

Pari-GP reference card

(PARI-GP version 2.15.3)

Note: optional arguments are surrounded by braces {}.

To start the calculator, type its name in the terminal: **gp**

To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

Help

| | |
|--|----------------|
| describe function | ?function |
| extended description | ??keyword |
| list of relevant help topics | ???pattern |
| name of GP-1.39 function f in GP-2.* | whatnow(f) |

Input/Output

| | |
|--|------------------------|
| previous result, the result before | %, %`, %`` , etc. |
| n -th result since startup | % n |
| separate multiple statements on line | ; |
| extend statement on additional lines | \ |
| extend statements on several lines | { seq_1 ; seq_2 ;} |
| comment | /* ... */ |
| one-line comment, rest of line ignored | \\ ... |

Metacommands & Defaults

| | |
|--|----------------------------|
| set default d to val | default({ d },{ val }) |
| toggle timer on/off | # |
| print time for last result | ## |
| print defaults | \d |
| set debug level to n | \g n |
| set memory debug level to n | \gm n |
| set n significant digits / bits | \p n , \pb n |
| set n terms in series | \ps n |
| quit GP | \q |
| print the list of PARI types | \t |
| print the list of user-defined functions | \u |
| read file into GP | \r $filename$ |
| set debuglevel for domain D to n | setdebug(D,n) |

Debugger / break loop

| | |
|--|---------------------------|
| get out of break loop | break or <C-D> |
| go up/down n frames | dbg_up({ n }), dbg_down |
| set break point | breakpoint() |
| examine object o | dbg_x(o) |
| current error data | dbg_err() |
| number of objects on heap and their size | getheap() |
| total size of objects on PARI stack | getstack() |

PARI Types & Input Formats

| | |
|---|---------------------------------------|
| t_INT . Integers; hex, binary | ± 31 ; $\pm 0x1F$, $\pm 0b101$ |
| t_REAL . Reals | ± 3.14 , 6.022 E23 |
| t_INTMOD . Integers modulo m | Mod(n,m) |
| t_FRAC . Rational Numbers | n/m |
| t_FFELT . Elt in finite field \mathbf{F}_q | ffgen(q , 't) |
| t_COMPLEX . Complex Numbers | $x + y * I$ |
| t_PADIC . p -adic Numbers | $x + O(p^k)$ |
| t_QUAD . Quadratic Numbers | $x + y * \text{quadgen}(D, \{ 'w \})$ |
| t_POLMOD . Polynomials modulo g | Mod(f,g) |
| t_POL . Polynomials | $a * x^n + \dots + b$ |
| t_SER . Power Series | $f + O(x^k)$ |
| t_RFRAC . Rational Functions | f/g |
| t_QFB . Binary quadratic form | Qfb(a,b,c) |
| t_VEC/t_COL . Row/Column Vectors | [x,y,z], [x,y,z]~ |
| t_VEC integer range | [1..10] |

| | |
|--|-----------------------|
| t_VECSMALL . Vector of small ints | Vecsmall([x,y,z]) |
| t_MAT . Matrices | [$a,b;c,d$] |
| t_LIST . Lists | List([x,y,z]) |
| t_STR . Strings | "abc" |
| t_INFINITY . $\pm\infty$ | +oo, -oo |

Reserved Variable Names

| | |
|---|-----------------------|
| $\pi \approx 3.14$, $\gamma \approx 0.57$, $C \approx 0.91$, $I = \sqrt{-1}$ | Pi, Euler, Catalan, I |
| Landau's big-oh notation | O |

Information about an Object, Precision

| | |
|---------------------------------------|---------------------------------------|
| PARI type of object x | type(x) |
| length of x / size of x in memory | # x , sizebyte(x) |
| real precision / bit precision of x | precision(x), bitprecision(x) |
| p -adic, series prec. of x | padicprec(x,p), serprec(x,v) |
| current dynamic precision | getlocalprec, getlocalbitprec |

Operators

| | |
|--|--|
| basic operations | +, - , *, / , ^ , sqr |
| $i \leftarrow i+1$, $i \leftarrow i-1$, $i \leftarrow i*j$, ... | i++, i--, i*=j,... |
| Euclidean quotient, remainder | $x \backslash y$, $x \backslash y$, $x \% y$, divrem(x,y) |
| shift x left or right n bits | $x << n$, $x >> n$ or shift($x, \pm n$) |
| multiply by 2^n | shiftmul(x,n) |
| comparison operators | <=, <, >=, >, ==, !=, ==, lex, cmp |
| boolean operators (or, and, not) | , &&, ! |
| bit operations | bitand, bitneg, bitor, bitxor, bitnegimply |
| maximum/minimum of x and y | max(x,y), min(x,y) |
| sign of x (gives $-1, 0, 1$) | sign(x) |
| binary exponent of x | exponent(x) |
| derivative of f , 2nd derivative, etc. | f' , f'' , ... |
| differential operator | diffop($f,v,d,\{n=1\}$) |
| quote operator (formal variable) | 'x |
| assignment | x = value |
| simultaneous assignment $x \leftarrow v[1]$, $y \leftarrow v[2]$ | [x,y] = v |

Select Components

| | |
|---|--------------------------------------|
| <i>Caveat</i> : components start at index $n = 1$. | |
| n -th component of x | component(x,n) |
| n -th component of vector/list x | $x[n]$ |
| components $a, a+1, \dots, b$ of vector x | $x[a..b]$ |
| (m,n) -th component of matrix x | $x[m,n]$ |
| row m or column n of matrix x | $x[m,]$, $x[,n]$ |
| numerator/denominator of x | numerator(x), denominator(x) |

