the model accordingly, we now draw a section of line $\Delta(x)$ long as shown in Figure 1.10 (Real Line Diagram). Taking the same voltage loop and current sum that we did back in the discussion of transmission lines⁷, we come up with the following version of the **telegrapher's equations**.

$$\frac{\partial V(x,t)}{\partial x} = (-R)I(x,t) - L\frac{\partial I(x,t)}{\partial t} \quad (1.20)$$

and

$$\frac{\partial I(x,t)}{\partial x} = -\left(GV(x,t) + C\frac{\partial V(x,t)}{\partial t}\right) \quad (1.21)$$

Real Line Diagram

Image not finished

Figure 1.10: A model for a line with losses.

Clearly, we would like to simplify things if we can. Let's again make a sinusoidal time excitation assumption, and let $I(x,t)$ and $V(x,t)$ become phasors. Since the time variation is now represented by a simple $e^{j\omega L}$ the time derivatives become just $j\omega$. We have

$$\frac{\partial V(x)}{\partial x} = -((R + j\omega L)I(x)) \quad (1.22)$$

⁶This content is available online at <<http://cnx.org/content/m10095/2.4/>>.

⁷"Distributed Parameters" <<http://cnx.org/content/m1043/latest/>>

and

$$\frac{\partial I(x)}{\partial x} = -((G + j\omega C)V(x)) \quad (1.23)$$

The way to get a solution is, of course, just like we have always done. Take the derivative with respect to x of (1.22)

$$\frac{\partial^2 V(x)}{\partial x^2} = -\left((R + j\omega L)\frac{\partial I(x)}{\partial x}\right) \quad (1.24)$$

and then plug in (1.23)

$$\frac{\partial^2 V(x)}{\partial x^2} = (R + j\omega L)(G + j\omega C)V(x) \quad (1.25)$$

The obvious solution to this (See how easy this gets after you've done it once or twice) is

$$V(x) = V_0 e^{\pm(\gamma x)} \quad (1.26)$$

with

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \quad (1.27)$$

This number, γ is called **the complex propagation constant**. Obviously, in general, it will have both a real and an imaginary part:

$$\gamma = \alpha + j\beta \quad (1.28)$$

and we have

$$V(x) = V_0 e^{\pm((\alpha + j\beta)x)} \quad (1.29)$$

Let's choose the minus sign in the exponent, and write the two terms as a product.

$$V(x) = V_0 e^{-(\alpha x)} e^{-(j\beta x)} \quad (1.30)$$

We see we have something similar to what we had before, but with just a minor difference. The $e^{-(j\beta x)}$ term is the propagating term which tells us how the phase angle of the phasor changes as we move along the line, and acts just like the β term we had before. Thus

$$\beta = \frac{2\pi}{\lambda} \quad (1.31)$$

and

$$v_p = \frac{\omega}{\beta} \quad (1.32)$$

The α is called the **attenuation coefficient**, and obviously, the $e^{-(\alpha x)}$ term in (1.30) causes the amplitude of the wave to decrease as it moves down the line. Figure 1.11 (Wave Decay) is a sketch of what a wave would look like if it is both propagating down the transmission line and also being attenuated. In a distance $\frac{1}{\alpha}$ the amplitude of the propagating wave has fallen to e^{-1} of the value it had when it started.

Wave Decay

Image not finished

Figure 1.11: Sketch of a decaying wave on a transmission line.

Let's take the minus sign solution in (1.26) and substitute back into (1.22)

$$\begin{aligned}\frac{\partial V(x)}{\partial x} &= -\left(\gamma V_0 e^{-(\gamma x)}\right) \\ &= -((R + j\omega L)I(x))\end{aligned}\tag{1.33}$$

From which we get

$$\begin{aligned}I(x) &= \frac{\gamma}{R + j\omega L} V_0 e^{-(\gamma x)} \\ &= \frac{\sqrt{(R + j\omega L)(G + j\omega C)}}{R + j\omega L} V(x) \\ &= \sqrt{\frac{G + j\omega C}{R + j\omega L}} V(x)\end{aligned}\tag{1.34}$$

Thus we can say

$$V(x) = Z_0 I(x)\tag{1.35}$$

where

$$\begin{aligned}Z_0 &= \sqrt{\frac{R + j\omega L}{G + j\omega C}} \\ &= R_0 + jX_0\end{aligned}\tag{1.36}$$

In general, in order to find α , β , R_0 , and X_0 , we would have to find the square root given in (1.27) and (1.36) for specific values of R , L , G , and C . On the other hand, we could maybe come up with some reasonable approximations which might suffice for cases of real interest. Obviously, if a line is very lossy,

we would not be very interested in using it, and so except in some very special cases where an extremely lossy line is unavoidable (usually having to do with signals at very high frequencies) we might see if we can find a **low loss** approximation.

1.5 A/C Line Behavior⁸

If we are going to try to use phasors on a transmission line, then we have to allow for spatial variation as well. This is simple to do, if we just let the phasor be a function of x , so we have $\tilde{V}(x)$. How the phasor varies in x is one of the things we now have to find out.

