UNIVERSITY OF BOTSWANA

2008/2009 – EXAMINATIONS

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COURSE NUMBER MAT111 **DURATION** 2 hrs **DATE** NOV 2008
TITLE OF PAPER INTRODUCTORY MATHEMATICS I
SUBJECT MATHEMATICS **TITLE OF EXAMINATION** BSc I

MORNING/AFTERNOON

INSTRUCTIONS:

- ANSWER **ALL** QUESTIONS IN SECTION **A** AND ANY **TWO(2)** QUESTIONS FROM SECTION **B**.
- ALL MARKS ARE INDICATED IN BRACKETS [ ].

NUMBER OF PAGES INCLUDING COVER PAGE: 5

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Section A

Answer All Questions: Each Question carries 10 Marks

Question 1

(a) Solve $3^{x+1} = 4^{x-1}$. [5]

(b) Solve $\sqrt{x+3} + \sqrt{x-2} = 5$. [5]

Question 2

(a) Solve the inequality

$$1 < |3x - 1| \leq 5$$

[6]

(b) Find the set of values of x satisfying

$$(x - 2)(2x + 1) > 0$$

[4]

Question 3

(a) Find the domain and range of the following function

$$f(x) = \sqrt{x^2 - 16}$$

[4]

(b) Find the equation of the line that passes through $(1, 1)$ and the intersection of the line $2x + 3y - 4 = 0$ with the x-axis [6]

Question 4

(a) Given that

$$4\log\left(\frac{1}{y^2}\right) - 6\log\left(\frac{1}{x}\right) = 8\log(y^{\frac{1}{2}})$$

find y in terms of x. [4]

(b) When a polynomial $p(x)$ is divided by $(x-3)$ the remainder is 3 and x is a factor of $p(x)$.
Find the remainder when p(x) is divided by $x^2 - 3x$. [6]

Question 5

(a) Find the exact value of $\cos 2\alpha$ if $\sin\alpha = -\frac{3}{5}$ and $180^0 < \alpha < 270^0$. [5]

(b) Calculate the value of x in radians in the interval $0 < x < \frac{\pi}{2}$ that satisfies the equation

$$3^{(1-\sin x)} = 27^{\sin x - \frac{1}{3}}$$

[5]

Question 6

(a) The first term of an A. P. is -1 and the common difference is $\frac{1}{3}$. What is the sum of the first 10 terms? [3]

(b) The fifth term of a G. P. is 7 and the sixth term is 21. What is the sum of the first two terms? [3]

(c) Find the polar form of the complex number (-5 + i4) . [4]

Section B

Answer TWO(2) Questions OUT of FOUR:
Each Question carries 20 Marks

Question 7

(a) Put the surd $\sqrt{7+2\sqrt{12}}$ in the form $\sqrt{a}+\sqrt{b}$. [4]

(b) Solve $3^{x+1}-2(3^{-x})+5=0$. [8]

(c) If α and β are the roots of $x^2+x-6=0$, find the equation whose roots are $2\alpha-\beta$ and $2\beta-\alpha$. [8]

Question 8

(a) The functions F and G, each with domain $\mathbb{R}$ are defined as follows:

$$F: x \to 3x+2 \qquad G: x \to x^2+1$$

For each of F and G find the range of the function and give a reason to show whether the function is one-to-one. Obtain formulae for $(F\circ G)(x)$ and $(G\circ F)(x)$ and find the values of x for which $(F\circ G)(x)=(G\circ F)(x)$. [8]

(b) If

$$f(x)=\frac{x-1}{x-2}, \quad x\neq 2$$

find $f^{-1}(x)$ and state the domain of the function f^{-1}. [6]

(c) Sketch on the same axes the graphs of $y=|x^2-3|$ and $y=2$.
Solve the equation $|x^2-3|=2$
Hence write down the solution to the inequality $|x^2-3|\geq 2$ [6]

Question 9

(a) Find the values of the constants a and b if the polynomial $P(x) = 2x^3 - 15x^2 + ax + b - 2$ is divisible by x and $x - 1$. [8]

(b) Sketch the graph of the function defined by $g(x) = \frac{x+3}{x-1}$, clearly indicating intercepts with the coordinate axes and asymptotes. [8]

(c) Solve for x:

$$\frac{1}{3}\log x^6 = 5\log 3 - \log 27$$

[4]

Question 10

(a) Prove the identity:

$$\cot\theta - \tan\theta = \frac{2\cos 2\theta}{\sin 2\theta}$$

[5]

(b) Solve the equation for θ where $0 \leq \theta \leq \pi$:

$$4\sin\theta\cos\theta = -\sqrt{3}$$

[5]

(c) Find the sum of the first 10 terms of the geometric series $\frac{1}{4} - \frac{1}{2} + 1 - 2 + 4 - 8 + \ldots$. [5]

(d) Use DeMoivre's theorem to prove that

$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$$

[5]