Random Numbers

| | |
|---------------------------------|-----------------------------------|
| random integer/prime in $[0,N[$ | random(N), randomprime(N) |
| get/set random seed | getrand, setrand(s) |

Conversions

| | |
|--|------------------------|
| to vector, matrix, vec. of small ints | Col/Vec, Mat, Vecsmall |
| to list, set, map, string | List, Set, Map, Str |
| create $(x \bmod y)$ | Mod(x,y) |
| make x a polynomial of v | Pol($x,\{v\}$) |
| variants of Pol <i>et al.</i> , in reverse order | Polrev, Vecrev, Colrev |
| make x a power series of v | Ser($x,\{v\}$) |
| convert x to simplest possible type | simplify(x) |
| object x with real precision n | precision(x,n) |
| object x with bit precision n | bitprecision(x,n) |
| set precision to p digits in dynamic scope | localprec(p) |
| set precision to p bits in dynamic scope | localbitprec(p) |

Character strings

| | |
|---|-------------------------|
| convert to TeX representation | strtex(x) |
| string from bytes / from format+args | strchr, sprintf |
| split string / join strings | strsplit, strjoin |
| convert time t ms. to h, m, s, ms format | strtime(t) |
| Conjugates and Lifts | |
| conjugate of a number x | conj(x) |
| norm of x , product with conjugate | norm(x) |
| L^p norm of x (L^∞ if no p) | normlp($x,\{p\}$) |
| square of L^2 norm of x | norml2(x) |
| lift of x from Mods and p -adics | lift, centerlift(x) |
| recursive lift | liftall |
| lift all t_INT and t_PADIC (\rightarrow t_INT) | liftint |
| lift all t_POLMOD (\rightarrow t_POL) | liftpol |

Lists, Sets & Maps

| | |
|--|-----------------------------|
| Sets (= row vector with strictly increasing entries w.r.t. cmp) | |
| intersection of sets x and y | setintersect(x,y) |
| set of elements in x not belonging to y | setminus(x,y) |
| symmetric difference $x \Delta y$ | setdelta(x,y) |
| union of sets x and y | setunion(x,y) |
| does y belong to the set x | setsearch($x,y,\{flag\}$) |
| set of all $f(x,y)$, $x \in X$, $y \in Y$ | setbinop(f,X,Y) |
| is x a set ? | setisset(x) |

| | |
|---|-------------------------------|
| Lists . create empty list: $L = \text{List}()$ | |
| append x to list L | listput($L,x,\{i\}$) |
| remove i -th component from list L | listpop($L,\{i\}$) |
| insert x in list L at position i | listinsert(L,x,i) |
| sort the list L in place | listsort($L,\{flag\}$) |
| Maps . create empty dictionary: $M = \text{Map}()$ | |
| attach value v to key k | mapput(M,k,v) |
| recover value attach to key k or error | mapget(M,k) |
| is key k in the dict? (set v to $M(k)$) | mapisdefined($M,k,\{\&v\}$) |
| remove k from map domain | mapdelete(M,k) |

GP Programming

User functions and closures

x,y are formal parameters; y defaults to Pi if parameter omitted; z,t are local variables (lexical scope), z initialized to 1.

fun(x, y=Pi) = my(z=1, t); seq

fun = (x, y=Pi) -> my(z=1, t); seq

| | |
|--|---------------------------------|
| attach help message h to s | addhelp(s,h) |
| undefine symbol s (also kills help) | kill(s) |
| Control Statements (X : formal parameter in expression seq) | |
| if $a \neq 0$, evaluate seq_1 , else seq_2 | if($a,\{seq_1\},\{seq_2\}$) |
| eval. seq for $a \leq X \leq b$ | for($X = a,b,seq$) |
| ...for $X \in v$ | foreach(v,X,seq) |
| ...for primes $a \leq X \leq b$ | forprime($X = a,b,seq$) |
| ...for primes $\equiv a \pmod q$ | forprimestep($X = a,b,q,seq$) |
| ...for composites $a \leq X \leq b$ | forcomposite($X = a,b,seq$) |
| ...for $a \leq X \leq b$ stepping s | forstep($X = a,b,s,seq$) |
| ...for X dividing n | fordiv(n,X,seq) |
| ... $X = [n, factor(n)]$, $a \leq n \leq b$ | forfactored($X = a,b,seq$) |
| ...as above, n squarefree | forsquarefree($X = a,b,seq$) |
| ... $X = [d, factor(d)]$, $d n$ | fordivfactored(n,X,seq) |
| multivariable for, lex ordering | forvec($X = v,seq$) |

loop over partitions of n
... permutations of S
... subsets of $\{1, \dots, n\}$
... k -subsets of $\{1, \dots, n\}$
... vectors v , $q(v) \leq B$; $q > 0$
... $H < G$ finite abelian group
evaluate seq until $a \neq 0$
while $a \neq 0$, evaluate seq
exit n innermost enclosing loops
start new iteration of n -th enclosing loop
return x from current subroutine

Exceptions, warnings
raise an exception / warning
type of error message E
try seq_1 , evaluate seq_2 on error

Functions with closure arguments / results
number of arguments of f
select from v according to f
apply f to all entries in v
evaluate $f(a_1, \dots, a_n)$
evaluate $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$
calling function as closure

Sums & Products
sum $X = a$ to $X = b$, initialized at x
sum entries of vector v
product of all vector entries
sum $expr$ over divisors of n
... assuming $expr$ multiplicative
product $a \leq X \leq b$, initialized at x
product over primes $a \leq X \leq b$

Sorting
sort x by k -th component
min. m of x ($m = x[i]$), max.
does y belong to x , sorted wrt. f
 $\prod g^x \rightarrow$ factorization (\Rightarrow sorted, unique g)