Let's start with the **Telegrapher's Equations** again.

$$\frac{\partial V(x,t)}{\partial x} = (-\mathbf{L}) \frac{\partial I(x,t)}{\partial x} \quad (1.37)$$

$$\frac{\partial I(x,t)}{\partial x} = (-\mathbf{C}) \frac{\partial V(x,t)}{\partial x} \quad (1.38)$$

For $V(x,t)$ we can now substitute $\tilde{V}(x)e^{j\omega t}$ and for $I(x,t)$ we plug in $\tilde{I}(x)e^{j\omega t}$. So we get:

$$\frac{\partial (\tilde{V}(x)e^{j\omega t})}{\partial x} = -\frac{\partial e^{j\omega t}}{\partial t} \quad (1.39)$$

and

$$\frac{\partial (\tilde{I}(x)e^{j\omega t})}{\partial x} = -\frac{\partial e^{j\omega t}}{\partial t} \quad (1.40)$$

We take the derivative with respect to time, which brings down a $j\omega$ and then we cancel the $e^{j\omega t}$ from both sides of each equation:

$$\frac{\partial \tilde{V}(x)}{\partial x} = -\left(j\omega\mathbf{L}\tilde{I}(x)\right) \quad (1.41)$$

and

$$\frac{\partial \tilde{I}(x)}{\partial x} = -\left(j\omega\mathbf{C}\tilde{V}(x)\right) \quad (1.42)$$

Viola! In one simple motion, we have completely eliminated the time variable, t , from our equations! It is not really gone, of course, for once we figure out what $\tilde{V}(x)$ is, we have to multiply it by $e^{j\omega t}$ and then take the real part before

⁸This content is available online at <<http://cnx.org/content/m11382/1.2/>>.

we can extract once again, the actual $V(x, t)$ that we want. Nonetheless, insofar as the telegrapher's equations are concerned, t has disappeared from the radar screen.

To solve these we do just as we did with the transient problem. We take a derivative with respect to x of (1.41), which gives us a $\frac{\partial \tilde{I}(x)}{\partial x}$ on the right hand side, for which we can substitute (1.42), which leaves us with

$$\frac{\partial^2 \tilde{V}(x)}{\partial x^2} = -\tilde{V}(x) \quad (1.43)$$

(- times - is +, but $jj = -1$ and so we have a - in front of the ω^2). We then re-write (1.43) as

$$\frac{\partial^2 \tilde{V}(x)}{\partial x^2} + \omega^2 \mathbf{LC} \tilde{V}(x) = 0 \quad (1.44)$$

The simplest solution to this equation is

$$\tilde{V}(x) = V_0 e^{\pm(j\omega\sqrt{\mathbf{LC}}x)} \quad (1.45)$$

from which we can then get the actual voltage signal

$$\begin{aligned} V(x, t) &= \tilde{V}(x) e^{j\omega t} \\ &= V_0 e^{j(\omega t \pm \omega\sqrt{\mathbf{LC}}x)} \end{aligned} \quad (1.46)$$

Note that we could factor out an $e^{j\omega\sqrt{\mathbf{LC}}}$, from the exponent, which, since it is just a constant, we could include in V_0 (and call it V_0' , switch the order of x and t , and write (1.46) as

$$V(x, t) = V_0' e^{j\left(x \pm \frac{1}{\sqrt{\mathbf{LC}}}t\right)} \quad (1.47)$$

which looks a lot like the "general" $f(x \pm vt)$ solution we were talking about earlier⁹!

The number $\omega\sqrt{\mathbf{LC}}$ is special. It is usually represented with a Greek letter β and is called the **propagation coefficient**. Thus we have

$$V(x, t) = V_0 e^{j(\omega t \pm \beta x)} \quad (1.48)$$

As previously, a point on the wave of constant phase requires that the argument inside the parenthesis remains constant. Thus if $V(x_1, t_1)$ is going to equal $V(x_2, t_2)$ (i.e. what was at point x_1 at t_1 is now at x_2 at time t_2 it must be that

$$\omega t_1 \pm \beta x_1 = \omega t_2 \pm \beta x_2 \quad (1.49)$$

⁹"Transmission Line Equation", (6) <<http://cnx.org/content/m11375/latest/#eqn6>>

or

$$\frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta(x)}{\Delta(t)} = \pm \left(\frac{\omega}{\beta} \right) = \pm \left(\frac{\omega}{\omega \sqrt{LC}} \right) = \pm \left(\frac{1}{\sqrt{LC}} \right) \equiv v_p \quad (1.50)$$

Which one again, defines the **phase velocity** of the wave. Other relationships to keep in mind are

$$\beta = \frac{2\pi}{\lambda} \quad (1.51)$$

$$\begin{aligned} \lambda &= \frac{v_p}{f} \\ &= \frac{\omega}{\beta} \\ &= \frac{2\pi}{\beta} \end{aligned} \quad (1.52)$$

The first comes from the fact that the wave varies in x as $e^{j\beta x}$. Thus when $x = \gamma$, the wavelength, $\beta\gamma$ just increases by 2π , to get the phasor to go through one full rotation. Note also, as before, the choice of the minus sign in the \pm in (1.48) represents a wave going in the x direction, while the choice of the $+$ sign will give a wave going in the $-x$ direction. Clearly, by starting out taking the x -derivative of the equation for $I(x, t)$ we would end up with

$$I(x, t) = I_0 e^{j(\omega t \pm \beta x)} \quad (1.53)$$

Let's consider the two phasors then, and define the voltage phasor associated with the positive going voltage wave as

$$\tilde{V}_{\text{plus}}(x) = V^+ e^{-(j\beta x)} \quad (1.54)$$

and the negative voltage phasor as

$$\tilde{V}_{\text{minus}}(x) = V^- e^{j\beta x} \quad (1.55)$$

We should keep in mind that both V^+ and V^- can be, and probably are, complex numbers. (From now on we will drop the little \sim over the variables because its very tedious to get it to show up with this word processor. You will just have to keep in mind that any variable we do not explicitly put inside absolute value markers (*i.e.* $|V^+|$) is going to be, in general, a complex number). We will, of course, have similar expressions for the positive and negative going current waves.

