UNIVERSITY OF BOTSWANA

2006/2007 --- EXAMINATIONS

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COURSE NUMBER MAT111 **DURATION** 2 HRS. **DATE** NOV. 2006

TITLE OF PAPER INTRODUCTORY MATHEMATICS I

SUBJECT MATHEMATICS **TITLE OF EXAMINATION** BSc I

MORNING/AFTERNOON

INSTRUCTIONS:

- ANSWER **ALL** QUESTIONS IN SECTION **A** AND ANY **TWO (2)** FROM SECTION **B**.
- ALL MARKS ARE INDICATED IN [ ]

NUMBER OF PAGES

5

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SECTION A (60 marks)

Candidates may attempt ALL questions being careful to number them A1 to A6.

A1. (a) Simplify

$$\frac{3\,(2^{n+1}) - 4\,(2^{n-1})}{2^{n+1} - 2^n}$$

[4]

(b) Solve for x in the following equations,

(i) $2\log_3 x = \log_3(x+6)$. [3]

(ii) $2^{x^2} = 16^{x-1}$. [3]

A2. (a) Prove the identity

$$\tan x = \frac{\sin 2x}{1+\cos 2x},$$

for $\cos 2x \neq -1$. [5]

(b) Given that $\cos 2x + \sin x = 0$, find all the possible values of $\sin x$. [5]

A3. (a) Solve the inequality $|2x+3| \leq 5$. [5]

(b) Find the maximal domain and the range of $f(x) = \sqrt{15-3x}$. [5]

A4. Given that the functions f and g are defined by

$$f : x \mapsto 2x - 5, \quad x \in \mathbf{R}, \qquad g :\mapsto \frac{4}{2-x}, \quad x \in \mathbf{R}, \quad x \neq 2.$$

(a) find $f \circ g(x)$, [3]

(b) find $f^{-1}(x)$ and $g^{-1}(x)$. [4]

(c) Sketch, on a single diagram, the graphs of $y = f(x)$ and $f^{-1}(x)$ clearly making the relationship between these two graphs. [3]

A5. Given the polynomial function

$$f(x) = 2x^3 - 15x^2 + ax + b$$

(a) find the values of a and b for which $f(x)$ is divisible by $x - 4$ and $2x - 1$. [7]

(b) Hence find the third factor of $f(x)$. [3]

A6. If α and β are the roots of

$$x^2 - 3x - 2 = 0$$

find the quadratic equation whose roots are $2\alpha - \beta, 2\beta - \alpha$. [10]

SECTION B (40 marks)

Candidates may attempt TWO questions being careful to number them B7 to B10.

B7. (a) Find the coordinates of the center and radius of the circle

$$x^2 + y^2 - 10x + 12y = 0.$$

[10]

(b) Find the equation to the circle which passes through the origin and cuts both the circles

$$x^2 + y^2 - 6x + 8 = 0 \quad \text{and} \quad x^2 + y^2 - 2x - 2y = 7$$

orthogonally. [10]

B8. (a) (i) Evaluate $\sin\left(\frac{\pi}{6}\right)$ using the half angle formula. [5]

(ii) If $\sin\alpha = \frac{2}{3}$, $\cos\beta = -\frac{2}{7}$, $0 < \theta < \frac{\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$,

find the value of $\cos(\alpha + \beta)$. [5]

(b) Solve the trigonometric equation

$$3\sin 2x = 2\tan x,$$

for $0^o \leq x \leq 360^o$. [10]

B9. The functions $f(x)$ and $g(x)$ are defined by

$$f(x) = \frac{x^2 - 2x - 3}{x - 1}, \qquad \text{and} \qquad g(x) : \mathbf{R} \setminus \{1\} \longrightarrow \mathbf{R} \text{ with } x \longmapsto \frac{3}{x-1}.$$

(a) State the maximal domain of $f(x)$ and find the range of $g(x)$. [4]

(b) Solve the equation $f(x) + g(x) = 0$. [3]

(c) Show that the function $g(x)$ is injective. [3]

(d) Sketch the graph of $f(x)$, clearly labelling the intercepts and asymptotes. [10]

B10. Solve the inequalities

(a)

$$1 < |x - 3| \leq 5$$

[7]

(b)

$$\frac{(x-2)(x+1)}{(x+4)} \geq 0$$

[8]

(c) Find the set of values of x for which

$$20 + 8x - x^2 \geq 0$$

[5]

END OF QUESTION PAPER