Input/Output
print with/without $\backslash n$, TeX format
pretty print matrix
print fields with separator
formatted printing
write $args$ to file
write x in binary format
read file into GP
... return as vector of lines
... return as vector of strings
read a string from keyboard

Files and file descriptors
File descriptors allow efficient small consecutive reads or writes from or to a given file. The argument n below is always a descriptor, attached to a file in **r**(ead), **w**(rite) or **a**(ppend) mode.
get descriptor n for file $path$ in given $mode$
... from shell cmd output (pipe)

close descriptor
commit pending write operations
read logical line from file
... raw line from file
write $s \backslash n$ to file
... write s to file

forpart($p = n, seq$)
forperm(S, p, seq)
forsubset(n, p, seq)
forsubset($[n, k], p, seq$)
forqfvec(v, q, b, seq)
forsubgroup($H = G$)
until(a, seq)
while(a, seq)
break($\{n\}$)
next($\{n\}$)
return($\{x\}$)

error(), warning()
errname(E)
iferr(seq_1, E, seq_2)

arity(f)
select(f, v)
apply(f, v)
call(f, a)
fold(f, a)
self()

sum($X = a, b, expr, \{x\}$)
vecsum(v)
vecprod(v)
sumdiv($n, X, expr$)
sumdivmult($n, X, expr$)
prod($X = a, b, expr, \{x\}$)
prodeuler($X = a, b, expr$)

vecsrt($x, \{k\}, \{fl = 0\}$)
vecmin($x, \{\&i\}$), vecmax
vecsearch($x, y, \{f\}$)
matreduce(m)

print, print1, printtex
printp
printsep(sep, \dots), printsep1
printf()
write, write1, writetex($file, args$)
writebin($file, x$)
read($\{file\}$)
readvec($\{file\}$)
readstr($\{file\}$)
input()

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Timers
CPU time in ms and reset timer
CPU time in ms since gp startup
time in ms since UNIX Epoch
timeout command after s seconds

Interface with system
allocates a new stack of s bytes
alias old to new
install function from library
execute system command a
... and feed result to GP
... returning GP string
get \$VAR from environment
expand env. variable in string

gettime()
getabstime()
getwalltime()
alarm($s, expr$)

allocatemem($\{s\}$)
alias(new, old)
install($f, code, \{gpf\}, \{lib\}$)
system(a)
extern(a)
externstr(a)
getenv("VAR")
strexpand(x)

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables (use **export** for this) and must be free of side effects. Enabled if threading engine is not *single* in gp header.

evaluate f on $x[1], \dots, x[n]$
evaluate closures $f[1], \dots, f[n]$
as **select**
as **sum**
as **vector**
eval f for $i = a, \dots, b$
... for each element x in v
... for p prime in $[a, b]$
... for $p = a \bmod q$
... multivariate
export x to parallel world
... all dynamic variables
frees exported value x
... all exported values

parapply(f, x)
pareval(f)
parselect($f, A, \{flag\}$)
parsum($i = a, b, expr$)
parvector($n, i, \{expr\}$)
parfor($i = a, \{b\}, f, \{r\}, \{f_2\}$)
parforeach($v, x, f, \{r\}, \{f_2\}$)
parforprime($p = a, \{b\}, f, \{r\}, \{f_2\}$)
parforprimestep($p = a, \{b\}, q, f, \{r\}, \{f_2\}$)
parforvec($X = v, f, \{r\}, \{f_2\}, \{flag\}$)

export(x)
exportall()
unexport(x)
unexportall()

Linear Algebra

dimensions of matrix x
multiply two matrices
... assuming result is diagonal
concatenation of x and y
extract components of x
transpose of vector or matrix x
adjoint of the matrix x
eigenvectors/values of matrix x
characteristic/minimal polynomial of x
trace/determinant of matrix x
permanent of matrix x
Frobenius form of x
QR decomposition
apply **matqr**'s transform to v

Constructors & Special Matrices
 $\{g(x): x \in v \text{ s.t. } f(x)\}$
 $\{x: x \in v \text{ s.t. } f(x)\}$
 $\{g(x): x \in v\}$
row vec. of $expr$ eval'ed at $1 \leq i \leq n$
col. vec. of $expr$ eval'ed at $1 \leq i \leq n$
vector of small ints

matsize(x)
 $x * y$
matmultodiagonal(x, y)
concat($x, \{y\}$)
vecextract($x, y, \{z\}$)
 $x \sim$, mattranspose(x)
matadjoint(x)
mateigen(x)
charpoly(x), minpoly(x)
trace(x), matdet(x)
matpermanent(x)
matfrobenius(x)
matqr(x)
mathouseholder(Q, v)

[g(x) | x <- v, f(x)]
[x | x <- v, f(x)]
[g(x) | x <- v]
vector($n, \{i\}, \{expr\}$)
vectorv($n, \{i\}, \{expr\}$)
vectorsmall($n, \{i\}, \{expr\}$)

$[c, c \cdot x, \dots, c \cdot x^n]$
 $[1, 2^x, \dots, n^x]$
matrix $1 \leq i \leq m, 1 \leq j \leq n$
define matrix by blocks
diagonal matrix with diagonal x
is x diagonal?
 $x \cdot \text{matdiagonal}(d)$
 $n \times n$ identity matrix
Hessenberg form of square matrix x
 $n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$
 $n \times n$ Pascal triangle
companion matrix to polynomial x
Sylvester matrix of x and y