Let's consider the positive going current and voltage waves, and plug them into (1.41).

$$\frac{\partial \left(V^+ e^{-(j\beta x)} \right)}{\partial x} = - \left(j\omega L I^+ e^{-(j\beta x)} \right) \quad (1.56)$$

The x-derivative brings down a $-(j\beta)$, the $e^{-(j\beta x)}$'s cancel, and we have

$$V^+ = \frac{j\omega L}{j\beta} I^+ \quad (1.57)$$

But, since $\beta = \omega\sqrt{LC}$ we have

$$V^+ = \sqrt{\frac{L}{C}} I^+ \equiv Z_0 I^+ \quad (1.58)$$

as we had before.

So, what has changed? Not much from the case of transients on a line. We will now assume we have a **steady state** problem. This means we turned on the generator a long time ago. We assume that it has been connected to the line long enough so that all transient behavior has died away, and that voltages and currents are not changing any more (except oscillating at frequency ω , of course).

If the line is semi-infinite (or matched with a load equal to Z_0) Figure 1.12 (A Wave On a Semi-Infinite Line) then it is pretty obvious that

$$V^+ = \frac{Z_0}{Z_0 + Z_g} V_g \quad (1.59)$$

where Z_g is the source impedance, and V_g is the source voltage phasor.

A Wave On a Semi-Infinite Line

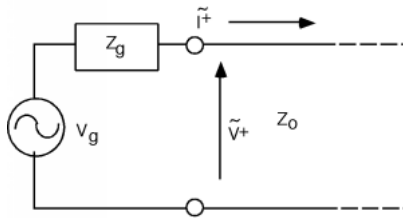


Figure 1.12

1.6 Terminated Lines¹⁰

If all we did was launch signals down semi-infinite transmission lines, we would not get very much useful work done. We really need to have a finite length line,

¹⁰This content is available online at <<http://cnx.org/content/m11378/1.2/>>.

and put something at the end...like a **termination**. So let's take a look at a terminated transmission line Figure 1.13 (At the Load End of the Line).

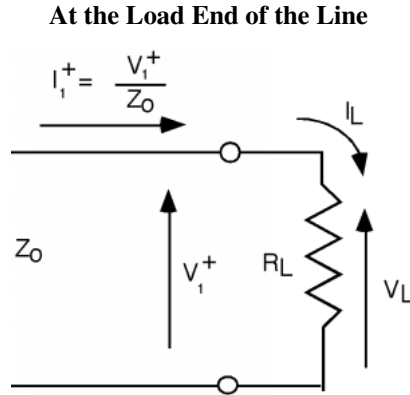


Figure 1.13

The line has characteristic impedance Z_0 and we assume that it is terminated with a load resistor R_L . If we have connected a source to the other end of the line, then we will have launched a voltage wave V_1^+ and a current wave I_1^+ down the line. If the line is L long, then it will take a time $\tau = \frac{L}{v_p}$, where $v_p = \frac{1}{\sqrt{LC}}$, for the signal to get to the end of the line.

What happens when the signal gets to the load? We can assume some voltage V_L will appear across the load resistor, and hence a current I_L will flow through it. The most logical thing to assume would be that $V_L = V_1^+$. **But**, we quickly run into a contradiction. If $V_L = V_1^+$, then

$$\begin{aligned} I_L &= \frac{V_L}{R_L} \\ &= \frac{V_1^+}{R_L} \end{aligned} \quad (1.60)$$

But Kirchoff says the sum of the currents into the output terminal must equal zero, thus I_1^+ must equal I_L . This then says

$$\begin{aligned} I_1^+ &= \frac{V_1^+}{Z_0} \\ &= I_L \\ &= \frac{V_1^+}{R_L} \end{aligned} \quad (1.61)$$

which can only be true if $Z_L = Z_0$, which while possible, will not be the case in general.

What are we to do? We have an obvious contradiction. The telegrapher's equations permit **two** solutions to the transmission line problem: a signal going in the x direction, V_1^+ , and a signal going in the $-x$ direction, V_1^- . The **only** way out of our problem is to assume that when the V_1^+ signal gets to the load, a **new signal**, going in the $-x$ direction, is created, which then heads back towards the load. So, let's put in a V_1^- and I_1^- Figure 1.14 (Reflected Waves are Generated).

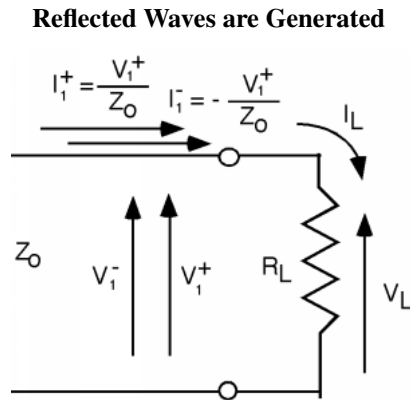


Figure 1.14

Now there is a little problem with signs here that we have to deal with. We can either draw I_1^- so that it points in the $-x$ direction, and say

$$I_1^- = \frac{V_1^-}{Z_0} \quad (1.62)$$

or we can draw I_1^- going in the x direction, and say

$$I_1^- = -\frac{V_1^-}{Z_0} \quad (1.63)$$

Since most of the time, it is better if we define **all** currents going in the **same** direction, and since this equation¹¹ says that V^- and I^- are related by $-Z_0$ the latter choice seems to be the better one.