Gaussian elimination
kernel of matrix x
intersection of column spaces of x and y
solve $MX = B$ (M invertible)
one sol of $M * X = B$
basis for image of matrix x
columns of x *not* in **matimage**
supplement columns of x to get basis
rows, cols to extract invertible matrix
rank of the matrix x
solve $MX = B \bmod D$
image mod D
kernel mod D
inverse mod D
determinant mod D

powers($x, n, \{c = 1\}$)
dirpowers(n, x)
matrix($m, n, \{i\}, \{j\}, \{expr\}$)
matconcat(B)
matdiagonal(x)
matisdiagonal(x)
matmuldiagonal(x, d)
matid(n)
mathess(x)
mathilbert(n)
matpascal($n - 1$)
matcompanion(x)
polsylvestermatrix(x, y)

matker($x, \{flag\}$)
matintersect(x, y)
matsolve(M, B)
matinverseimage(M, B)
matimage(x)
matimagecompl(x)
matsupplement(x)
matindexrank(x)
matrank(x)
matsolvemod(M, D, B)
matimagemod(M, D)
matkermod(M, D)
matinvmod(M, D)
matdetmod(M, D)

Lattices & Quadratic Forms

Quadratic forms
evaluate ${}^t x Q y$
evaluate ${}^t x Q x$
signature of quad form ${}^t y * x * y$
decomp into squares of ${}^t y * x * y$
eigenvalues/vectors for real symmetric x

HNF and SNF
upper triangular Hermite Normal Form
HNF of x where d is a multiple of $\det(x)$
multiple of $\det(x)$
HNF of $(x \mid \text{diagonal}(D))$
elementary divisors of x
 q -rank from elementary divisors
elementary divisors of $\mathbf{Z}[a]/(f'(a))$
integer kernel of x
 \mathbf{Z} -module \leftrightarrow \mathbf{Q} -vector space

Lattices
LLL-algorithm applied to columns of x
... for Gram matrix of lattice
find up to m sols of **qfnorm**(x, y) $\leq b$
 $v, v[i] :=$ number of y s.t. **qfnorm**(x, y) = i
perfection rank of x
find isomorphism between q and Q
precompute for isomorphism test with q
automorphism group of q

qfeval($\{Q = id\}, x, y$)
qfeval($\{Q = id\}, x$)
qfsign(x)
qfgaussred(x)
qfjacobi(x)

mathnf(x)
mathnfmod(x, d)
matdetint(x)
mathnfmodid(x, D)
matsnf(x)
snfrank(v, q)
poldiscreduced(f)
matkerint(x)
matrixqz(x, p)

qflll($x, \{flag\}$)
qflllgram($x, \{flag\}$)
qfminim(x, b, m)
qfrep($x, B, \{flag\}$)
qfperfection(x)
qfisom(q, Q)
qfisominit(q)
qfauto(q)

convert `qfauto` for GAP/Magma `qfautoexport($G, \{flag\}$)`
orbits of V under $G \subset \mathrm{GL}(V)$ `qforbits(G, V)`

Polynomials & Rational Functions

all defined polynomial variables `variables()`
get var. of highest priority (higher than v) `varhigher($name, \{v\}$)`
... of lowest priority (lower than v) `varlower($name, \{v\}$)`

Coefficients, variables and basic operators

degree of f `poldegree(f)`
coef. of degree n of f , leading coef. `polcoef(f, n), pollead`
main variable / all variables in f `variable(f), variables(f)`
replace x by y in f `subst(f, x, y)`
evaluate f replacing vars by their value `eval(f)`
replace polynomial expr. $T(x)$ by y in f `substpol(f, T, y)`
replace x_1, \dots, x_n by y_1, \dots, y_n in f `substvec(f, x, y)`

$f \in A[x]$; reciprocal polynomial $x^{\deg f} f\left(\frac{1}{x}\right)$ `polrecip(f)`
gcd of coefficients of f `content(f)`
derivative of f w.r.t. x `deriv($f, \{x\}$)`
... n -th derivative of f `derivn($f, n, \{x\}$)`
formal integral of f w.r.t. x `intformal($f, \{x\}$)`
formal sum of f w.r.t. x `sumformal($f, \{x\}$)`

Constructors & Special Polynomials

interpolation polynomial at $(x[1], y[1]), \dots, (x[n], y[n])$, evaluated at t , with error estimate e `polinterpolate($x, \{y\}, \{t\}, \{&e\}$)`
 $T_n/U_n, H_n$ `polchebyshev(n), polhermite(n)`
 $P_n, L_n^{(\alpha)}$ `pollegendre(n), pollaguerre(n, a)`
 n -th cyclotomic polynomial Φ_n `polcyclo(n)`
return n if $f = \Phi_n$, else 0 `poliscyclo(f)`
is f a product of cyclotomic polynomials? `poliscycloprod(f)`
Zagier's polynomial of index (n, m) `polzagier(n, m)`

Resultant, elimination

discriminant of polynomial f `poldisc(f)`
find factors of `poldisc(f)` `poldiscfactors(f)`
resultant $R = \mathrm{Res}_v(f, g)$ `polresultant($f, g, \{v\}$)`
 $[u, v, R], xu + yv = \mathrm{Res}_v(f, g)$ `polresultantext($x, y, \{v\}$)`
solve Thue equation $f(x, y) = a$ `thue($t, a, \{sol\}$)`
initialize t for Thue equation solver `thueinit(f)`

Roots and Factorization (Complex/Real)

complex roots of f `polroots(f)`
bound complex roots of f `polrootsbound(f)`
number of real roots of f (in $[a, b]$) `polsturm($f, \{[a, b]\}$)`
real roots of f (in $[a, b]$) `polrootsreal($f, \{[a, b]\}$)`
complex embeddings of `t_POLMOD` z `conjvec(z)`

Roots and Factorization (Finite fields)

factor f mod p , roots `factormod(f, p), polrootsmod`
factor f over $\mathbf{F}_p[x]/(T)$, roots `factormod($f, [T, p]$), polrootsmod`
squarefree factorization of f in $\mathbf{F}_q[x]$ `factormodSQF($f, \{D\}$)`
distinct degree factorization of f in $\mathbf{F}_q[x]$ `factormodDDF($f, \{D\}$)`
factor n -th cyclotomic pol. Φ_n mod p `factormodcyclo(n, p)`

Roots and Factorization (p -adic fields)

factor f over \mathbf{Q}_p , roots `factorpadic(f, p, r), polrootspadic`
 p -adic root of f congruent to a mod p `padicappr(f, a)`
Newton polygon of f for prime p `newtonpoly(f, p)`
Hensel lift $A/\mathrm{lc}(A) = \prod_i B[i]$ mod p^e `polhensellift(A, B, p, e)`
 $T = \prod (x - z_i) \mapsto \prod [x - \omega(z_i)] \in \mathbf{Z}_p[x]$ `polteichmuller(T, p, e)`
extensions of \mathbf{Q}_p of degree N `padicfields(p, N)`

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Roots and Factorization (Miscellaneous)

symmetric powers of roots of f up to n `polsym(f, n)`
Graeffe transform of $f, g(x^2) = f(x)f(-x)$ `polgraeffe(f)`
factor f over coefficient field `factor(f)`
cyclotomic factors of $f \in \mathbf{Q}[X]$ `polcyclofactors(f)`

Finite Fields

A finite field is encoded by any element (`t_FFELT`).
find irreducible $T \in \mathbf{F}_p[x]$, $\deg T = n$ `ffinit($p, n, \{x\}$)`
Create t in $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$ `t = ffgen($T, 't$)`
... indirectly, with implicit T `t = ffgen($q, 't$); T = t.mod`
map m from $\mathbf{F}_q \ni a$ to $\mathbf{F}_{q^k} \ni b$ `m = ffembed(a, b)`
build $K = \mathbf{F}_q[x]/(P)$ extending $\mathbf{F}_q \ni a$, `ffextend(a, P)`
evaluate map m on x `ffmap(m, x)`
inverse map of m `ffinvmap(m)`
compose maps $m \circ n$ `ffcompomap(m, n)`
 x as polmod over codomain of map m `ffmaprel(m, x)`
 F^n over $\mathbf{F}_q \ni a$ `fffrobenius(a, n)`
 $\#$ {monic irred. $T \in \mathbf{F}_q[x]$, $\deg T = n$ } `ffnbirred(q, n)`

Formal & p -adic Series

truncate power series or p -adic number `truncate(x)`
valuation of x at p `valuation(x, p)`
Dirichlet and Power Series
Taylor expansion around 0 of f w.r.t. x `taylor(f, x)`
Laurent series of closure F up to x^k `laurentseries(f, k)`
 $\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$ `serconvol(a, b)`
 $f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$ `serlaplace(f)`
reverse power series F so $F(f(x)) = x$ `serreverse(f)`
remove terms of degree $< n$ in f `serchop(f, n)`
Dirichlet series multiplication / division `dirmul, dirdiv(x, y)`
Dirichlet Euler product (b terms) `direuler($p = a, b, expr$)`

Transcendental and p -adic Functions

real, imaginary part of x `real(x), imag(x)`
absolute value, argument of x `abs(x), arg(x)`
square/ n th root of x `sqrt(x), sqrtn($x, n, \{&z\}$)`
all n -th roots of 1 `rootsof1(n)`
FFT of $[f_0, \dots, f_{n-1}]$ `w = fftinit(n), fft/fftinw(w, f)`
trig functions `sin, cos, tan, cotan, sinc`
inverse trig functions `asin, acos, atan`
hyperbolic functions `sinh, cosh, tanh, cotanh`
inverse hyperbolic functions `asinh, acosh, atanh`
 $\log(x), \log(1+x), e^x, e^x - 1$ `log, loglp, exp, expm1`
Euler Γ function, $\log \Gamma, \Gamma'/\Gamma$ `gamma, lngamma, psi`
half-integer gamma function $\Gamma(n+1/2)$ `gammah(n)`
Riemann's zeta $\zeta(s) = \sum n^{-s}$ `zeta(s)`
 $\sum_{1 \leq n \leq N} n^s$ `dirpowerssum(N, s)`
Hurwitz's $\zeta(s, x) = \sum (n+x)^{-s}$ `zetahurwitz(s, x)`
Lerch $\Phi(z, s, x) = \sum z^n (n+x)^{-s}$ `lerchphi(z, s, x)`
Lerch $L(s, x, t) = \Phi(e^{2i\pi t}, s, x)$ `lerchzeta(s, x, t)`
multiple zeta value (MZV), $\zeta(s_1, \dots, s_k)$ `zetamult($s, \{T\}$)`
all MZVs for weight $\sum s_i = n$ `zetamultall(n)`
convert MZV id to $[s_1, \dots, s_k]$ `zetamultconvert($f, \{flag\}$)`
MZV dual sequence `zetamultdual(s)`
multiple polylog $Li_{s_1, \dots, s_k}(z_1, \dots, z_k)$ `polylogmult(s, z)`