¹¹"Transmission Line Equation", (20) <<http://cnx.org/content/m11375/latest/#eqn20>>

Going by that convention then, we equate voltage on either side of the termination

$$V_1^+ + V_1^- = V_L \quad (1.64)$$

and we sum currents into the output terminal

$$I_1^+ + I_1^- = I_L \quad (1.65)$$

or

$$\frac{V_1^+}{Z_0} - \frac{V_1^-}{Z_0} = \frac{V_L}{R_L}$$

This makes

$$V_L = \frac{R_L}{Z_0} (V_1^+ - V_1^-) \quad (1.66)$$

which we can substitute into (1.64) to get:

$$V_1^+ + V_1^- = \frac{R_L}{Z_0} (V_1^+ - V_1^-) \quad (1.67)$$

or

$$V_1^+ \left(1 - \frac{R_L}{Z_0}\right) = V_1^- \left(\left(-\frac{R_L}{Z_0}\right) - 1\right)$$

and this can be solved for V_1^- as

$$V_1^- = \frac{R_L - Z_0}{R_L + Z_0} V_1^+ \equiv \Gamma_{vL} V_1^+ \quad (1.68)$$

where Γ_{vL} is called the load **voltage reflection coefficient**.

We could also have solved for I_1^+ in terms of I_1^- and we would have found:

$$I_1^- = \frac{Z_0 - R_L}{Z_0 + R_L} V_1^+ \equiv \Gamma_{iL} V_1^+ \quad (1.69)$$

where Γ_{iL} is the load **current reflection coefficient**. Note that $\Gamma_{iL} = -\Gamma_{vL}$.

Let's take a break from equation manipulating, and think about what we have here. First of all, although the result we have obtained is very important, the method we used to get there was even more so. What did we do? We postulated a voltage and current on the line, and then took a look to see if that solution resulted in a reasonable result. In this case it did not. We were in gross violation of Kirchoff Laws! We rescued ourselves by taking the only possible escape route: we added an additional voltage and current to the solution. Since V^- and I^- are related to each other in a different manner than V^+ and I^+ (by $-Z_0$ rather than Z_0), this gave us an additional degree of freedom so that we could simultaneously satisfy **both** the transmission line I-V relationships as well as the load I-V relationship.

Let's take a look at Γ_{vL} for a minute. Over what range can it vary? A glance at (1.68) shows that it depends on the range of R_L . If we exclude the possibility of negative resistance (a reasonable exclusion) then $0 \leq R_L \leq \infty$. As R_L varies over this range, the voltage reflection coefficient goes from $-1 \leq \Gamma_{vL} \leq 1$. When R_L is **less** than Z_0 , the reflection coefficient is negative. When R_L is **greater** than Z_0 , the reflection coefficient is positive. When $R_L = Z_0$ the reflection coefficient is zero, and the line is said to be **matched**. In a matched transmission line, a signal traveling down it is completely absorbed by the load, and nothing is reflected from it. In this case we can have the incident voltage and current signals just equal load voltage and currents without the need to add reflected waves. For an unmatched transmission line, a signal incident on the load is (partially) reflected, and a new signal starts moving back down the line towards the source. An example of a transmission line is a buss in a computer. What would be the implications of unmatched terminations on various connections between the buss and the computer circuitry?

Two special cases of terminated transmission lines that are of interest are $R_L = 0$ (a shorted line) and $R_L = \infty$ (an open line). For $R_L = 0$, $\Gamma_{vL} = -1$ and so a signal with the same amplitude but opposite sign as the incident wave, is reflected back down the line. For $R_L = \infty$, a signal with the **same** amplitude and sign gets reflected back down the line. It is easy to remember which is which, and also to make sure you have the right order in the equation for the reflection coefficient, when you keep in mind that the voltage across the load is the sum of V_1^+ and V_1^- . If the line is terminated with a short circuit, V_L must equal 0. Since

$$V_1^- = \Gamma_{vL} V_1^+ \quad (1.70)$$

and

$$\Gamma_{vL} = \frac{R_L - Z_0}{R_L + Z_0}$$

From these equations we can see that with $R_L = 0$ (a short), Γ_{vL} does indeed -1 , and so $V_1^- = -V_1^+$ and hence $V_L = V_1^+ + V_1^- = 0$ as it should.

Now, let's go back to our terminated transmission line. What is going to happen to V_1^- after it leaves the load? It obviously travels back down the line towards the generator. What happens when it gets back there? Time to look at Figure 1.15 (Back at the Generator).

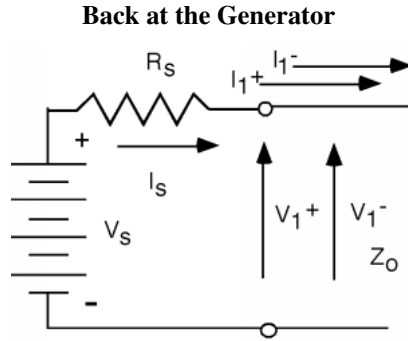


Figure 1.15

At the generator end of the line we can not leave out V_1^+ , as it is still here, so long as the source remains connected. Once it is launched, it stays on the line forever. We just have to add in the new V_1^- . What happens now, if we test Kirchoff's voltage law here at the generator end?

$$V_S - I_S R_S - V_1^+ - V_1^- = 0 \quad (1.71)$$

Substituting $I_S = I_1^+ + I_1^-$ and then using the impedance relationship between the current and the voltages we get:

$$V_S - \frac{R_S}{Z_0} (V_1^+ - V_1^-) - V_1^+ - V_1^- = 0 \quad (1.72)$$

Note that again, we used $\frac{V_1^+}{Z_0}$ for I_1^+ and $-\frac{V_1^-}{Z_0}$ for I_1^- .

We know

$$V_1^+ = \frac{Z_0}{Z_0 + R_S} V_S \quad (1.73)$$

which if we substitute into (1.72) and re-arrange a little bit we get

$$V_S \left(1 - \frac{R_S}{Z_0 + R_S} - \frac{Z_0}{Z_0 + R_S} \right) + \frac{R_S}{Z_0} V_1^- - V_1^- = 0 \quad (1.74)$$

The stuff inside the big parentheses sums to zero (as in fact it should, this is just the solution to the initial V_1^+ generation problem) and we are left with the uncomfortable conclusion that V_1^- must be zero! What are we going to do? We will just have to add **another** wave V_2^+ to the solution! (Figure 1.16 (Yet Another Wave is Formed))

Yet Another Wave is Formed

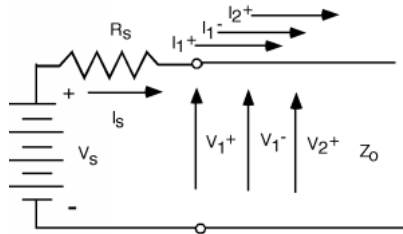


Figure 1.16

Now we have two ways to proceed from here. The first would be the dumb way, and try to solve this whole problem. But then we could also be smart and note that all of the equations we have relating voltages and currents in this problem are **linear equations** and hence we can use the principle of **superposition** Figure 1.17 (Using Superposition).

Using Superposition

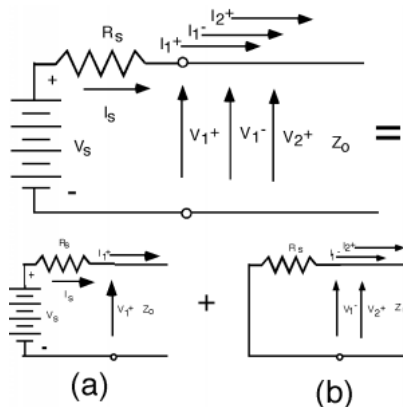


Figure 1.17

Thus, the top sketch in Figure 1.17 (Using Superposition) can be broken into the **two** circuits (a) and (b) at the bottom of the figure. Then, everything is trivial. We have already solved (a) - that was just our initial V_1^+ launching

problem. Part (b) is just a reflection problem again, with a wave of V_1^- incident on a load of value R_S through a transmission line with characteristic impedance Z_0 . The reflected wave in this instance is V_2^+ , which is why we had to use numbered subscripts in the first place.

If you do not believe using superposition is valid, you can try doing the problem over again, but it should be pretty obvious that we can write

$$V_2^+ = \frac{R_S - Z_0}{R_S + Z_0} V_1^- \equiv \Gamma_{vS} V_1^- \quad (1.75)$$

with

$$\Gamma_{vS} = \frac{R_S - Z_0}{R_S + Z_0} \quad (1.76)$$

Chapter 2

Filters

2.1 Practical Filters¹

An ideal filter² simply removes all unwanted frequencies, preserving the remaining frequencies exactly. This would resemble some sort of a finite rectangle function in frequency. However, a simple, finite rectangle function in frequency is an infinite sinc function in time. This is a problem. A sinc function is an infinite length signal in both the positive and negative directions, making it impossible to create in the real-world. This leads us to ask what would happen if we just made this sinc function causal by "chopping it off" somewhere. What we find when we do this is that the frequency domain representation is no longer a perfect rectangle: it now does not fall off immediately and shows some wiggling where it was flat before.

¹This content is available online at <<http://cnx.org/content/m10126/2.6/>>.

²"Ideal Filters" <<http://cnx.org/content/m10103/latest/>>

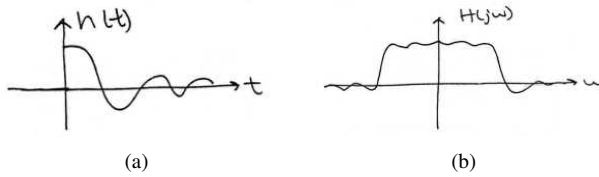


Figure 2.1: (a) A truncated sinc function in time... (b) ... leads to a smooth, wiggly, "rectangle" in frequency.

2.1.1 The Bands

An ideal filter has two types of bands: the **stop band** defines the region of frequencies that are eliminated by the filter, while the **pass band** defines the region of frequencies that the filter allows through. Practical filters add one more, the **transition band**. This is the area where the filter is moving between the stop band and pass band.

2.1.2 Filter Design Specifications

In our look at filter design specifications, we will use the example of a lowpass filter. The extension to the other kinds of filters should be fairly straightforward. Figure 2.2 shows the parameters for a lowpass filter in the frequency domain.

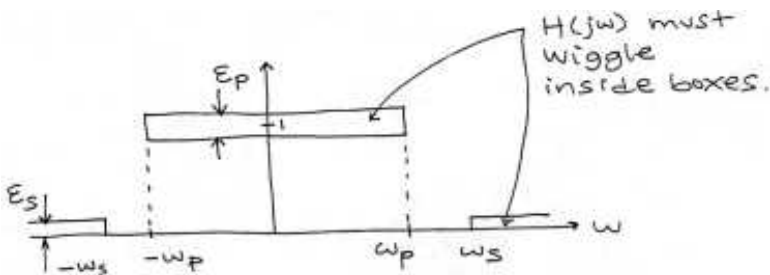


Figure 2.2

From this image, the passband is the region from $-\omega_p$ to ω_p , the transition bands are the regions from $-\omega_s$ to $-\omega_p$ and from ω_p to ω_s while the stop band is the region less than $-\omega_s$ or greater than ω_s .

In the figure above, e_p and e_s represent the acceptable tolerance (or error) around the desired level that the passband and stopband respectively may vary. The behavior within the transition band is not specified, allowing anything to occur there, as long as the width is within specifications.

Example 2.1

This example will look at a moving average system.