incomplete Γ function ($y = \Gamma(s)$) `incgam($s, x, \{y\}$)`
complementary incomplete Γ `incgamc(s, x)`
 $\int_x^\infty e^{-t} dt/t, (2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$ `eint1, erfc`
elliptic integral of 1st and 2nd kind `ellK(k), ellE(k)`
dilogarithm of x `dilog(x)`
 m -th polylogarithm of x `polylog($m, x, \{flag\}$)`
 U -confluent hypergeometric function `hyperu(a, b, u)`
Hypergeometric ${}_pF_q(A, B; z)$ `hypergeom(A, B, z)`
Bessel $J_n(x), J_{n+1/2}(x)$ `besselj(n, x), besseljh(n, x)`
Bessel $I_\nu, K_\nu, H_\nu^1, H_\nu^2, Y_\nu$ `(bessel)i, k, h1, h2, y`
 k -th zero of $J_\nu(x)$ `besseljzero($nu, \{k = 1\}$)`
 k -th zero of $Y_\nu(x)$ `besselyzero($nu, \{k = 1\}$)`
Airy functions $A_i(x), B_i(x)$ `airy(x)`
Lambert $W: x$ s.t. $xe^x = y$ `lambertw(y)`
Teichmuller character of p -adic x `teichmuller(x)`

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ($a \in \mathbf{C}$, regular) or $\pm\infty$ (decreasing at least as x^{-2}) or
 $(x-a)^{-\alpha}$ singularity `[a, α]`
exponential decrease $e^{-\alpha|x|}$ `[$\pm\infty, \alpha$], $\alpha > 0$`
slow decrease $|x|^\alpha$ `... $\alpha < -1$`
oscillating as $\cos(kx)$ `$\alpha = k\mathbf{I}, k > 0$`
oscillating as $\sin(kx)$ `$\alpha = -k\mathbf{I}, k > 0$`

numerical integration `intnum($x = a, b, f, \{T\}$)`
weights T for `intnum` `intnuminit($a, b, \{m\}$)`
weights T incl. kernel K `intfuncinit($t = a, b, K, \{m\}$)`
integrate $(2i\pi)^{-1} f$ on circle $|z-a| = R$ `intcirc($x = a, R, f, \{T\}$)`
Other integration methods

n -point Gauss-Legendre `intnumgauss($x = a, b, f, \{n\}$)`
weights for n -point Gauss-Legendre `intnumgaussinit($\{n\}$)`
quasi-periodic function, period $2H$ `intnumosc($x = a, f, H$)`
Romberg (low accuracy) `intnumromb($x = a, b, f, \{flag\}$)`

Numerical summation

sum of series $f(n), n \geq a$ (low accuracy) `suminf($n = a, expr$)`
sum of alternating/positive series `sumalt, sumpos`
sum of series using Euler-Maclaurin `sumnum($n = a, f, \{T\}$)`
... Sidi summation `sumnumsidi($n = a, f$)`
 $\sum_{n \geq a} F(n)$, F rational function `sumnumrat(F, a)`
... $\sum_{p \geq a} F(p^s)$ `sumeulerrat($F, \{s = 1\}, \{a = 2\}$)`
weights for `sumnum`, a as in DE `sumnuminit($\{\infty, a\}$)`
sum of series by Monien summation `sumnummonien($n = a, f, \{T\}$)`
weights for `sumnummonien` `sumnummonieninit($\{\infty, a\}$)`
sum of series using Abel-Plana `sumnumap($n = a, f, \{T\}$)`
weights for `sumnumap`, a as in DE `sumnumapinit($\{\infty, a\}$)`
sum of series using Lagrange `sumnumlagrange($n = a, f, \{T\}$)`
weights for `sumnumlagrange` `sumnumlagrangeinit`

Products

product $a \leq X \leq b$, initialized at x `prod($X = a, b, expr, \{x\}$)`
product over primes $a \leq X \leq b$ `prodeuler($X = a, b, expr$)`
infinite product $a \leq X \leq \infty$ `prodinf($X = a, expr$)`
 $\prod_{n \geq a} F(n)$, F rational function `prodnumrat(F, a)`
 $\prod_{p \geq a} F(p^s)$ `prodeulerrat($F, \{s = 1\}, \{a = 2\}$)`

Other numerical methods

| | |
|---|---|
| real root of f in $[a, b]$; bracketed root | <code>solve($X = a, b, f$)</code> |
| ...interval splitting, step s | <code>solvestep($X = a, b, s, f, \{flag = 0\}$)</code> |
| limit of $f(t)$, $t \rightarrow \infty$ | <code>limitnum($f, \{\alpha\}$)</code> |
| asymptotic expansion of f (rational) | <code>asypnum($f, \{\alpha\}$)</code> |
| ... $N + 1$ terms as floats | <code>asypnumraw($f, N, \{\alpha\}$)</code> |
| numerical derivation w.r.t x : $f'(a)$ | <code>derivnum($x = a, f$)</code> |
| evaluate continued fraction F at t | <code>contfraceval($F, t, \{L\}$)</code> |
| power series to cont. fraction (L terms) | <code>contfracinit($S, \{L\}$)</code> |
| Padé approximant (deg. denom. $\leq B$) | <code>bestapprPade($S, \{B\}$)</code> |

Elementary Arithmetic Functions

| | |
|--|---|
| vector of binary digits of $ x $ | <code>binary(x)</code> |
| bit number n of integer x | <code>bittest(x, n)</code> |
| Hamming weight of integer x | <code>hammingweight(x)</code> |
| digits of integer x in base B | <code>digits($x, \{B = 10\}$)</code> |
| sum of digits of integer x in base B | <code>sumdigits($x, \{B = 10\}$)</code> |
| integer from digits | <code>fromdigits($v, \{B = 10\}$)</code> |
| ceiling/floor/fractional part | <code>ceil, floor, frac</code> |
| round x to nearest integer | <code>round($x, \{\&e\}$)</code> |
| truncate x | <code>truncate($x, \{\&e\}$)</code> |
| gcd/LCM of x and y | <code>gcd(x, y), lcm(x, y)</code> |
| gcd of entries of a vector/matrix | <code>content(x)</code> |

Primes and Factorization

| | |
|--|---|
| extra prime table | <code>addprimes()</code> |
| add primes in v to prime table | <code>addprimes(v)</code> |
| remove primes from prime table | <code>removeprimes(v)</code> |
| Chebyshev $\pi(x)$, n -th prime p_n | <code>primepi(x), prime(n)</code> |
| vector of first n primes | <code>primes(n)</code> |
| smallest prime $\geq x$ | <code>nextprime(x)</code> |
| largest prime $\leq x$ | <code>precprime(x)</code> |
| factorization of x | <code>factor($x, \{lim\}$)</code> |
| ...selecting specific algorithms | <code>factorint($x, \{flag = 0\}$)</code> |
| $n = df^2$, d squarefree/fundamental | <code>core($n, \{fl\}$), coredisc</code> |
| certificate for (prime) N | <code>primecert(N)</code> |
| verifies a certificate c | <code>primecertisvalid(c)</code> |
| convert certificate to Magma/PRIMO | <code>primecertexport</code> |
| recover x from its factorization | <code>factorback($f, \{e\}$)</code> |
| $x \in \mathbf{Z}$, $ x \leq X$, $\gcd(N, P(x)) \geq N$ | <code>zncoppersmith($P, N, X, \{B\}$)</code> |
| divisors of N in residue class r mod s | <code>divisorslensstra(N, r, s)</code> |

Divisors and multiplicative functions

| | |
|--|---|
| number of prime divisors $\omega(n)$ / $\Omega(n)$ | <code>omega(n), bigomega</code> |
| divisors of n / number of divisors $\tau(n)$ | <code>divisors(n), numdiv</code> |
| sum of (k -th powers of) divisors of n | <code>sigma($n, \{k\}$)</code> |
| Möbius μ -function | <code>moebius(x)</code> |
| Ramanujan's τ -function | <code>ramanujantau(x)</code> |

Combinatorics

| | |
|---|---|
| factorial of x | <code>x!</code> or <code>factorial(x)</code> |
| binomial coefficient $\binom{x}{k}$ | <code>binomial($x, \{k\}$)</code> |
| Bernoulli number B_n as real/rational | <code>bernreal(n), bernfrac</code> |
| $[B_0, B_2, \dots B_{2k}]$ | <code>bernvec(k)</code> |
| Bernoulli polynomial $B_n(x)$ | <code>bernpol($n, \{x\}$)</code> |
| Euler numbers | <code>eulerfrac, eulerreal, eulervec</code> |
| Euler polynomial $E_n(x)$ | <code>eulerpol($n, \{x\}$)</code> |
| Eulerian polynomial $A_n(x)$ | <code>eulerianpol</code> |
| Fibonacci number F_n | <code>fibonacci(n)</code> |
| Harmonic number $H_{n,r} = 1^{-r} + \dots + n^{-r}$ | <code>harmonic(n, r)</code> |
| Stirling numbers $s(n, k)$ and $S(n, k)$ | <code>stirling($n, k, \{flag\}$)</code> |

Pari-GP reference card

(PARI-GP version 2.15.3)

| | |
|---|---|
| number of partitions of n | <code>numbpart(n)</code> |
| k -th permutation on n letters | <code>numtoperm(n, k)</code> |
| ...index k of permutation v | <code>permtotnum(v)</code> |
| order of permutation p | <code>permorder(p)</code> |
| signature of permutation p | <code>permsign(p)</code> |
| cyclic decomposition of permutation p | <code>permcycles(p)</code> |

Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, \mathbf{F}_q^*

| | |
|---|--|
| Euler ϕ -function | <code>eulerphi(x)</code> |
| multiplicative order of x (divides ϕ) | <code>znorder($x, \{o\}$), fforder</code> |
| primitive root mod q / x .mod | <code>znprimroot(q), fprimroot(x)</code> |
| structure of $(\mathbf{Z}/n\mathbf{Z})^*$ | <code>znstar(n)</code> |
| discrete logarithm of x in base g | <code>znlog($x, g, \{o\}$), ffflog</code> |
| Kronecker-Legendre symbol $(\frac{x}{y})$ | <code>kronecker(x, y)</code> |
| quadratic Hilbert symbol (at p) | <code>hilbert($x, y, \{p\}$)</code> |

Euclidean algorithm, continued fractions

| | |
|--|---|
| CRT: solve $z \equiv x$ and $z \equiv y$ | <code>chinese(x, y)</code> |
| minimal u, v so $xu + yv = \gcd(x, y)$ | <code>gcdext(x, y)</code> |
| half-gcd algorithm | <code>halfgcd(x, y)</code> |
| continued fraction of x | <code>confrac($x, \{b\}, \{lmax\}$)</code> |
| last convergent of continued fraction x | <code>confracpnqn(x)</code> |
| rational approximation to x (den. $\leq B$) | <code>bestappr($x, \{B\}$)</code> |
| recognize $x \in \mathbf{C}$ as polmod mod $T \in \mathbf{Z}[X]$ | <code>bestapprnf(x, T)</code> |

Miscellaneous

| | |
|---|---|
| integer square / n -th root of x | <code>sqrtnint(x, n)</code> |
| largest integer e s.t. $b^e \leq x$, $e = \lfloor \log_b(x) \rfloor$ | <code>logint($x, b, \{\&z\}$)</code> |