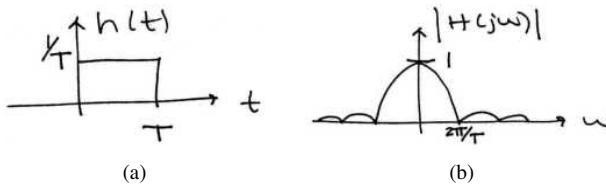


Figure 2.3: (a) Time domain representation of the moving average. (b) The frequency domain representation of the moving average system is a sinc function. $H(j\omega) = e^{-j\omega \frac{T}{2}} \frac{\sin(\omega \frac{T}{2})}{\omega \frac{T}{2}}$

Some notes about this system:

- It is lowpass
- It has linear phase with jumps of π radians when the sinc function changes sign
- The duration of the filter is inversely proportional to its bandwidth
- This filter is finite impulse response (FIR)
- It cannot be built with passive R, L, C circuits
- We do not have independent control over all four design specifications

We are going to design a moving average filter with the following design specs: $\omega_p = 100\pi$, $e_s = 0.1$, $e_p = 0.1$

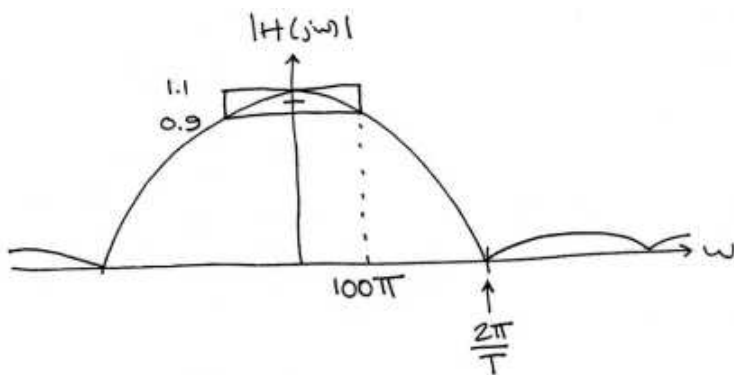


Figure 2.4

With this specification, we are allowing ω_s to be a dependant variable (since we need one). We can now find the equation for this moving average system.

We begin with

$$|H(j\omega)| = 1.1 \frac{\sin\left(\frac{\omega T}{2}\right)}{\frac{\omega T}{2}} \quad (2.1)$$

We will now solve for T with

$$\begin{aligned} |H(j100\pi)| &= 1.1 \frac{\sin(50(\pi T))}{50\pi T} \\ &= 0.9 \end{aligned} \quad (2.2)$$

For these specs, $T \simeq 0.007$. This means that $|H(j\omega)|$ does not stay below $e_s = 0.1$ until $\omega_s \simeq 771\pi$.

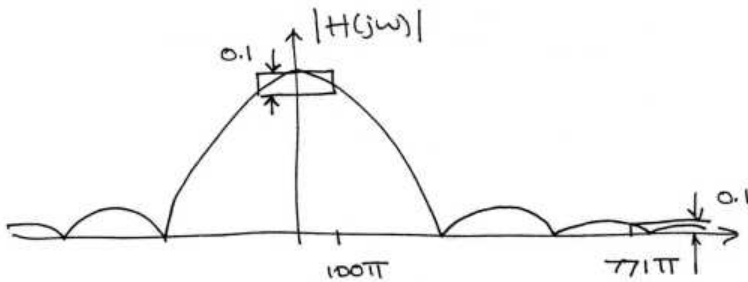


Figure 2.5: A graphical look at the transfer function of this lowpass filter with the passband and stopband noted.

It is very clear from this representation that the transition band is huge (671π). This is a very bad filter, especially when you consider that it cannot be implemented with passive circuitry. Fortunately better filters (e.g. Butterworth (Section 2.2), Chebyshev (Section 2.3) and Elliptical³) do exist.

2.1.3 Beyond Lowpass Filter Design

In the discussion of the different filters (Butterworth, Chebyshev and Elliptical) is common to see explanations based on lowpass filters. This explanation is very nice when first learning about them, because it is sufficient to understand the fundamentals of each of them. It is acceptable, because there exist fairly straightforward techniques to convert these lowpass filters into highpass, bandpass or bandstop filters. These techniques are the lowpass to highpass transformation, lowpass to bandpass transformation and lowpass to bandstop transformation.

2.2 Butterworth Filters⁴

The Butterworth filter is a filter that can be constructed out of passive R, L, C circuits. The magnitude of the transfer function for this filter is

³"Elliptic Filters" <<http://cnx.org/content/m10105/latest/>>

⁴This content is available online at <<http://cnx.org/content/m10127/2.10/>>.

Magnitude of Butterworth Filter Transfer Function

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^{2n}}} \quad (2.3)$$

where n is the **order** of the filter and ω_c is the **cutoff frequency**. The cutoff frequency is the frequency where the magnitude experiences a 3 dB dropoff (where $|H(j\omega)| = \frac{1}{\sqrt{2}}$).

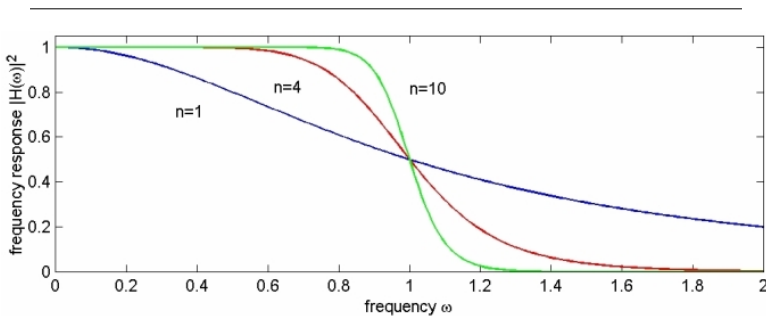


Figure 2.6: Three different orders of lowpass Butterworth analog filters: $n = \{1, 4, 10\}$. As n increases, the filter more closely approximates an ideal brickwall lowpass response.

The important aspects of Figure 2.6 are that it does not ripple in the passband or stopband as other filters tend to, and that the larger n , the sharper the cutoff (the smaller the transition band (Section 2.1)).

This transfer function is often seen in its normalized form of

Magnitude of Normalized Transfer Function for Lowpass Butterworth Filter

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \omega^{2n}}} \quad (2.4)$$

Butterworth filters give transfer functions ($H(j\omega)$ and $H(s)$) that are **rational functions**. They also have only poles⁵, resulting in a transfer function of the form

$$\frac{1}{(s - s_1)(s - s_2) \cdots (s - s_n)} \quad (2.5)$$

⁵"Poles and Zeros" <<http://cnx.org/content/m10112/latest/>>

and a pole-zero plot of

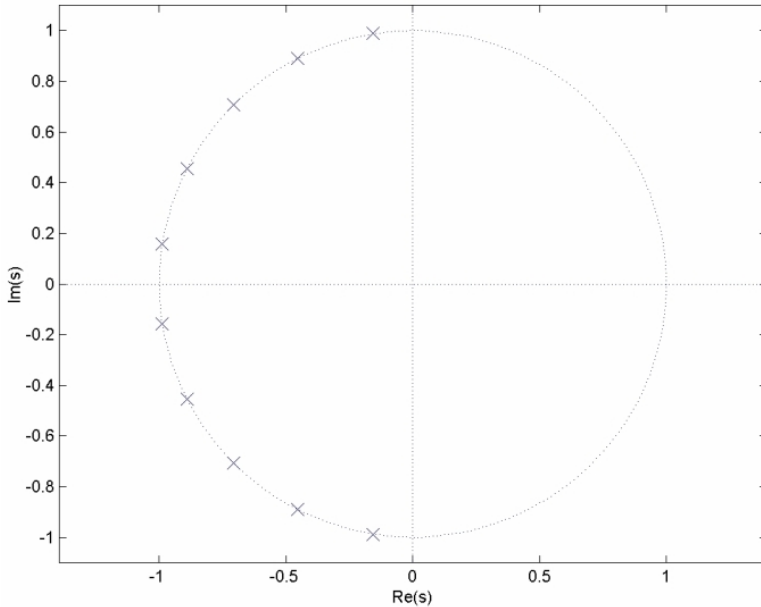


Figure 2.7: Poles of a 10th-order ($n = 5$) lowpass Butterworth filter.

Note that the poles lie along a circle in the s -plane.

2.2.1 Designing a Butterworth Filter

Designing a Butterworth filter is a trivial task. Since we know that the filter contains only poles, we know that we can write it as

$$H(s) = \frac{1}{s^n + a_{n-1}s^{n-1} + \dots + a_1s + 1} \quad (2.6)$$

From this, we may look up the a_i from a table (like the one below) for any desired n . We can also find them in Matlab by using the `buttap` command. The real challenge of designing a Butterworth filter comes with figuring out the optimal characteristics for the given application.

n	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
2	1.414214								
3	2.000000	0.000000							
4	2.613126	1.414214	1.414214	2.613126					
5	3.236068	2.236068	2.236068	3.236068					
6	3.863703	2.464102	1.416204	2.464102	3.863703				
7	4.493959	2.097835	1.591794	1.591794	2.097835	4.493959			
8	5.125831	1.370218	1.846135	1.370218	5.125831				
9	5.758770	1.581731	1.163443	1.986386	1.163443	1.581731	5.758770		
10	6.392453	1.431729	1.802064	1.882376	1.233409	1.882376	1.802064	1.431729	6.392453

Table 2.1

Exercise 2.1*(Solution on p. 37.)*

Design a Butterworth filter with a passband gain between 1 and 0.891 (-1 dB gain) for $0 < \omega < 10$ and a stopband not to exceed 0.0316 (-30 dB gain) for $\omega \geq 20$.

2.3 Chebyshev Filters⁶

This module will cover Chebyshev filters with an assumed understanding of Butterworth filters (Section 2.2). This module will also examine the lowpass example of these filters, leaving conversion to other types of filters for the Lowpass to Highpass Transformation, Lowpass to Bandpass Transformation and Lowpass to Bandstop Transformation modules.

Like Butterworth filters, Chebyshev filters contain only poles. However, while the poles of the Butterworth filter lie on a circle in the s-plane, those of the Chebyshev filter lie on an **ellipse**.

⁶This content is available online at <<http://cnx.org/content/m10104/2.10/>>.

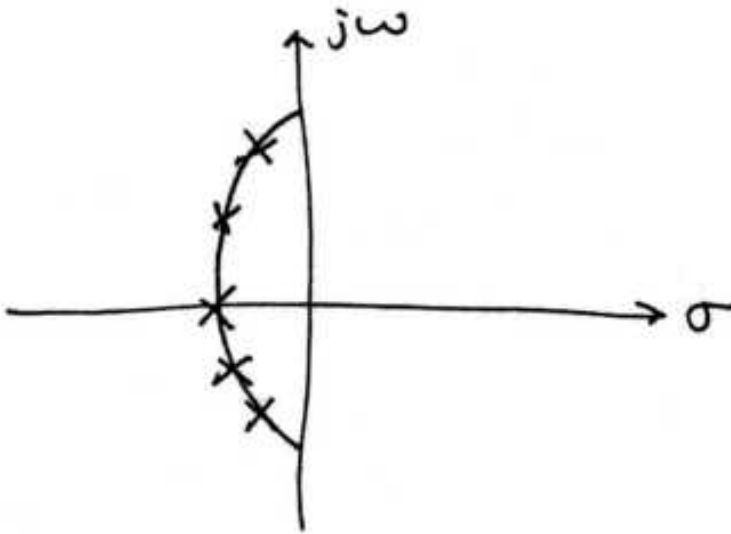


Figure 2.8: A pole-zero plot of a lowpass Chebyshev filter. The poles are equally spaced around an ellipse in the left half of the complex plane.