Characters

Let $\chi = [d_1, \dots, d_k]$ represent an abelian group $G = \oplus (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$ or any structure G affording a `.cyc` method; e.g. `znstar($q, 1$)` for Dirichlet characters. A character χ is coded by $[c_1, \dots, c_k]$ such that $\chi(g_j) = e(n_j/d_j)$.
 $\chi \cdot \psi$; χ^{-1} ; $\chi \cdot \psi^{-1}$; χ^k `charmul, charconj, chardiv, charpow`
order of χ `charorder(cyc, χ)`
kernel of χ `charker(cyc, χ)`
 $\chi(x)$, G a GP group structure `chareval($G, \chi, x, \{z\}$)`
Galois orbits of characters `chargalois(G)`

Dirichlet Characters

| | |
|---|---|
| initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$ | <code>G = znstar($q, 1$)</code> |
| convert datum D to $[G, \chi]$ | <code>znchar(D)</code> |
| is χ odd? | <code>zncharisodd(G, χ)</code> |
| real $\chi \rightarrow$ Kronecker symbol (D/\cdot) | <code>znchartokronecker(G, χ)</code> |
| conductor of χ | <code>zncharconductor(G, χ)</code> |
| $[G_0, \chi_0]$ primitive attached to χ | <code>znchartoprimitive(G, χ)</code> |
| induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$ | <code>zncharinduce(G, χ, N)</code> |
| χp | <code>znchardecompose(G, χ, p)</code> |
| $\prod_p (\chi, N) \chi p$ | <code>znchardecompose(G, χ, Q)</code> |
| complex Gauss sum $G_a(\chi)$ | <code>znchargauss(G, χ)</code> |

Conrey labelling

| | |
|---|--|
| Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \rightarrow$ character | <code>znconreychar(G, m)</code> |
| character \rightarrow Conrey label | <code>znconreyexp(G, χ)</code> |
| log on Conrey generators | <code>znconreylog(G, m)</code> |
| conductor of χ (χ_0 primitive) | <code>znconreyconductor($G, \chi, \{\chi_0\}$)</code> |

True-False Tests

| | |
|--|--|
| is x the disc. of a quadratic field? | <code>isfundamental(x)</code> |
| is x a prime? | <code>isprime(x)</code> |
| is x a strong pseudo-prime? | <code>ispseudoprime(x)</code> |
| is x square-free? | <code>issquarefree(x)</code> |
| is x a square? | <code>issquare($x, \{\&n\}$)</code> |
| is x a perfect power? | <code>ispower($x, \{k\}, \{\&n\}$)</code> |
| is x a perfect power of a prime? ($x = p^n$) | <code>isprimepower($x, \&n$)</code> |
| ... of a pseudoprime? | <code>ispseudoprimepower($x, \&n$)</code> |
| is x powerful? | <code>ispowerful(x)</code> |
| is x a totient? ($x = \varphi(n)$) | <code>istotient($x, \{\&n\}$)</code> |
| is x a polygonal number? ($x = P(s, n)$) | <code>ispolygonal($x, s, \{\&n\}$)</code> |
| is pol irreducible? | <code>polisirreducible(pol)</code> |

Graphic Functions

| | |
|---|---|
| crude graph of $expr$ between a and b | <code>plot($X = a, b, expr$)</code> |
| High-resolution plot (immediate plot) | |
| plot $expr$ between a and b | <code>plotoh($X = a, b, expr, \{flag\}, \{n\}$)</code> |
| plot points given by lists lx, ly | <code>plotthraw($lx, ly, \{flag\}$)</code> |
| terminal dimensions | <code>plotsizes()</code> |

Rectwindow functions

| | |
|--|---|
| init window w , with size x, y | <code>plotinit(w, x, y)</code> |
| erase window w | <code>plotkill(w)</code> |
| copy w to w_2 with offset (dx, dy) | <code>plotcopy(w, w_2, dx, dy)</code> |
| slice contents of w | <code>plotclip(w)</code> |
| scale coordinates in w | <code>plotscale(w, x_1, x_2, y_1, y_2)</code> |
| plotoh in w | <code>plotrecth($w, X = a, b, expr, \{flag\}, \{n\}$)</code> |
| plotthraw in w | <code>plotrectthraw($w, data, \{flag\}$)</code> |
| draw window w_1 at $(x_1, y_1), \dots$ | <code>plotdraw($[[w_1, x_1, y_1], \dots]$)</code> |

Low-level Rectwindow Functions

| | |
|---|--|
| set current drawing color in w to c | <code>plotcolor(w, c)</code> |
| current position of cursor in w | <code>plotcursor(w)</code> |
| write s at cursor's position | <code>plotstring(w, s)</code> |
| move cursor to (x, y) | <code>plotmove(w, x, y)</code> |
| move cursor to $(x + dx, y + dy)$ | <code>plotrmove(w, dx, dy)</code> |
| draw a box to (x_2, y_2) | <code>plotbox(w, x_2, y_2)</code> |
| draw a box to $(x + dx, y + dy)$ | <code>plotrbox(w, dx, dy)</code> |
| draw polygon | <code>plotlines($w, lx, ly, \{flag\}$)</code> |
| draw points | <code>plotpoints(w, lx, ly)</code> |
| draw line to $(x + dx, y + dy)$ | <code>plotrline(w, dx, dy)</code> |
| draw point $(x + dx, y + dy)$ | <code>plotrpoint(w, dx, dy)</code> |

Convert to Postscript or Scalable Vector Graphics

| | |
|---|--|
| The format f is either "ps" or "svg". | |
| as plotoh | <code>plotexport($f, X = a, b, expr, \{flag\}, \{n\}$)</code> |
| as plotthraw | <code>plotthrawexport($f, lx, ly, \{flag\}$)</code> |
| as plotdraw | <code>plotexport($f, [[w_1, x_1, y_1], \dots]$)</code> |

Based on an earlier version by Joseph H. Silverman
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