The result of this repositioning of poles is a "rippling effect" in the passband of the magnitude of the transfer function. Since each local maximum in this rippling reaches the same value and each local minimum reaches the same value, this rippling is described as **equal ripple**. It is important to notice that there is no rippling in the stopband and to be aware that this transition band will be narrower than a comparable Butterworth filter.

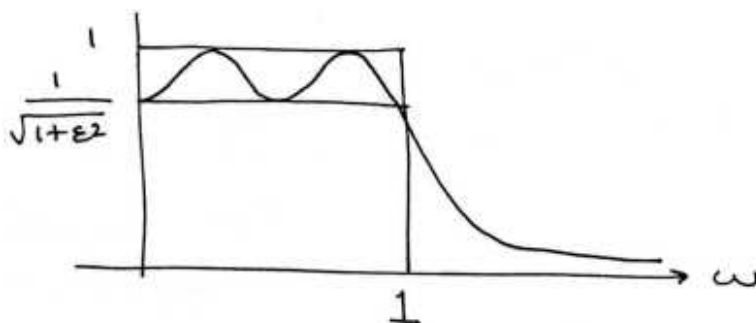


Figure 2.9: A sketch of the magnitude of a lowpass Chebyshev filter. Notice that the "equal rippling" in the passband and not in the stopband.

The magnitude of the transfer function of a Chebyshev filter takes the form

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 C_n^2(\omega)}} \quad (2.7)$$

where the polynomial, $C_n(\omega) = \cos(n \arccos(\omega))$ is known as the **Chebyshev polynomial**.

The design of Chebyshev filters is generally done the same way Butterworth filters are (with tables and Matlab). The most relevant Matlab command is `cheb1ap`, but there are others.

Solutions to Exercises in Chapter 2

Solution to Exercise 2.1 (p. 34)

The first step is to determine n . To do this, we must solve for n using the passband and stopband criteria. We begin by finding the equation for the gain in the passband in dB,

$$\begin{aligned}\hat{G}_p &= 20\log|H(j\omega)| \\ &= -10\log\left(1 + \left(\frac{\omega_p}{\omega_c}\right)^{2n}\right)\end{aligned}\quad (2.8)$$

and for the stopband in dB,

$$\begin{aligned}\hat{G}_s &= 20\log|H(j\omega)| \\ &= -10\log\left(1 + \left(\frac{\omega_s}{\omega_c}\right)^{2n}\right)\end{aligned}\quad (2.9)$$

these equations can also take the form

$$\left(\frac{\omega_x}{\omega_c}\right)^{2n} = 10^{\frac{-\hat{G}_x}{10}} - 1 \quad (2.10)$$

In this form, we may divide the passband equation by the stopband equation to get rid of the ω_c . From there, we can solve for n to get

$$n = \frac{\log\frac{10^{\frac{-\hat{G}_s}{10}} - 1}{10^{\frac{-\hat{G}_p}{10}} - 1}}{2\log\frac{\omega_s}{\omega_p}} \quad (2.11)$$

By plugging in, we find $n = 5.9569$. However, since n must be an integer, we round this up to $n = 6$

The next step is to find ω_c . We can do this by substituting $n = 6$ into the equations for the passband and stopband and solving for ω_c . This yields $\omega_c = 11.1919$ for the passband equation and $\omega_c = 11.2478$ for the stopband equation. The difference in these solutions is a result of n needing to be an integer. If we choose the solution from the passband equation, the passband will meet its requirements exactly, and the stopband will surpass its requirements. If we choose the solution from the stopband equation instead, the stopband requirements will be met exactly, while we will exceed the passband requirements. Therefore, we may choose either value or any value in between. For this example, we will choose $\omega_c = 11.2478$.

Now, we can find the normalized transfer function. Since we know this to be a sixth-order Butterworth, we can determine from the table that

$$H(s) = \frac{1}{s^6 + 3.863703s^5 + 7.464102s^4 + 9.141620s^3 + 7.464102s^2 + 3.863703s + 1} \quad (2.12)$$

Finally, we can determine the final transfer function.

$$H(s) = \frac{1}{\left(\frac{s}{11.2478}\right)^6 + 3.863703\left(\frac{s}{11.2478}\right)^5 + 7.464102\left(\frac{s}{11.2478}\right)^4 + 9.141620\left(\frac{s}{11.2478}\right)^3 + 7.464102\left(\frac{s}{11.2478}\right)^2 + 3.863703\left(\frac{s}{11.2478}\right) + 1} \quad (2.13)$$

Rather than multiplying this out and factoring, we will leave it in this form for readability, since the numbers can get quite large otherwise.

Glossary

D distributed parameter

A distributed parameter is a parameter which is spread throughout a structure and is not confined to a lumped element such as a coil of wire.

Example: For instance, we will hereby define \mathbf{L} as the **distributed inductance** for the transmission line. It has units of Henrys/meter. If we have a length of transmission line x_0 meters long, and if that line has a distributed inductance of \mathbf{L} H/m, then the inductance L of that length of line is just $L = \mathbf{L}x_0$.

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 allpass, § 2.3(34)
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- B** bandpass, § 2.1(27), § 2.3(34)
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- C** chebyshev filter, § 2.3(34)
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